Improving Quantum Hardware: Building New Superconducting Qubits and Couplers

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Abstract

Over the past 10 years, improvements to the fundamental components in superconducting qubits and the realization of novel circuit topologies have increased the lifetimes of qubits and catapulted this architecture to become one of the leading hardware platforms for universal quantum computation. Despite the progress that has been made in increasing the lifetime of the charge qubit by almost six orders of magnitude, further improvements must be made to climb over the threshold for fault tolerant quantum computation. Two complementary approaches towards achieving this goal are investigating and improving upon existing qubit designs, and looking for new types of superconducting qubits which would offer some intrinsic improvements over existing designs. This thesis will explore both of these directions through a detailed study of new materials, circuit designs, and coupling schemes for superconducting qubits. In the first experiment, we explore the use of disordered superconducting films, specifically Niobium Titanium Nitride, as the inductive element in a fluxonium qubit and measure the loss mechanisms limiting the qubit lifetime. In the second experiment, we work towards the experimental realization of the $0 - \pi$ qubit architecture, which offers the promise of intrinsic protection in lifetime and decoherence compared to existing superconducting qubits. In the final experiment, we design and measure a two qubit device where the static $\sigma_z \otimes \sigma_z$ crosstalk between the two qubits is eliminated via destructive interference. The use of multiple coupling elements removes the $\sigma_z \otimes \sigma_z$ crosstalk while maintaining the large $\sigma_z \otimes \sigma_x$ interaction needed to perform two qubit gates.
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Chapter 1

Introduction

As originally conceived [1], quantum computers would be most useful for problems physicists are interested in. It can be argued that the study of very small scale quantum “processors” in the past 30 years has led to a deeper understanding of these systems themselves rather than performing complex calculations. One generality of quantum systems is that they are extremely fragile, in particular, two level quantum bits, or qubits, face an underlying tension between isolation and control. Qubits can be made nearly isolated from the universe and can have very long lifetimes, but this makes it impossible to control them in a deterministic way in order to perform useful computations. Modern “Quantum Mechanics and Engineers” continue to improve how qubits are constructed, using better materials and removing extraneous couplings to the environment. A large portion of these efforts involve improving qubit energy relaxation time, $T_1$. Relaxation times in state of the art superconducting qubits pose a significant challenge for near term quantum computation. Assuming gate operations and state preparation can be made perfect, a rough approximation for the maximum number of qubit gates can be expressed as $1/T_1$. With current $T_1$ values of $\sim 100$ $\mu$s in superconducting qubits and two qubit gate times of $\sim 100$ ns, without error correction, this puts an upper limit of about $10^3$ gates before all useful
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information about the system will be lost. Therefore, understanding and mitigating the sources of relaxation in qubits is an active topic of research \([2, 3]\). Even if no further advances to qubit lifetimes or gate speeds are made, the implementation of error correcting surface codes \([4]\) could allow for the operation of a fault tolerant quantum computer. Although these codes allow for errors at the present level, there is a cost to be paid in that the number of physical qubits to construct a single logical qubit is on the order of \(10^4\) and that a vast majority of the gate operations and computation time would be spent correcting faulty qubits. With that in mind, we take the approach of improving existing qubits and coupling between qubits as a means towards the realization of a fault-tolerant quantum computer.

1.1 Quantum computing with qubits

A qubit is the quantum analog of a classical bit, that is, a two state quantum system that can be in an arbitrary superposition of the two states. A quantum computer \([5]\), will likely contain a physical qubit layer consisting of qubits, controls and readout circuitry, as well as a logical layer containing a “quantum compiler” and controller to run algorithms \([6]\). The DiVincenzo criteria lay out a set of general guidelines for the construction of a quantum computer, namely, the underlying system should be comprised of physically “scalable” qubits, all the qubits need to be initialized in a known state, the decoherence and relaxation times in the system must be “long”, there must be a universal set of quantum gates, and the qubits must be able to be measured with high fidelity \([7]\). It is interesting to note that two of the criteria, the system scalability and coherence times, are particularly vague and continue to be redefined as new hardware and error correcting schemes are realized. Based on these five criteria, the last two decades have seen an incredible amount of advancement
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in different physical implementations of qubits and small-scale quantum computers, including superconducting qubits [8, 9], quantum dots [10, 11], bulk NMR systems [12, 13], photonic systems [14, 15], ultracold atoms [16, 17], and trapped ions [18, 19].

Among these physical systems, superconducting qubits have emerged as one of the most promising candidates in constructing a gate-based quantum computer. The building blocks of superconducting circuits include inductors, capacitors, and Josephson junctions (JJs). With this small set of circuit elements, the field of circuit Quantum Electrodynamics (cQED) has expanded and explored a wide variety of qubits with different properties. In particular, the transmon [20], essentially a slightly anharmonic oscillator, has emerged as the most ubiquitous superconducting qubit, due to its simple level structure, long coherence, and ease of manufacture [21]. Despite these properties, the transmon might not be the qubit in a final quantum computer if its coherence and lifetime cannot continue to be improved. Making improvements to transmons and looking for “life after the transmon” are two topics explored in this work.

1.2 Thesis overview

The work in this thesis is presented as follows. Chapter 2 presents a brief overview of superconducting qubits. Chapter 3 describes the experimental methods and setup used for measuring superconducting devices. Chapter 4 details our work in measuring a fluxonium qubit with an inductive element formed by a thin nanowire of NbTiN. Chapter 5 explores experiments on a protected $0 - \pi$ qubit. In Chapter 6, we describe a two qubit, two coupler scheme to eliminate static crosstalk between qubits. A brief summary and outlook of the thesis is presented in Chapter 7, and several additional detailed discussions can be found in the Appendices.
Chapter 2

Theoretical background

In this section, we consider some of the building blocks of superconducting qubits. We describe the method of circuit quantization which allows us to understand the eigenmodes of general quantum circuits. We discuss several types of superconducting qubits as well as their distinguishing characteristics. The framework for qubit coupling and relaxation is introduced. We conclude with a description of kinetic inductance.

2.1 Josephson Junctions

If two superconductors are placed into contact with a thin insulating layer between, a Josephson Junction (JJ) is formed. This type of junction is known as a superconductor-insulator-superconductor (S-I-S) junction in which the insulating layer between the two superconductors needs to be thin enough to allow Cooper pairs to tunnel between the superconductors. In what is known as the first Josephson relation [22], a zero-voltage current, $I$, will flow across the junction, and is given by

\[ I = I_c \sin \varphi, \quad (2.1) \]
CHAPTER 2. THEORETICAL BACKGROUND

where \( \varphi \) is the difference in phase of the Ginzburg-Landau wavefunction between the two superconductors and \( I_c \) is the critical current of the junction \([22, 23]\). The second Josephson relation states that if a voltage difference \( V \) is maintained across the junction, the time evolution of the phase difference across the junction is given by:

\[
\frac{d(\Delta \varphi)}{dt} = \frac{2eV}{\hbar}.
\]  

(2.2)

The energy stored in a JJ can be obtained by integrating the electrical work done by a current source in changing the phase across the junction:

\[
\int I_s V dt = \int I_s (\hbar/2e) d(\Delta \varphi) \to \text{const.} - E_J \cos(\Delta \varphi),
\]  

(2.3)

where \( E_J \equiv \hbar I_c/2e \). \( I_c \) can be related to the normal state resistance of the JJ through the Ambegaokar-Baratoff relation \([24]\)

\[
I_c R_n = \frac{\pi\Delta}{2e} \tanh\left( \frac{\Delta}{2k_B T} \right),
\]  

(2.4)

where \( \Delta \) is the energy gap of the superconductor at \( T = 0 \) K. Each JJ will have an additional charging energy, which can be thought to arise from the capacitance between the two superconducting leads, and is given by \( E_{CJ} = e^2/2C_J \) where \( e \) is the electron charge and \( C_J \) is the capacitance between the leads. The JJ has two properties that make it desirable for constructing qubits, first, its intrinsic non-linearity from the sinusoidal current relation, and second, the junction itself is free of dissipation. These features will allow us to construct circuits with a wide range of properties from simple arrangements of JJs, linear inductors and capacitors.
2.2 Circuit Quantization

One of the most powerful theoretical tools at our disposal is circuit quantization \([25–27]\). This allows us to take macroscopic objects with many degrees of freedom and reduce the collective behavior into a small number of parameters that characterize the circuit. Our goal is to obtain the circuit Hamiltonian and its corresponding eigenenergies based on a small number of input parameters (capacitance in the circuit, Josephson energy, etc.). To that end, we begin by defining a magnetic flux variable, \(\Phi\),

\[
\Phi (t) = \int_{-\infty}^{t} V (\tau) d\tau
\]

as the time integral of the voltage, \(V\) across a circuit element, as well as the charge \(Q\),

\[
Q (t) = \int_{-\infty}^{t} I (\tau) d\tau
\]

as the time integral of the current flowing through a circuit element. Let’s now consider a simple LC oscillator consisting of an inductor, with inductance \(L\), placed in parallel with a capacitor, with capacitance \(C\) (Figure 2.1). We can define the two voltage nodes on either side of the inductor, and make use of gauge invariance to set the potential at one of the nodes to ground. This leaves the circuit with one degree of freedom (dof). To generalize, a circuit with \(n\) voltage nodes will be left with \(n - 1\) dof’s. We make use of Kirchoff’s law that the sum of currents into and out of a node must be equal,

\[
I_L + I_C = 0 = \frac{\Phi}{L} + \dot{Q}
\]

\[
0 = \frac{\Phi}{L} + C\ddot{\Phi},
\]

\(6\)
Figure 2.1: The quantum LC oscillator consisting of a parallel combination of an inductor and capacitor. The two voltage nodes of the circuit are indicated with dots. The oscillator has an eigenspectrum with equally spaced levels, $\hbar \omega$ apart.

also utilizing the expressions for current through an inductor, $I_L \equiv \Phi/L$, and the time derivative of charge as current, $I \equiv \dot{Q}$. We can write the Euler-Lagrange equations of motion for the circuit to find

$$0 = \frac{d}{dt} \left( \frac{dL}{d\dot{\Phi}_i} \right) - \frac{dL}{d\Phi_i}$$

$$\Rightarrow L = \frac{C}{2} \dot{\Phi}_i^2 - \frac{1}{2L} \Phi_i^2,$$

where the subscript $i$ denotes a particular node in the circuit. Using the Legendre transform,

$$H = \sum_{i=1}^{N} Q_i \dot{\Phi}_i - \mathcal{L},$$

where the charge variable, $Q_i$, is the conjugate to the flux and $Q_i = d\mathcal{L}/d\dot{\Phi}_i$. The Hamiltonian can now be written as,

$$H = \frac{1}{2C} Q^2 + \frac{1}{2L} \dot{\Phi}^2.$$
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One additional definition that can be made for convenience is the impedance of the oscillator, \( Z_0 \equiv \sqrt{L/C} \). We can now quantize the circuit by introducing the creation and annihilation operators, defined as

\[
\hat{a} = \sum_{n=0}^{\infty} \sqrt{n+1} |n\rangle \langle n+1|
\]

(2.11)

\[
\hat{a}^\dagger = \sum_{n=0}^{\infty} \sqrt{n} |n+1\rangle \langle n|
\]

(2.12)

where the states \( |n\rangle \) are the harmonic oscillator eigenstates and the commutation relation between \( \hat{a} \) and \( \hat{a}^\dagger \) is given by \( [\hat{a}, \hat{a}^\dagger] = 1 \), and write the phase and charge variables as operators to find

\[
Q = \sqrt{\frac{\hbar}{2Z_0}} i (\hat{a}^\dagger - \hat{a}) \quad \Phi = \sqrt{\frac{\hbar Z_0}{2}} (\hat{a}^\dagger + \hat{a})
\]

(2.13)

which results in a Hamiltonian of the form

\[
H = \hbar \omega_r \left( \hat{a}^\dagger \hat{a} + 1/2 \right)
\]

(2.14)

where \( \omega_r = 1/\sqrt{LC} \). It will become useful to draw the analogy between the electrical and mechanical harmonic oscillators, where the capacitive energy is equivalent to the kinetic energy of the oscillator and the inductive energy is equivalent to the potential energy. A harmonic oscillator by itself makes a poor qubit due to the equal spacing between the Fock states [28] and so generally, the linear inductor is replaced with a non-linear JJ, allowing the Hilbert space to be truncated to the first two levels of the system, from which we can make a qubit.
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2.3 Types of superconducting qubits

As the number of degrees of freedom in superconducting circuits is small, they are often treated as “artificial atoms”. Unlike true atomic systems, artificial superconducting atoms have no fundamental characteristic which uniquely identifies its type. The superconducting qubit community has historically applied different names to circuits with varying levels of differences between them. Broadly speaking, most qubits to date have two voltage nodes and consist of a Josephson junction, or pair of junctions in a SQUID loop, where the two sides of the junction are either capacitively or inductively shunted. Most of the different types of qubits come from different limits of Josephson energy $E_J$, the charging energy $E_C$, and inductive energy $E_L$. Here we will provide a brief, and by no means exhaustive, list of superconducting circuits and their salient features.

2.3.1 Cooper pair box

A Cooper pair box (CPB) [29–31] consists of a single (or two) junction(s) with a small capacitance between the superconducting leads. The Hamiltonian is given by

$$H = 4E_C (\hat{n} - n_g)^2 - E_J \cos \hat{\phi},$$  \hspace{1cm} (2.15)

where $\hat{n}$ is the integer number of excess Cooper pairs across the JJ, $n_g$ is the dimensionless offset charge between the islands, the charging energy $E_C = e^2/2C$, and $E_J/E_C$ is typically of order unity. The voltage on a nearby gate electrode is used to tune $n_g$. A diagram of two styles of CPB along with the energy spectra for $E_J/E_C = 1$ and $E_J/E_C = 5$ are shown in Figure 2.2. The charge dispersion, $\epsilon_m = dE_m/dn_g$, for a CPB is large away from integer and half integer values of $n_g$, making it extremely
CHAPTER 2. THEORETICAL BACKGROUND

Figure 2.2: A CPB (top left) and a CPB formed with a SQUID loop (top right). The introduction of the SQUID loop gives a tunable $E_{J, \text{left}} = 2E_J | \cos (\pi \Phi_{\text{ext}} / \Phi_0) |$. The energy levels for a cooper pair box with $E_J = 1 \text{ GHz}$ (bottom left) and $E_J = 5 \text{ GHz}$ (bottom right) with $E_C = 1 \text{ GHz}$ in both cases. Importantly, the energy bands of the CPB flatten as $E_J / E_C$ is increased.
susceptible to charge noise, and even when operated at charge sweet spots ($n_g$ values where $dE_m/dn_g = 0$), the coherence times of the CPB typically are less that 1 µs.

2.3.2 Transmon

A transmon [20, 32] is a CPB in the limit that $E_J/E_C \gg 1$. Using the analogy of $E_C$ as inverse mass, the transmon is a “heavy” CPB. In the transmon regime, the charge dispersion of the $m^{th}$ energy level can be expressed as,

$$
\epsilon_m \simeq E_C \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \frac{E_J}{2E_C}^{m+\frac{3}{4}} e^{-\sqrt{8E_J/E_C}} \tag{2.16}
$$

which leads to an exponential suppression of charge noise sensitivity as the ratio of $E_J/E_C$ is increased. In this regime, the qubit frequency is given by,

$$
\hbar \omega_{01} \simeq \sqrt{8E_J E_C - E_C} \tag{2.17}
$$

The difference between the 0→1 and 1→2 transition frequencies, or anharmonicity, is defined as $\alpha \equiv \omega_{12} - \omega_{01} \simeq E_C$, with typical transmons having $\alpha$ values between -200 to -400 MHz, which makes the transmon a weakly anharmonic oscillator. The size of $\alpha$ is important as it sets a “speed limit” on the controlling gates, that is to say, short gate lengths will have larger high frequency Fourier components causing excitations of higher qubit levels. Figure 2.3 shows the energy levels and wavefunctions for a transmon with $E_J/E_C=50$. Although the CPB/transmon has made tremendous improvements in coherence and lifetimes in the last 20 years [33, 34], we can see that the phase matrix element between the ground and excited states will always be of order unity, given that the wavefunctions are highly overlapping, which has implications for the ultimate limit of $T_1$ as discussed in Section 2.5.
2.3.3 Flux qubit

A flux qubit [35] consists of a single junction shunted by two or more smaller junctions. Although there is no strict definition for the number of shunting junctions, flux qubits are usually fabricated with less than 2-5, and the value of \( d \), a unit-less area scale factor, is picked such that \( E_L \sim E_J \). The phase difference across the large junctions is much smaller than the single small junction, and therefore the shunting junctions can be treated as a single linear inductor, which adds an additional term to the circuit Hamiltonian, \( E_L \) which will be the inverse sum of the \( E_J/d \)'s,

\[
H = 4E_C\hat{n}^2 - E_J \cos \hat{\phi} + \frac{1}{2}E_L \left( \hat{\phi} + 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right)^2. \tag{2.18}
\]

The \( n_g \) has been removed through the dc shunt across the junction [36], which makes the qubit insensitive to charge fluctuations. In addition to the quadratic \( \varphi \) term from the inductance, we also now have to consider the flux through the closed loop formed

Figure 2.3: The spectrum (left) of the transmon, qubit vs. \( n_g \) for \( E_J = 15 \) GHz, and \( E_C = 0.3 \) GHz. The charge dispersion has been reduced to \( \epsilon_1 = 2.2 \) kHz, making the qubit nearly immune to charge noise. The square of the first few wavefunctions plotted in the cosine Josephson potential (right).
CHAPTER 2. THEORETICAL BACKGROUND

Figure 2.4: The circuit diagram of a flux qubit (left) along with the excited state spectrum near half a flux quantum (center). The first three wavefunctions near half a flux quantum are shown on the right. The qubit parameters are $E_J=100$ GHz, $E_L=60$ GHz, and $E_C = 3$ GHz.

by the inductor and the JJ, $\Phi_{\text{ext}}$, and by changing flux variables, this flux dependence can lumped into the cosine term. In contrast to capacitively shunted qubits, the inductive shunt between the ends of the single JJ breaks the compactness of the phase across the junction, that is, $\varphi \neq \varphi + 2\pi$. This allows for disjoint wavefunctions, eigenstates which are localized in different wells of the cosine potential. Flux qubits are operated with half a flux quantum through the qubit loop. The wavefunctions near half flux are plotted in Figure 2.4. Due to the large value of $E_L$, flux qubits are very sensitive to flux noise, although recent work has shown that with a large capacitive shunt, the coherence and lifetimes can approach that of a transmon [37].

2.3.4 Fluxonium

$E_L$ is typically of order 100 GHz in a flux qubit, which makes it very susceptible to flux noise. We can decrease the sensitivity to flux noise by making a large array of JJs, and by having the array JJs sufficiently large (large $E_J/E_{C,J}$) such that charge fluctuations across the small junction are divided by the number of array junctions.
Figure 2.5: A fluxonium, consisting of a single small “black sheep” junction shunted by an array of large junctions. The array junctions must be large to keep the ratio of $E_J/E_{CJ} \gg 1$ for each individual junction, to suppress Cooper pair tunneling across the array junctions.

Fluxonium [38] exists in this regime, where $E_J/E_L \gg 1$ and $E_J/E_C \sim 1$, sharing the circuit Hamiltonian of the flux qubit. Importantly, the impedance of the shunting inductance must exceed the quantum of resistance $R_Q \simeq \hbar/(2e)^2 = 6.5$ kΩ. As a result of the large asymmetry between $\varepsilon_0$ and $\mu_0$, making high impedance elements must be achieved through JJ arrays [39, 40] or high kinetic inductance, $L_k$, materials [41–43]. With an $E_L \ll E_J$, several low lying excited states can reside in each of the wells created by the cosine potential. We refer to transitions between states in a single well as “plasmon” transitions as they resemble plasma oscillations in a single JJ and transitions between states in different wells are labeled as “fluxon” transitions, which correspond to alternating directions of persistent current in the flux loop. Fluxon transition frequencies are strongly coupled to $\Phi_{\text{ext}}$ and will have a first order insensitive sweet spot at zero and half a flux quantum. When operated at the half-flux sweet spot, the qubit frequency is typically detuned from the readout resonator by $\sim 7$ GHz, which drastically reduces relaxation due to the Purcell effect [45] when compared to a transmon. Recent advances have shown that fluxonium
Figure 2.6: The energies for a fluxonium qubit (top left), with $E_J=8$, $E_C=2$, and $E_L=0.5$ GHz and the corresponding matrix elements (top right) between the ground and first excited fluxon for the operators $\hat{\phi}$ (blue) and $\sin\frac{\hat{\phi}}{2}$ (orange). The two matrix elements, $\hat{\phi}$ and $\sin\frac{\hat{\phi}}{2}$ are used to calculate relaxation from capacitive/inductive losses and quasi-particle loss [44]. The lower plots show the first few wavefunctions at zero flux (bottom left) and half a flux quantum (bottom right).
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devices are less sensitive to dielectric losses and can reproducibly achieve relaxation times of 200 $\mu$s with $T_2 = 2T_1$ [46].

Heavy Fluxonium

The charging energy plays the role of effective mass in the circuit, and by decreasing $E_C$, we can effectively make the circuit “heavier”, which can be thought of as a localization of wavefunctions in different cosine potential wells. Traditional fluxonium circuits use a ratio of $E_J/E_C$ between 1 and 5. As $E_C$ is further reduced, we enter the regime of “heavy fluxonium”. The idea of disjoint wavefunctions becomes explorable, as the effective mass of the qubit is increased and the fluxon states localize in different potential wells. This has recently been explored [47, 48], and record superconducting qubit lifetimes exceeding 8 ms have been measured. This can be understood by looking at the decrease in the phase matrix element between the fluxon states. When comparing the matrix elements in Figure 2.6 to Figure 2.7, we see that the matrix element is many orders of magnitude lower at all values of external flux. This opens up several potential and exciting ideas, chief among them the use of dielectrics necessary for multi-layer qubit fabrication [49, 50]. As the number of qubits continues to increase, it is certain that multi-layer connectivities will have to be created to avoid parasitic box modes [51]. Generally the capacitive quality factor of the dielectrics used for multilayer chips is low enough to substantially impact the $T_1$ of the qubit. With heavy fluxonium however, we now have a qubit which can be made insensitive to the quality of the dielectrics used on the chips.

There are several additional considerations that need to be addressed in order for heavy fluxonium to gain traction as a replacement to the ubiquitous transmon qubit. Practically speaking, the decrease in $E_C$ leads to a frequency crowding between the levels in different fluxon wells. Fundamentally, this is not an insurmountable problem,
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but it does require careful parameter choices for $E_J$, $E_C$, and $E_L$ to avoid frequency collisions between different fluxon and plasmon states. The suppression of the matrix elements between the fluxon states also makes direct operations between the two disjoint states challenging, but this can be overcome through the use of multi-tone Raman transitions \[47\] or other control schemes for many level systems. The largest challenge for heavy fluxonium is answering the question “At which flux should the qubit be operated?”. Traditionally, fluxonium has shown the best $T_1$ and $T_2$ values when operated at the low frequency flux sweet spot \[44, 46, 52\]. However, as the qubit is made heavier, hybridization between the fluxon states at this point is reduced, and the transition frequency will tend to 0. From eqn. \ref{eqn:coth}, we note that $\coth \left( \frac{\hbar \omega_q}{2 k_B T} \right)$ term will diverge, and the relaxation rate will drastically increase, negating the benefit of the disjoint wavefunctions. The qubit could be operated at the zero flux point where the frequency difference is finite, however, here the energy gap between the $e_{-1}$ and $e_1$ fluxon states closes as the qubit is made heavier, and the gate times become prohibitively long to prevent leakage from the computational space. All intermediate flux values are also undesirable as the fluxon transition has a linear flux dispersion and is sensitive to flux noise through the qubit loop. More theoretical work is needed to determine if it is possible to find a two node circuit with a set of disjoint and flux insensitive transitions exists. If these criteria can be simultaneously satisfied, then we would have a suitable replacement for the transmon.

2.3.5 The $0 - \pi$ qubit

All the qubits considered so far consist of two nodes, and a single degree of freedom. The simple structure of these circuits allows for analytic solutions to the spectra and wavefunctions which is why most of the superconducting qubit work in the last 20 years has focused on two node circuit topologies. However, by increasing number of
Figure 2.7: The energy spectrum (top left) for a heavy fluxonium with $E_J=8$ GHz, $E_C=0.5$ GHz, and $E_L=0.5$ GHz. The matrix elements (top right) are substantially lower than those of a traditional fluxonium (Figure 2.6) except for the increase when the fluxon and plasmon states hybridize near 5 GHz. The lower panels show the wavefunctions at zero flux (left) and half a flux quantum (right).
Figure 2.8: The circuit diagram for the $0 - \pi$ qubit. The expected values to be in the protected regime are $E_J=10$ GHz, $E_{CJ}=5$ GHz, $E_L=50$ MHz, and $E_C=50$ MHz.

nodes, interesting behaviors and spectral features emerge. These include topologically protected qubits [53], different native gate sets and operations [54] and longitudinal qubit couplings [55]. Here we consider the protected $0 - \pi$ qubit as first proposed by Kitaev and subsequently revisited [56]. The circuit consists of two JJ’s opposite each other in a four legged circuit with two superinductors, and opposite corner nodes connected with large capacitors, as shown in Figure 2.8. To gain an understanding of the eigenstates and energies of the system, we follow the approach detailed in [57].

The four node fluxes around the structure are labeled $\varphi_i$ for each node and given by

$$\varphi_i = \int_{-\infty}^{t} d\tau V_i(\tau)/\Phi_0 \quad \text{with} \quad \Phi_0 = \hbar/2e.$$  

From the recipe outlined in Section 2.2, the kinetic energy can be written as

$$T = \frac{1}{2} C_J (\dot{\varphi}_2 - \dot{\varphi}_1)^2 + \frac{1}{2} C_J (\dot{\varphi}_4 - \dot{\varphi}_3)^2 + \frac{1}{2} C (\dot{\varphi}_3 - \dot{\varphi}_1)^2 + \frac{1}{2} C (\dot{\varphi}_4 - \dot{\varphi}_2)^2, \quad (2.19)$$
where $C_J$ is the small JJ capacitance, and $C$ is the capacitance between nodes 1-3 and 2-4. The two superinductors and single JJ’s give the potential terms

$$U = -E_J \cos (\varphi_4 - \varphi_3 - \varphi_{ext}/2) - E_J \cos (\varphi_2 - \varphi_1 - \varphi_{ext}/2)$$

$$+ \frac{1}{2} E_L (\varphi_2 - \varphi_3)^2 + \frac{1}{2} E_L (\varphi_4 - \varphi_1)^2,$$

where $E_J$ is the Josephson energy of the single junctions, and $E_L$ is the inductive energy of the superinductors. For notational convenience, we have absorbed the flux quantum constant into the capacitance and flux variables, $C = C\Phi_0^2$ and $\varphi_{ext} = \Phi_{ext}/\Phi_0$. To gain an intuition for the spectra, we can decompose the circuit into four “modes” of motion, shown in Figure 2.9. By making a change of variables to diagonalize the kinetic energy term

$$2\dot{\phi} = (\varphi_2 - \varphi_3) + (\varphi_4 - \varphi_1)$$

$$2\dot{\theta} = (\varphi_2 - \varphi_1) - (\varphi_4 - \varphi_3)$$

$$2\dot{\chi} = (\varphi_2 - \varphi_3) - (\varphi_4 - \varphi_1)$$

$$\Sigma = \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4,$$

we can rewrite the kinetic and potential energy terms as

$$T = C_J \dot{\phi}^2 + (C_J + C) \dot{\theta}^2 + C \dot{\chi}^2,$$

and

$$U = -2E_J \cos \theta \cos (\phi - \varphi_{ext}/2) + E_L \dot{\phi}^2 + E_L \dot{\chi}^2.$$

By making the variable transformation, we have eliminated one degree of freedom, the $\Sigma$ mode, which represents a global shift of the potential at all nodes, leaving us with the correct, $n - 1 = 3$, degrees of freedom. This choice of variables also highlights
Figure 2.9: Decomposition of $0 - \pi$ circuit into the modes which diagonalize the kinetic energy term of the circuit Hamiltonian.
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that one of the modes of motion, the $\chi$ mode, is decoupled from the small JJ’s, that is, it’s a purely harmonic variable that oscillates with frequency $\Omega_\chi = \sqrt{8E_L E_C}/\hbar$. In the absence of disorder, the $\chi$ modes will remain uncoupled from the rest of the spectrum. Experimentally, we expect some degree of disorder from device fabrication variations; however, as long as the relevant qubit frequencies are kept far from the $\chi$ modes, there is no expected degradation in the lifetime and coherence for the qubit [58]. The Lagrangian with the remaining two modes of motion for the perfectly symmetric device is therefore given by

$$\mathcal{L} = C_J \dot{\phi}^2 + C_\Sigma \dot{\theta}^2 + 2E_J \cos \theta \cos (\phi - \varphi_{\text{ext}})/2 - E_L \phi^2,$$

(2.24)

where $C_\Sigma \equiv (C_J + C)$. Performing the Legendre transform and canonical quantization yields

$$H = -2E_C \phi^2 - 2E_{C\Sigma} \theta^2 - 2E_J \cos \theta \cos (\phi - \varphi_{\text{ext}})/2 + E_L \phi^2 + 2E_J,$$

(2.25)

where the additional factor of $2E_J$ is included for convenience to have the energy spectrum always positive. The effective potential of the system is given by $V(\theta, \phi) = -2E_J \cos \theta \cos (\phi - \varphi_{\text{ext}})/2 + E_L \phi^2 + 2E_J$ and is shown in Figure 2.10. The circuit receives its name from the two potential wells in the $\theta$ direction, with minima at 0 and $\pi$. As originally conceived, the circuit parameters are chosen such that the ground state and first excited state exist as wavefunctions that are localized in a single $\theta$ well but delocalized over many $\varphi$ wells. As originally conceived the four constraints to be in the $0 - \pi$ regime are:

1. $E_{CJ} \gg E_{C\Sigma}$

2. $E_J \gg E_{C\Sigma}$
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Figure 2.10: The effective potential, $V(\theta, \phi)$, for the $0-\pi$ qubit at $\varphi_{\text{ext}} = 0$.

3. $E_J \gg E_L$

4. $E_{CJ} \gg E_L$

Note that $E_L$ is the smallest energy scale which must be on the order of 50 MHz or \( \sim 3 \mu\text{H} \). These constraints, the seemingly contradictory light $\phi$ and heavy $\theta$ modes and superinductance values that are in excess of 1 $\mu\text{H}$, have made the experimental realization challenging.

Despite the fabrication challenges and complex level structure, the potential increase in $T_1$ and $T_2$ of the $0-\pi$ over existing two node circuits has garnered experimental interest in recent years. As was recently reported [58], both the $T_1$ and $T_2$ times are expected to be several orders of magnitude larger than state of the art transmon values.

To understand the origin of the protection in qubit lifetime and coherence, we consider the ground and first excited state wavefunctions and the $0-\pi$ energy spectrum. As shown in Figure 2.11, the ground and excited states are extremely disjoint, that is to say, the overlap of the wavefunctions is vanishingly small. This small overlap
Figure 2.11: The first few eigenenergies of $0 - \pi$ plotted as a function of external flux through the qubit loop (top). The $|0\rangle$ and $|1\rangle$ states are highlighted in gold and green respectively. The wavefunctions for the $|0\rangle$ (middle) and $|1\rangle$ (bottom) states, note the localization in the $0$ and $\pi$, $\theta$ wells.
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results in a protection of the excited state lifetime (as is discussed in greater detail in Section 2.5). The protection in $T_2$ arises from the near degeneracy of the $|0\rangle$ and $|1\rangle$ states, as the pure dephasing rate between two levels is given by $\partial E_{01}/\partial \lambda$ where $E_{01}$ is the transition energy between the levels and $\lambda$ is some external parameter (ie. flux or charge). This near degeneracy between the states is achieved through constraints 3 and 4.

2.4 Qubit-resonator coupling

To perform state initialization, coherent manipulation, and measurement, we must couple a probe to our qubits. In traditional cavity QED experiments an atom is coupled to the field of an single mode in an optical resonator. In the case of circuit QED, we typically make use of a planar structure, such as a coplanar waveguide resonator, to capacitively or inductively couple to qubits. Superconducting qubits can be made in the strong coupling regime, where the coupling strength between the resonator and the qubit is much greater than either the photon loss rate of the cavity or the decay rate of the qubit [59, 60]. A qubit coupled to a resonator can be described by the following Hamiltonian

$$H_{\text{tot}} = H_q + H_r + H_c,$$

where $H_q$ describes the qubit subsystem, $H_r$ the resonator subsystem, and $H_c$ describes the coupling between the two. We follow the theoretical description of such a system with a resonator that is capacitively coupled to a superconducting resonator.
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[61] to find

\[ H = \sum_j \hbar \omega_j a_j^\dagger a_j + \sum_l E_l |l\rangle \langle l| + \sum_j \sum_{l,l'} g_{j;l'} |l\rangle \langle l'| \left( a_j + a_j^\dagger \right), \]  

(2.27)

where \( \omega_j \) is the photon mode frequency, \( E_l \) are the energy levels of the states \(|l\rangle\) of a multilevel “qudit” system, and the coupling coefficients, \( g_{j;l'} \) are given by

\[ g_{j;l'} = g_j \langle l| \hat{n} |l'\rangle, \]  

(2.28)

where \( g_j = 2eV_j^{\text{rms}} C_q / C_\Sigma \). In the dispersive regime of cQED [62], the Hamiltonian can be approximated to second order by

\[ H_{\text{eff}} = \sum_j \hbar \omega_j a_j^\dagger a_j + \sum_l E_l |l\rangle \langle l| + \sum_{j,l} \chi_{j;l} a_j^\dagger a_j |l\rangle \langle l| + \sum_l \kappa_l |l\rangle \langle l|, \]  

(2.29)

where the third and fourth terms represent the ac-Stark and Lamb shifts of the qudit energy levels. We can sum over the partial dispersive shifts \( \chi_{j;l} \equiv |g_{j;l'}|^2 / \Delta_{j;l'} \), where \( \Delta_{j;l'} \) are the resonator mode and qudit energy level detunings, to find the energy correction in eqn. 2.29,

\[ \chi_{j;l} = \sum_{l'} (\chi_{j;l'} - \chi_{j;l}) . \]  

(2.30)

When the qubit state changes, the change in \( \chi_{j;l} \) will cause the a shift in the microwave resonator frequency which can be measured through standard microwave transmission or reflection measurements [63], allowing us to determine the state of the qubit. Typically, this many level treatment can be simplified for CPB and transmon qubits where the system can more accurately be approximated with fewer qubit levels, how-
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Figure 2.12: A simplified circuit model for relaxation in the qubit, consisting of a current noise source, $I(t)$ and an admittance $Y(\omega)$ placed in parallel with the qubit.

ever keeping these corrections is important in understanding highly multilevel systems such as fluxonium or $0 - \pi$.

2.5 Relaxation in qubits

The energy relaxation rate, or $T_1$, is the average rate at which a qubit decays from its excited state back to the ground state [5]. Understanding relaxation in qubits is important as it ultimately sets the bound on qubit coherence; $T_2 = 2T_1$ in the absence of dephasing. Relaxation can arise from many different microscopic causes, but can generally be thought of as coupling between the qubit and a lossy environment, which will take energy from the qubit. Figure 2.12 shows a simplified general circuit model for situation, in which the qubit can be placed in parallel to an admittance $Y(\omega)$ consisting of a frequency dependent resistor $R$, a coupling capacitance $C_C$, and a fluctuating current, $I$. This coupling can be expressed in a similar manner as the resonator coupling in the previous section:

$$H_{\text{env}} = \hat{\Phi} \hat{I} = \varphi_0 \hat{\varphi} \hat{I}. \tag{2.31}$$
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From Fermi’s golden rule [64], we can express the transition rate between two states, \( |\alpha\rangle \) and \( |\beta\rangle \) as
\[
\Gamma_{\alpha\beta} = \frac{1}{4e^2} |\langle \alpha | \hat{\gamma} | \beta \rangle|^2 S_{\text{env}}^{\text{II}}(\omega_{\alpha\beta}),
\] (2.32)
where
\[
S_{\text{env}}^{\text{II}}(\omega_{\alpha\beta}) = \hbar \omega_{\alpha\beta} \Re \left[ Y(\omega_{\alpha\beta}) \right] \left( \coth \left( \frac{\hbar \omega_{\alpha\beta}}{2k_B T} \right) + 1 \right)
\] (2.33)
is the quantum current spectral noise density [65, 66] and \( \omega_{\alpha\beta} \) is the transition frequency. \( Y(\omega) \) is given by the series combination of \( R \) and \( C_C \),
\[
Y(\omega) = \left( R + (j \omega C_C)^{-1} \right)^{-1}
\]
\[
\Re \left[ Y(\omega) \right] = \frac{1}{R \left( 1 + (R \omega C_C)^2 \right)},
\] (2.34)
which in the limit of long lifetimes (small \( R \)) and weak coupling (small \( C_C \)) simplifies to \( (R \omega C_C)^2 / R \).

Without knowing the microscopic details for particular loss mechanisms, we can treat each of these couplings as a separate dissipation source attached in parallel to the qubit and add the relaxation rates
\[
\Gamma_{\text{total}} = \Gamma_{\text{inductive}} + \Gamma_{\text{capacitive}} + \Gamma_{\text{Purcell}} + \ldots
\] (2.35)
A fluxonium qubit has previously been used to explore different loss mechanisms as the qubit frequency is changed over a large frequency range [44]. In Chapter 4, we also make use of a nanowire fluxonium to explore loss arising from capacitive and inductive sources using the qubit as a \( T_1 \) “spectrometer”.

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2.6 Kinetic Inductance

Inductance has become an increasingly important tool in superconducting circuits, in particular, the use of superinductors, inductors with \( Z > R_Q \), in fluxonium and \( 0 - \pi \) circuits. Previous implementations of superinductors [38] use of arrays of JJ as the superinductive element. However these carry with them the challenges of self resonance modes [39], and phase slips across the array junctions limiting \( T_2 \) [52]. An alternative superinductor construction consists of a high kinetic inductance nanowire. Kinetic inductance is the inductive like energy that arises from the motion of charge carriers [23]. All materials have kinetic inductance, however, in practice,

![Figure 2.13: In a thin disordered superconducting film, the motion of Cooper pairs gives rise to an increased inductance of the wire. Examples of materials with large \( L_k \) include NbN [67], TiN [68], NbTiN [43], and granular Al [69]](image)

for normal metals the mean free path of the electrons makes this contribution an insignificant fraction compared to the geometric (magnetic) inductance. However, in disordered superconductors with very low densities of Cooper pairs, this kinetic inductance contribution can dominate over the geometric inductance of a nanowire or thin film. The kinetic inductance, \( L_k \), can be expressed as
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\[ L_k = \left( \frac{m}{2e^2 n_s} \right) \left( \frac{l}{wd} \right), \]  

(2.36)

where \( n_s \) is the density of Cooper pairs; \( m \) is the electron mass; \( e \) is the electron charge; and \( l, w, d \) the length, width, and thickness of the wire respectively [67]. \( L_k \) can also be obtained using BCS theory [23] and expressed as

\[ L_k = \left( \frac{l}{w} \right) \left( \frac{R_{\square}h}{2\pi^2 \Delta} \right) \left( \frac{1}{\tanh \left( \frac{\Delta}{2k_B T} \right)} \right). \]  

(2.37)

Although equivalent, the BCS expression is more useful in terms of parameters which are easily to experimentally measure, namely, the sheet resistivity, \( R_{\square} \), and the superconducting gap, \( \Delta \).
Chapter 3

Experimental Techniques

Running a cQED experiment requires knowledge and control of systems that range in length scales from $\sim 100$ nm (the nanofabricated JJ’s) to meters (the dilution refrigerator and room temperature electronics). In this chapter, we provide a description of the fabrication techniques used to fabricate and package qubits, details of the measurement setup, and a brief overview of the microwave measurement techniques used to measure devices. We conclude with a characterization of the NbTiN films used in constructing the nanowire superductance fluxonium qubit (NSFQ).

3.1 Fabrication

Here we provide an overview of the device fabrication process, for the NSFQ, $0 - \pi$ and the zero ZZ devices, with a more detailed list of cleanroom recipes provided in Appendix. A. The fabrication processes are nominally identical for the NSFQ and $0 - \pi$ compared to the zero ZZ device, with an additional set of steps added for the patterning of the NbTiN nanowires. Careful cleaning is performed before and after different fabrication steps to limit the amount of metal and organic residues on the surface of the device. The main process involves a TAMI (Toluene, Acetone, Methanol, Isopropanol) clean, sonicating the chips for 2 minutes in each of the in-
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dividual solvents. For the NSFQ and 0 – π chips, we start with a 500 µm thick C-plane sapphire with a film of either 10 or 15 nm of NbTiN deposited on the surface (provided by StarCryo). The chips are covered in UV light sensitive resist (S1811) and a pattern is directly written on the chip using a Heidelberg DWL-66+ mask writer in the cleanroom. The exposed resist is removed with a weak base (MF-319) and the unwanted NbTiN is removed with a fluorine based (SF₆:Ar) etch. A second photolithography step covers the remaining NbTiN, followed by sputtering an additional layer of 200 nm thick Nb for the readout resonators, with one additional photolithography step and fluorine etch to pattern the Nb. Although in principle the resonators could have been fabricated from the NbTiN, we added the additional fabrication steps to eliminate any self-Kerr non-linearities from the resonator which might complicate the device measurements. In future designs, this process could be modified, and even taken advantage of to make more compact resonators based on their large kinetic inductance. Once the resonators are patterned and etched, we perform e-beam lithography with the Elionix 125 kV e-beam writer, to expose a double layer of MMA EL13/ PMMA950-A3 e-beam resist with a 40 nm thick Al anti-charge layer on top of the resist. The MMA has a lower clearing dose than the PMMA and provides an “undercut” for shadow evaporation of the junctions. Once the e-beam pattern is written, the anti-charge layer is removed along with the exposed resist, and the devices are loaded into the Plassys evaporator for evaporation of the JJs. After evaporation and oxidation of the JJ’s the extraneous Al is removed by soaking the chips in N-Methyl-2-pyrrolidone for several hours. For fabrication of the zero ZZ devices, we perform all the steps starting from the deposition of Nb onto a cleaned sapphire substrate.
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3.1.1 Junction types

There are different styles of JJ’s, which we will briefly mention: Dolan bridge junctions [70], cliff junctions [71], and Manhattan junctions [72, 73]. At the time of this writing, there is no experimental evidence indicating if any of these junction styles is superior to the others, although future comprehensive studies might help to elucidate underlying loss mechanisms present in the design of JJ itself. The choice of junction is usually based on design considerations, but all three are fabricated on a similar principle of tilting the sample, evaporating Al, oxidizing the Al, tilting the sample a different direction and evaporating a second layer of Al. Figure 3.1 shows the resist patterns used for the different junction styles. Cliff junctions tend to have a small amount of extraneous metal and can be made very large (> 0.5 \( \mu \text{m}^2 \)), however the sensitivity to the undercut and full clear doses make it difficult to fabricate small junctions with \( E_J < 8 \text{ GHz} \). Manhattan and Dolan junctions can be reliably fabricated with \( E_J \) down to 2 GHz. NSFQ devices 1 and 2 were fabricated with a single cliff junction each, where as all other \( 0 - \pi \) and zero ZZ devices were fabricated with Manhattan junctions.

3.1.2 Junction resistance

From the Ambegaokar-Baratoff relation [24], we can relate the measured test junction resistance at room temperature to estimate the \( E_J \) of the devices. As the oxide thickness and resistance of the junction depends on the fabrication parameters (oxidation length, partial pressure of oxygen, quality of Al, etc.), the \( E_J \cdot R_n \) constant needs to be calibrated for each process. Effectively, this is a calibration for variations in \( \Delta \) from eqn. 2.4, which would lead to differences from the theoretically predicted 132 k\( \Omega \)-GHz. Several junctions with different areas are measured to extract an expected
Figure 3.1: Several types of JJ’s. The red areas of the schematic are exposed at an e-beam dose high enough to remove both the PMMA and MMA layers of resist (typically \( \sim 2000 \, \mu \text{C/cm}^2 \)). Blue areas are exposed at roughly one-fifth the dose, clearing the bottom MMA layer but leaving the top layer intact resulting in a resist “undercut”. The arrows indicate the direction of the evaporation angles, with the sample holder additionally tilted by 40° out of plane for each evaporation. The scanning electron micrograph (SEM) images have red highlighted segments showing the JJ area.
R\textsubscript{n} per area. Fits of transmon or fluxonium energy spectra are used to verify the correspondence between \( E_J \) and \( R_n \). Table 3.1 has several resistance values for the witness Manhattan junctions on NSFQ dev 3 chip.

<table>
<thead>
<tr>
<th>Junction linear dimensions (nm x nm)</th>
<th>( R ) (kΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100x100</td>
<td>33.6</td>
</tr>
<tr>
<td>100x100</td>
<td>35.3</td>
</tr>
<tr>
<td>100x100</td>
<td>35.0</td>
</tr>
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<td>125x125</td>
<td>24.2</td>
</tr>
<tr>
<td>150x150</td>
<td>16.8</td>
</tr>
<tr>
<td>150x150</td>
<td>16.6</td>
</tr>
<tr>
<td>150x150</td>
<td>17.5</td>
</tr>
<tr>
<td>175x175</td>
<td>11.9</td>
</tr>
<tr>
<td>175x175</td>
<td>11.5</td>
</tr>
<tr>
<td>175x175</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table 3.1: Measured test junction resistances at room temperature. From the experimentally measured value of \( E_J R_n \approx 120 \text{ kΩ-GHz} \), this range of areas results in \( E_J \) values between 3.5 - 10 GHz.

### 3.2 Packaging and shielding

Once the sample fabrication is completed, the sample is mounted into a copper disk (referred to as a “penny”) which has a 7 \times 7 mm slot cutout for the chip. Several of these copper pennies have a \( \sim 1 \text{ mm} \) deep trench below the sample to reduce the capacitance between the chip and the sample holder, and to disrupt box modes. The chip is secured onto the penny with conductive silver paste (Ted Pella 16032) and mounted with an indium ring pressed onto the printed circuit board. For the fluxonium and 0 – \( \pi \) samples, a copper wire coil mounted to a spool is screwed onto the PCB above the surface of the chip, providing external magnetic flux. The completed sample holder is mounted inside a set of shields consisting of an aluminum can inside one or two high permeability \( \mu \)-metal shields. The interior of the aluminum
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shield is coated with Stycast 2850 (catalyst 23LV) mixed (3:1 by weight) with Silicon Carbide #16 and #320 grit, to lower the emissivity of the shield interior, reducing the blackbody radiation emitted from these shields to the sample. The exterior of all shields is wrapped with aluminized polyethylene terephthalate (trade name of Mylar) to increase the emissivity of the exterior of the shields. The shields and sample are attached to the mixing chamber plate of the dilution refrigerator with an OFHC (oxygen free high conductivity) copper rod. In order to maintain a “light-tight” shielding enclosure, the measurement wires pass through the shields via several SMA bulkheads attached to the outer µ-metal can.

3.3 Measurement setup

3.3.1 Dilution Refrigerator

All the measurements presented in this thesis were performed in a BlueFors dilution refrigerator. The fridge is separated into several temperature stages from room temperature down to 10 mK. A multi-stage pulse tube cools the first two stages of the fridge down to ~50 K and ~4 K. A mixture of $^3$He/$^4$He circulates in a separate closed loop through the still, $T = 850$ mK, quasi-cold stage, $T \sim 150$ mK, and the mixing chamber, $T = 10$-20 mK. We make use of the different temperature stages as thermal anchors for different portions of the measurement setup, including the filters/attenuators and high electron mobility transistor (HEMT) amplifiers.

3.3.2 Wiring

Keeping qubits isolated from the environment needs careful engineering of the wiring and microwave setup. The qubits are strongly coupled to microwave cavity resonators
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for control and readout which implies that microwave photons in the cavity can have a significant back-action on the qubits themselves. Thermal photons have been shown to significantly degrade qubit coherence times as they create Stark shifts in the qubit energy levels [74, 75]. A lossless transmission line connected from room temperature down to a device on the mixing chamber sitting at 10 mK would result in a 300 K noise temperature. To reduce the number of thermal photons, attenuation on the rf input coaxial cables is configured in such a way as to reduce the effective temperature of the input line at the resonator, in effect, lowering the temperature of the blackbody source attached to the resonator. One constraint is the relatively small cooling powers, $\sim \text{mW}$ and $\sim \mu\text{W}$, at the still and mixing chamber respectively. Additional concerns have been raised that the center pin of the attenuators is not in thermal equilibrium with the dilution refrigerator, that dissipating too much power can cause a much higher effective temperature at the resonator, and that some commercial attenuators are ineffective at blocking thermal radiation above 20 GHz [76]. Therefore, K&L 12 GHz low pass filters are added to the input/output lines on the mixing chamber as well as custom Ecosorb absorptive filters. When selecting attenuators, the three important metrics we consider are total attenuation, final theoretical temperature, and the power dissipated by the attenuation on the mixing chamber plate. Table 3.2 lists several different possible attenuation setups.

In addition to the cavity input/output lines, the NSFQ and $0-\pi$ devices need an additional line for biasing the magnetic flux coil. The attenuation setup for the flux bias line is shown in Figure 3.2. The lines are low pass filtered with Minicircuits SLP-1.9+ MHz low pass filters. We can calculate the expected magnitude of the integrated flux noise assuming a Johnson noise source at an some temperature $S_V = \sqrt{4k_BTR\delta f}$, (3.1)
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<table>
<thead>
<tr>
<th>Attenu. config (dB)</th>
<th>( P_D ) (( \mu \text{W}, 0 \text{ dBm input} ))</th>
<th>( T_{\text{out eff}} ) (mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10, 10, 10, 0, 30</td>
<td>900, 90, 9, 0, 0.99</td>
<td>17.2</td>
</tr>
<tr>
<td>0, 10, 10, 0, 40</td>
<td>0, 900, 90, 0, 9.9</td>
<td>15.5</td>
</tr>
<tr>
<td>0, 0, 20, 20, 20</td>
<td>0, 0, 990, 9.9, 0.099</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 3.2: Several fridge attenuator configurations. The sequence of numbers in the attenuator configuration and power dissipation, \( P_D \), columns represent the 50 K, 4 K, Still, quasi-cold, and mixing chamber stages, assuming temperatures of 50 K, 4 K, 850 mK, 150 mK and 15 mK respectively. Note how similar effective temperatures are achieved, but that for the case of 0, 10, 10, 0, 40 dB the power dissipated at the mixing chamber is nearly 10 \( \mu \text{W} \).

where \( T \) is the effective temperature of the \( R = 50 \Omega \) resistor with a system bandwidth of \( \delta f \). Assuming a flux loop area of \( 10^4 \mu \text{m}^2 \), a noise bandwidth of 2 MHz, a magnet field constant of 0.1 Gauss/V, and an effective noise temperature of 2 K, we expect a total integrated flux noise of 5 \( \mu \Phi_0 \) due to Johnson noise, which is acceptably small.

On the output, we need to keep the cavity isolated as well as collect measurement photons with as minimal loss as possible before the first amplifier. Based on the Frii’s equation [77], any loss present between the signal generator, the output port of the measurement resonator in this case, and the receiver, such as the HEMT amplifier, will effectively increase the noise temperature of the subsequent amplifier, proportional to the loss. As an example, 3 dB of loss between the measurement resonator and a low noise HEMT with a noise temperature of 3 K will effectively increase the noise temperature of the HEMT to 6 K. It is therefore undesirable to have attenuation on the output line before the amplifier for thermalization purposes, and we instead use isolators mounted on the mixing chamber to prevent noise photons from traveling back to the device. These ferrite based isolators are only frequency matched over a narrow band, typically 4-8 GHz, and so additional K&L low pass and Ecosorb filters are added to the lines as well. The coaxial cables from the isolators to the first HEMT amplifier are constructed with Nb to further reduce the line loss. As long as
3.4 Microwave measurement techniques

As discussed in Chapter 2, superconducting qubits are coupled to microwave resonators for state preparation, manipulation and readout. Typical resonator frequencies are between 4-10 GHz corresponding to a wavelength of order 10 mm. Here we
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describe the methods used in understanding simple microwave networks and measurements.

3.4.1 Microwave networks

A microwave network can be characterized by the voltages \( V_n \) at, and currents \( I_n \) flowing in/out, of each of the \( n \)-ports of the network [78], that is,

\[
V_n = V_n^+ + V_n^-
\]
\[
I_n = I_n^+ + I_n^-.
\]

(3.2)

The generalized multi-port impedance matrix for an \( n \)-port network can be written as

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1N} \\
Z_{21} & \ddots & \vdots \\
\vdots & \ddots & \ddots \\
Z_{N1} & \cdots & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix},
\]

(3.3)

where an element of the impedance matrix, \( Z_{ij} = V_i/I_j \mid I_k = 0, \text{ for } k \neq j \). \( Z_{ij} \) can be found by driving port \( j \) with current \( I_j \) with open loads at all other ports, and measuring the voltage at port \( i \). It is often easier to only measure voltage at each port of a network, and so we can define a \( n \times n \) matrix, known as the scattering or \( S \) matrix whose elements relate the incoming and reflected voltages at each port

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
\vdots \\
V_n^-
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1N} \\
S_{21} & \ddots & \vdots \\
\vdots & \ddots & \ddots \\
S_{N1} & \cdots & S_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
\vdots \\
V_n^+
\end{bmatrix},
\]

(3.4)
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where an S-parameter \( S_{ij} = \frac{V_i^-}{V_j^+} \big|_{V_k^+ = 0} \), for \( k \neq j \). One subtle modification to measuring S-parameters is that all unused ports must be terminated with matched loads, typically \( Z_0 = 50 \, \Omega \), in order to enforce \( V_k^+ = 0 \). A vector network analyzer is used to measure the amplitude and phase of the incident and reflected voltages at the different ports to measure the S-parameters of a device. A typical microwave setup might consist of a \( \lambda/2 \) coplanar waveguide resonator with coupling capacitors on the ends of the resonator. In this situation, the number of ports for each of the elements in the circuit is two, but having three cascaded elements makes analyzing the circuit with S-parameters and impedance matrices is quite cumbersome. One additional useful definition, is the transmission or ABCD matrix [78], which for a two-port network is defined as

\[
V_1 = AV_2 + BI_2 \\
I_1 = CV_2 + DI_2,
\]

or in matrix form

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}.
\] (3.6)

The utility of the ABCD matrix formalism is that the equivalent ABCD matrix for the cascaded network will be the product of the matrices for each of the components. Several useful ABCD matrices include a series impedance

\[
\begin{pmatrix}
1 & Z \\
0 & 1
\end{pmatrix},
\] (3.7)
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where $Z$ is the complex impedance of the circuit element, a shunt admittance

$$
\begin{pmatrix}
1 & 0 \\
Y & 1
\end{pmatrix},
$$

(3.8)

where $Y$ is the complex admittance of the circuit element, and a transmission line

$$
\begin{pmatrix}
\cosh(\alpha + \beta) & jZ_0 \sinh(\alpha + \beta)l \\
jY_0 \sinh(\alpha + \beta) & \cosh(\alpha + \beta)l
\end{pmatrix},
$$

(3.9)

where $Z_0$ is the characteristic impedance of the transmission line, $Y_0 = 1/Z_0$ and $\beta$ and $\alpha$ are the propagation and attenuation constants respectively. Once the transmission matrix for a cascaded network has been obtained, the S-parameters can be expressed in terms of the ABCD matrix,

$$
\begin{align*}
S_{11} &= \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \\
S_{12} &= \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \\
S_{21} &= \frac{2}{A + B/Z_0 + CZ_0 + D} \\
S_{22} &= \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}.
\end{align*}
$$

(3.10)

For qubit measurements, the S-parameters $S_{11}$, reflected voltage from 1 port, and $S_{21}$, the transmitted voltage transmitted from port 1 to 2, are particularly important. Note that in general, the S-parameters can be complex and we typically will be interested in either the amplitude of the power, $|S_{ij}|^2$, or the phase, $\text{Arg}[S_{ij}]$. The ABCD formalism can also be quite useful in the design of on chip Purcell filters [79] and for multi-segment devices for bandgap engineering [80, 81].
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3.4.2 CPW resonators

A coplanar waveguide (CPW) consists of a thin superconducting “center pin”, with width $S$, placed in a gap with a width $W$ (see Figure 3.3) between two semi-infinite superconducting ground planes [82]. The system acts as a planar analog to a coaxial cable, supporting TEM-like propagating modes. The C-plane sapphire substrate has an anisotropic dielectric constant with $\varepsilon_r = 11.5$ for fields parallel to the C-axis and $\varepsilon_r = 9.5$ for perpendicular fields, and the effective dielectric constant will have an intermediate value based on the dimensions of the center pin width and gaps which dictate the electric field distribution in the substrate. The surface above the CPW is typically vacuum, $\varepsilon_r = 1$. Through conformal mapping techniques, both $\varepsilon_{\text{eff}}$ and $Z_0$ can be found and are given as

$$
\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_{\text{sub}} - 1}{2} \frac{K(k_1) K(k'_0)}{K(k'_1) K(k_0)}
$$

$$
Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}} K(k_0)},
$$

(3.11)

where $K(k_i)$ are the complete elliptic integrals of the first kind [82]. The arguments of the elliptic integrals are given by

$$
k_0 = \frac{S}{S + 2W} \quad k'_0 = \sqrt{1 - k_0^2},
$$

$$
k_1 = \frac{\sinh \left( \frac{\pi S}{4h} \right)}{\sinh \left( \frac{\pi (S+2W)}{4h} \right)} \quad k'_1 = \sqrt{1 - k_1^2},
$$

(3.12)
where \( h \) is the substrate thickness. From these expressions, we can obtain the characteristic capacitance and inductance per length of the CPW as

\[
\frac{L}{l} = \frac{\mu_0 K(k_0)}{\frac{4}{K(k_0)}}
\]

\[
\frac{C}{l} = 2\varepsilon_0 (\varepsilon_{\text{sub}} - 1) \frac{K(k_1)}{K(k_1')} + 4\varepsilon_0 \frac{K(k_0)}{K(k_0')}.
\]

Knowing the characteristic inductance of the CPW will be important in the analysis of the \( \lambda/4 \) NbTiN resonators to extract the fraction of kinetic inductance arising from the NbTiN film.

To turn the waveguide into a resonator, the center pin can be either cut in two locations or cut at one end and terminated via a short to the ground plane on the other end. These configurations produce standing waves in the center pin with either a voltage anti-node at each end or a voltage anti-node at one end and a node at the other end (see Figure 3.3). The gaps on both ends is referred to as a \( \lambda/2 \) resonator and the single gap with shorted termination is referred to as a \( \lambda/4 \) resonator. The resonance frequencies are given by

\[
l = \lambda/2 = \omega_r = \frac{c\pi}{l\sqrt{\varepsilon_{\text{eff}}}} \quad l = \lambda/4 = \omega_r = \frac{c\pi}{2l\sqrt{\varepsilon_{\text{eff}}}}.
\]

Additional standing wave modes exist at higher frequency harmonics, e.g. \( 3\lambda/2, 5\lambda/2, 7\lambda/2... \) or \( 3\lambda/4, 5\lambda/4, 7\lambda/4... \), which has important implications for relaxation via multi-mode Purcell effect [45], and leads to the general rule that the qubit frequencies should be placed below the fundamental frequency of a multi-mode resonator.

Near resonance, an open circuited \( \lambda/2 \) resonator behaves like a parallel RLC circuit, with a sharp resonance peak and the \( \lambda/4 \) resonator can be approximated as a series RLC circuit, near its resonance frequency with an anti-resonance (dip).
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Resonators can be “side coupled” to a microwave feedline, where a portion of the resonator is placed within a few µm of a CPW transmission line. Side coupling resonators allows for multiplexed readout of several devices on a single measurement line. In the case of a resonator side coupled to a microwave transmission line, the transmission can be expressed as

\[ S_{21} = 1 - \frac{Q_T}{Q_C} - 2iQ_T \frac{\delta \omega}{\omega_0}, \]  

(3.15)

where \( Q_T \) is the loaded quality factor of the resonator (\( 1/Q_T = 1/Q_i + 1/Q_C \) with \( Q_i \) as the internal quality factor of the resonator and \( Q_C \) as the external coupling quality factor), \( \omega_0 \) is the resonance frequency, and \( \delta \omega \) is a parameter that accounts for resonator asymmetry due to impedance mismatches in the microwave readout line [83–85]. \( Q_C \) is proportional to the size of the coupling capacitance, \( C_C \), between the resonator and microwave feedline. For a resonator with a single port coupled to the feedline [86], the coupling quality factor is given by

\[ Q_C = \frac{\pi}{2 (C_C Z_0 \omega_0)^2}. \]  

(3.16)

With knowledge of \( \omega_0 \) and \( Q_C \), we can also define \( \kappa \), the rate at which photons leave the cavity, as \( Q_C = \omega_0/\kappa \), or \( \kappa \approx 2\omega_0^3 C_C^2 Z_0^2 / \pi \). The addition of the external coupling capacitance and transmission line impedance “loads” the resonator, causing a downward shift in resonance frequency given by

\[ \omega' = \frac{\omega_0}{\sqrt{1 + C_C Z_0 \omega' \omega_0}}. \]  

(3.17)

For reasonably fast readout, \( Q_C \sim 10^3 - 10^4 \) and with \( Q_i \)'s of \( 10^5 - 10^6 \), the CPW resonators are over-coupled to the feedline. To keep the linear dimensions of the chip
Figure 3.3: $\lambda/2$ (top) and $\lambda/4$ (bottom) resonators. $S$ is the width of the center pin and $W$ the gap between the center pin and the ground planes. For capacitive coupling, the qubits should be placed at the voltage anti-nodes where $V_0$ will be the largest, indicated by the blue shaded voltage standing wave.

Small, the CPW meanders with a bend radius greater than at least $3(S + 2W)$, as a rule of thumb, to avoid impedance mismatches and to keep a large ground plane on the sides on the center pin. By having long sections of ground cut up by the slot for the center pin, the ground plane needs to be reconnected with wirebonds to the outer ground plane at the edge of the chip and PCB. The wirebonds themselves have a non-trivial inductance, about 1 nH/mm which can give rise to an appreciable impedance at microwave frequencies, $|Z| = 2\pi(5 \times 10^9 \text{ GHz})(10^{-9} \text{ nH/mm}) \approx 30 \Omega/\text{mm}$, which is why it’s important to include many additional bonds in parallel to keep the impedance between the ground planes as low as possible.
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3.4.3 Microwave spectroscopy

As discussed in Chapter 2, when a qubit is coupled to a microwave resonator and changes its state, the dispersive interaction will shift the resonator frequency. If the resonator is probed at its frequency when the qubit is in the ground state, the corresponding excited state shift will lead to a change in either the transmitted or reflected probe tone, resulting in a shift of the amplitude and phase of $S_{21}$ or $S_{11}$ [60].

In a typical experiment, the frequency of a single continuous wave (cw) probe tone is swept to locate the resonator. Once the resonator’s frequency is known, the probe tone is fixed at this frequency and the frequency of a second spectroscopic tone is swept to find the qubit [87]. When the frequency of the spectroscopic tone matches the qubit frequency, the steady-state population of the qubit changes from its ground state to some fraction in the excited state resulting in a change in the measured S-parameter. This “two-tone” qubit spectroscopy can be repeated for different values of some control parameter, such as $n_g$ or $\Phi_{\text{ext}}$ for voltage or flux tunable qubits, respectively. Once the qubit and cavity frequencies are known, the system can be setup for pulsed measurements.

Figure 3.4: The measurement setup for single and two-tone cw spectroscopy measurements. Port 3 of the network analyzer can be used for the swept spectroscopy tone, while measuring the forward transmission, $S_{21}$ from the other ports.
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Figure 3.5: A block diagram for pulsed measurements. For NSFQ and $0 - \pi$ samples, the M9018A AWG was replaced with a Tektronix 5014C. The additional 33250 10 MHz clock is needed to provide a $3 V_{pp}$ 10 MHz reference for the U1082A fast ADC. For the zero ZZ experiments, the LO phase is adjusted to put all the signal in one quadrature (for homodyne) so that both qubits can be simultaneously measured with the single ADC card.

3.4.4 Pulsed measurements

Pulsed measurements consist of separating the qubit excitation pulse and the readout pulse in time. This allows us to measure dynamical properties, such as qubit relaxation $T_1$ and dephasing $T_2$. A typical measurement cycle consists of a pulse sequence, eg. a $\pi$ rotation around the x-axis of the Bloch sphere ($X_{\pi}$), followed by a short (< 10 $\mu$s) measurement pulse at the resonator frequency. This cycle is typically repeated on the order of $10^4$ times, and the results are then averaged to achieve an acceptable SNR [88]. In a pulsed measurement, the readout signal from the fridge is amplified at room temperature and combined with a local oscillator (LO) signal and mixed down for homodyne (LO=rf) or heterodyne (LO=rf+Intermediate
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Frequency) measurements. After the frequency down conversion, the demodulated signal is further amplified to get an appropriately sized signal and digitized by a fast ADC. It’s important to consider the ADC full scale and bit resolution to avoid clipping, when the signal exceeds the full scale range of the ADC, and digitization noise, when the signal is comparable to the voltage bin size. In homodyne measurements, a few $\mu$s of data are averaged together to obtain a single amplitude (or phase), and for heterodyne, the measurement data is digitized and “digitally” down converted [89].

3.5 Characterization of NbTiN films

3.5.1 Sheet resistance and $T_c$

The NbTiN films are deposited on a 500 $\mu$m thick C-plane sapphire substrate from StarCryo, who provide an estimate of the film’s sheet resistance. In order to verify this measurement and determine how the resistance changes after processing the NbTiN films, we incorporate a set of test structures on the side of each chip, which consists of a set of wires with fixed length and varying width. The measured resistances are fit to the form

$$R = \frac{R_{\Box} l}{w + \Delta w},$$

(3.18)

where $l$ is the length of wire, $R_{\Box}$ is the wire sheet resistance, $w$ the width, and $\Delta w$ a parameter that reflects the difference in the designed width compared to the actual width. $\Delta w$ arises from the over/under exposure and developing of the e-beam resist and the run to run differences in the fluorine etch. For $w \gg \Delta w$, this correction is small, however for wires less than $\sim$100 nm in width, this correction is necessary to accurately estimate the kinetic inductance. Independent SEM images of the wires confirm that the typical $\Delta w$ is around 30 nm. These structures are valuable for rapid
feedback of fabrication variations and as a rough film characterization for development of different materials for high $L_K$ applications.

Figure 3.6 shows the characterization on two sets of NbTiN films with different thickness. To find the critical temperature, $T_C$, of the 15 nm thick film, one of the test structures was bonded into a PCB and cooled off in a helium flow cryostat and measured to find $T_C = 9.5 \text{ K}$. 

### 3.5.2 Resonators

In order to characterize the microwave properties of the 15 nm thick NbTiN film, we fabricated a set of side-coupled $\lambda/4$ CPW resonators. Transmission measurements at $T = 15 \text{ mK}$ give an $L_K$ of 14 pH/□. Based on the other material parameters, the extracted London penetration depth [23] is $\lambda = 440 \text{ nm}$. This is within 7% and 2% of the values extracted from the spectra of NSFQ 1 and 2 respectively. The numbers listed in Table 3.3 are taken at low power where the self-Kerr contribution
Figure 3.7: The normalized transmission (left) through the microwave feedline for the $S = 7.5 \, \mu m$ resonator at -40 dBm (red) and -10 dBm (blue). The resonator internal quality factor, $Q_i$, vs. applied microwave drive power (right). The increase in $Q_i$ is a result of additional measurement photons saturating lossy two level systems on the surface of the resonators. The data points for highly non-linear resonator responses at large powers have been omitted.

to the inductance is negligible. As the measurement power is increased, the resonator

$$
\begin{array}{ccccccc}
 l (\text{mm}) & S (\mu m) & W (\mu m) & \omega/2\pi (\text{GHz}) & L_k/(L_k + L) & Z_0 (\Omega) \\
3.866 & 20.0 & 8.40 & 4.365 & 0.67 & 85 \\
3.475 & 7.5 & 3.14 & 3.699 & 0.83 & 115 \\
3.084 & 3.5 & 1.47 & 3.041 & 0.91 & 160 \\
2.836 & 2.5 & 1.05 & 2.855 & 0.93 & 185 \\
2.588 & 1.5 & 0.62 & 2.417 & 0.96 & 240 \\
\end{array}
$$

Table 3.3: Parameters for the NbTiN $\lambda/4$ resonators.

frequencies decrease accompanied with an increase in $Q_i$, as has been observed in granular aluminum $L_k$ films [90]. In general, the single photon $Q_i$ for the resonators decreases with decreasing center pin width. We note however that in order to maintain a reasonably low resonator impedance, the CPW gap was also decreased, increasing the electric field in the gap. Several groups [91, 92] have recently investigated the
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contribution of loss mechanisms from different interfaces in these structures which could prove useful for determining the contribution of internal loss from the NbTiN.
Chapter 4

Nanowire Superinductance Fluxonium Qubit

The development of superinductors [38, 93] has received significant interest due to their potential to provide noise protection in superconducting qubits [53, 56, 94]. Moreover, inductively shunted Josephson junction based superconducting circuits are known to be immune to charge noise [38], and to flux noise in the limit of large inductances [47, 48, 57, 58]. Despite remarkable progress, the superinductances that have been so far reported in the literature are still small compared to those needed for qubit protection. Additionally, Josephson junction array based superinductors are susceptible to phase slips [52] and junction array modes [39] which complicate the level structure and potentially lead to additional sources of decoherence and relaxation.

A thin-film nanowire built from a disordered superconductor constitutes an alternative approach to reach the required superinductance regime. We demonstrate a fluxonium circuit integrating a NbTiN nanowire superinductance, and we characterize the effect of the nanowire modes on the qubit spectrum with a multimode circuit theory accounting for the distributed nature of the superinductance.
CHAPTER 4. NANOWIRE SUPERINDUCTANCE FLUXONIUM QUBIT

4.1 Device design

A simplified circuit schematic of the nanowire superinductance fluxonium is shown in Figure 4.1 (a). A small patch of NbTiN, where the nanowire will be fabricated later in the process, is protected with Microposit™ S1811 photoresist and the remaining NbTiN is removed with an SF$_6$:Ar dry etch. For the lumped element readout resonator and transmission line, 200 nm of Nb is sputtered over the areas of the chip which had no NbTiN and subsequently patterned and etched with another SF$_6$:Ar step. Next, a layer of ZEP520A (1:1 dilution in anisole) e-beam resist is spun on the chip and the nanowire pattern is exposed and developed with standard e-beam lithography techniques. Finally, an MMA/PMMA bilayer e-beam resist is placed on the chip and the Josephson junction layer is patterned with e-beam lithography. To ensure metallic contact between the junction and the NbTiN nanowire, a 400 V Ar ion beam milling process is applied for 45 seconds to remove the native oxide layer formed on the surface of the NbTiN film. The JJ layer is then immediately fabricated with a double angle evaporation of 30 nm and 60 nm of Al, with a 15 minute oxidation step in between the first and second evaporation angles to form the oxide layer of the junction.

We present data from measurements of three devices fabricated on two different films. The nanowires in devices 1 and 2 have widths of 110 and 40 nm, respectively, equal lengths of 730 $\mu$m and a film thickness of 15 nm. The nanowire in device 3 is fabricated on a 10 nm thick film, has a width of 100 nm and length of 630 $\mu$m. The qubit on devices 1 and 2 is capacitively coupled to a lumped element Nb resonator, with resonance frequency $\omega_r/2\pi = 6.08$ GHz and a loaded quality factor of $Q_T = 8,400$. The qubit on device 3 is coupled to a half-wavelength coplanar waveguide.
resonator with $Q_T = 14,800$ and $\omega_r/2\pi = 7.50$ GHz. An optical image of device 1 is shown in Figure 4.1 (c).

Figure 4.1: (a) The circuit diagram for the qubit, with the first antisymmetric standing wave nanowire mode in blue. $\psi(x,t)$ denotes the flux operator as a function of the dimensionless coordinate $x = x/l$. An off-chip coil generates the magnetic flux ($\Phi_{\text{ext}}$) that is threaded through the loop formed by the nanowire and the junction. $C_g$ and $C_0$ are the coupling capacitances to the readout resonator and to ground, respectively. (b) The first few fluxonium eigenstates plotted for $\Phi_{\text{ext}}/\varphi_0 = -0.38\pi$, and the respective qubit potential with wells around $\phi/\varphi_0 = -2\pi$ and $\phi/\varphi_0 = 0$, where $\varphi_0 = \hbar/2e$. (c) False colored image of the device with the NbTiN nanowire shown in blue, the single Josephson junction and gate capacitors in red, the readout resonator in purple and the input transmission line in green. Figure adapted from [43].
CHAPTER 4. NANOWIRE SUPERINDUCTANCE FLUXONIUM QUBIT

4.2 Spectrum measurement

The fluxonium energy spectrum is obtained by performing two-tone spectroscopy measurements as a function of the external magnetic flux, $\Phi_{\text{ext}}$. The amplitude of the transmitted power is monitored at the dressed cavity frequency while sweeping a second spectroscopic tone of frequency $\omega_{\text{spec}}/2\pi$. The measurement results are shown in Figure 4.2. Labeling the energy eigenstates within a single potential well as $|g_i⟩, |e_i⟩, |f_i⟩, \ldots$, where the index $i$ indicates the potential well to which these belong [see Figure 4.1 (b)], the fluxonium transitions are classified in two types: intra-well plasmons, such as $|g_0⟩ \rightarrow |e_0⟩$, and inter-well fluxons, such as $|g_0⟩ \rightarrow |g_{-1}\rangle$. Parity selection rules of the fluxonium circuit allow for transitions between adjacent plasmon states by absorption of a single photon. However, the direct transition $|g_0⟩ \rightarrow |f_0⟩$ can only be completed via a two-photon process in which $|e_0⟩$ serves as an intermediate virtual state. We note that devices 1 and 2 operate in a similar parameter regime to “heavy fluxonium” [47, 48], where the ratio between the Josephson ($E_J$) and charging ($E_C$) energies is large. As a consequence, transitions between the fluxonium potential wells are exponentially attenuated. Therefore, such excitations are most clearly visible in the regions where they hybridize with the plasmon energy levels.

Figure 4.2 (a) shows the presence of a second fluxonium mode for device 1 at 16.3 GHz. While similar characteristics have been observed in previous fluxonium devices, high-frequency modes have been so far phenomenologically modeled as harmonic oscillators linearly coupled to the qubit degree of freedom [38]. Here we go beyond such an approximation and derive a multimode Hamiltonian considering the complete device Lagrangian, which accounts for the distributed nature of the superinductance. Importantly, we find that the qubit spectrum is determined by the nonlinear interaction of the circuit modes which are antisymmetric at the Josephson junction ports.
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[see Figure 4.1 (a)]. The agreement with the measured data is excellent over a very large frequency range.

Figure 4.2: Two-tone spectroscopy of device 1 (a) and device 2 (b) as a function of $\Phi_{\text{ext}}$. The experimentally measured transition frequencies are indicated with blue markers. The result of a fit to the two-mode Hamiltonian in eqn. 4.4, is shown with red dashed lines corresponding to the fluxonium spectrum and with purple dashed lines indicating sideband transitions [95]. In (a), the inscription “JJ mode” (Josephson junction mode) identifies the second antisymmetric nanowire mode. Figure adapted from [43].

In contrast to standard fluxonium devices, where a lumped element inductor shunts the Josephson junction, our circuit model takes into account the fact that the nanowire superinductor is a high-impedance transmission line. The nanowire is described as a homogeneous transmission line with distributed capacitance $c = C_{\text{nw}}/2l$ and inductance $\ell = L_{\text{nw}}/2l$, where $C_{\text{nw}}$, $L_{\text{nw}}$ and $2l$ are, respectively, the total ground
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capacitance, inductance and length of the nanowire. Defining the flux operator \( \psi(x, t) \) in terms of the dimensionless coordinate \( x = x/l \), the nanowire Lagrangian can be written as

\[
\mathcal{L}_{nw} = \int_{-1}^{1} dx \frac{C_{nw}/2}{2} \dot{\psi}(x, t)^2 - \frac{1}{2(L_{nw}/2)} \dot{\psi}(x, t)^2. \tag{4.1}
\]

Additionally, we consider gate capacitances \( C_g \) placed at the two ports of the device \( (x_p = \pm 1) \) with respective driving voltages \( \{V_{x_p}\} \), as well as ground capacitances \( C_0 \). The Lagrangian of the inductively shunted Josephson junction then reads

\[
\mathcal{L} = \sum_{x_p} C_g \frac{\psi(x_p, t) - V_{x_p}}{2} \dot{\psi}(x_p, t)^2 + C_0 \frac{\psi(x_p, t)^2}{2} + \mathcal{L}_{nw} + \frac{C_J}{2} \dot{\delta\psi}(t)^2 + E_J \cos(\delta\psi(t)/\varphi_0), \tag{4.2}
\]

where

\[
\frac{\delta\psi(t)/\varphi_0}{\varphi_0} = (\Delta\psi(t) + \Phi_{\text{ext}})/\varphi_0, \tag{4.3}
\]

is the gauge-invariant superconducting phase difference across the junction, \( \Delta\psi(t) = \psi(1, t) - \psi(-1, t) \) is the flux operator difference at the boundaries of the superinductor, and \( E_J \) is the Josephson energy \([96, 97]\).

To obtain a tractable theoretical description of our device, we map eqn. 4.2 into the Lagrangian of an infinite number of nonlinearly interacting normal modes. We observe that modes which are symmetric at the junction ports are not coupled to the Josephson nonlinearity, and thus do not contribute to the qubit Hamiltonian. We therefore derive a multimode Hamiltonian for the antisymmetric normal modes, which is later truncated to a finite number of modes. The truncation is possible due to the fact that only few antisymmetric modes lie in the frequency range of interest. Furthermore, the effective normal mode impedance decreases quickly with the mode number such that high-frequency modes are only weakly anharmonic.
We find that the spectra of our devices can be accurately described by a two-mode Hamiltonian of the form

\[ H_{\text{two-mode}} = \frac{(q_0 - q_{g0})^2}{2C_0} + \frac{\phi_0^2}{2L_0} + \frac{(q_1 - q_{g1})^2}{2C_1} + \frac{\phi_1^2}{2L_1} - \frac{\phi_0 \phi_1}{L_J} - E_J \cos \left( \frac{\phi_0 + \phi_1}{\varphi_0} + \Phi_{\text{ext}} \right), \]  

(4.4)

where \( C_i \), \( L_i \) and \( q_{gi} \) are, respectively, the effective capacitance, inductance and offset charge corresponding to the first two antisymmetric modes labeled by \( i = \{0, 1\} \), and \( L_J = E_J/\varphi_0^2 \). In effect, the two-mode fit gives a copy of the lumped element Hamiltonian shifted higher in frequency by the nano-wire mode. The results in Figure 4.2 are obtained by numerical diagonalization of the complete Hamiltonian of the device, including eqn. 4.4, the resonator Hamiltonian and the interaction between the subsystems.

<table>
<thead>
<tr>
<th>Device</th>
<th>( C_g ) [fF]</th>
<th>( C_p ) [fF]</th>
<th>( C_J ) [fF]</th>
<th>( L_J ) [nH]</th>
<th>( C_{nw} ) [fF]</th>
<th>( L_{nw} ) [nH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.33</td>
<td>30.20</td>
<td>3.52</td>
<td>14.33</td>
<td>35.49</td>
<td>121.38</td>
</tr>
<tr>
<td>2</td>
<td>15.89</td>
<td>60.89</td>
<td>4.67</td>
<td>9.82</td>
<td>5.79</td>
<td>314.75</td>
</tr>
</tbody>
</table>

Table 4.1: Circuit element parameters as obtained from the eqn. 4.4 two-mode fit to the fluxonium qubit spectra presented in Figure 4.2. \( C_g \) is the gate capacitance, \( C_p \) is additional parasitic ground capacitance, \( C_J \) is the junction capacitance, \( L_J \) is the junction inductance, \( C_{nw} \) the nanowire total capacitance, and \( L_{nw} \) is the nanowire total inductance.

From the two-mode fit to the qubit spectrum, we find nanowire inductances of 121 nH, 314 nH and 309 nH for devices 1, 2 and 3, respectively, and corresponding characteristic impedances \( Z_{nw} = \sqrt{L_{nw}/C_{nw}} \) of about 1.85 k\( \Omega \), 7.38 k\( \Omega \) and 12.43 k\( \Omega \). The inductance values from the fit are within 7% of the theoretical prediction given by eqn. 2.37. Table 4.2 provides the Hamiltonian parameters extracted from a single-mode fit allowing direct comparison to previous implementations of JJ array based fluxonium devices [38, 44, 47, 48, 98].
### Table 4.2: Device parameter table obtained a single-mode approximate fit to the fluxonium qubit spectrum, for devices 1, 2, and 3.

<table>
<thead>
<tr>
<th>Device</th>
<th>$E_C$ [GHz]</th>
<th>$E_L$ [GHz]</th>
<th>$E_J$ [GHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.89</td>
<td>1.37</td>
<td>10.95</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>0.52</td>
<td>16.16</td>
</tr>
<tr>
<td>3</td>
<td>1.90</td>
<td>0.53</td>
<td>5.90</td>
</tr>
</tbody>
</table>

#### 4.3 Multilevel spectroscopy

In devices 1 and 2, the small dipole element between the fluxon states makes it experimentally challenging to directly drive the $|g_1⟩ → |g_0⟩$ transition. By using multiple drives, we are able to transfer the ground state population between the neighboring wells using the intermediate $|h_0⟩$ state, which is located close to the top of the barrier and has spectral weight in both wells. We apply three coherent and simultaneous drives of frequencies $\omega_\alpha/2\pi$, $\omega_\beta/2\pi$ and $\omega_\gamma/2\pi$, respectively, targeting the $|g_0⟩ → |f_0⟩$ (two-photon), the $|f_0⟩ → |h_0⟩$ (one-photon) and the $|h_0⟩ → |e_{-1}⟩$ (one-photon) transitions [see Figure 4.3(a)].

At $\Phi_{ext}/\varphi_0 = -0.46\pi$, we set $\Omega_\gamma = 0$ and simultaneously vary $\omega_\alpha/2\pi$ and $\omega_\beta/2\pi$ around the $|g_0⟩ → |f_0⟩$ and $|f_0⟩ → |h_0⟩$ transitions. We observe a vertical band corresponding to the $|g_0⟩ → |f_0⟩$ transition at 7.8 GHz, and a diagonal band with a slope of $\omega_\alpha/\omega_\beta = -1/2$, corresponding to the Raman transition between the $|g_0⟩$ and $|h_0⟩$ states [Figure 4.3 (a)]. Around the resonance condition ($2\hbar\omega_\alpha \approx E_{f_0} - E_{g_0}$ and $\hbar\omega_\beta \approx E_{h_0} - E_{f_0}$), the two bands exhibit an avoided crossing, which is the hallmark of the Autler-Townes doublet previously observed in other superconducting qubits [99–102]. Next, we fix the frequency of the $\alpha$ tone at $\Delta_\alpha/2\pi = 20$ MHz, turn on the $\gamma$ drive and simultaneously scan the frequencies $\omega_\beta/2\pi$ and $\omega_\gamma/2\pi$. Figure 4.3 (b) displays the resulting Autler-Townes splitting, where the Raman transition manifests itself here with a slope of $\omega_\gamma/\omega_\beta = +1$, corresponding to the three-drive Raman
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condition. This method allows us to experimentally determine the energy levels of the fluxonium qubit using population transfer.

The system can be modeled with a four-level Hamiltonian which, in the
\((|g_0\rangle, |f_0\rangle, |h_0\rangle, |e_{-1}\rangle)\) energy eigenbasis and in the absence of external drives, reads

\[
H_0 = E_{g_0} |g_0\rangle \langle g_0| + E_{f_0} |f_0\rangle \langle f_0| + \\
E_{h_0} |h_0\rangle \langle h_0| + E_{e_{-1}} |e_{-1}\rangle \langle e_{-1}|, \tag{4.5}
\]

where the groundstate energy is chosen to be \(E_{g_0} = 0\), and the energies of excited levels satisfy the relations \(E_{g_0} < E_{f_0}, E_{e_{-1}} < E_{h_0}\) [see Figure 4.3 (c)]. We work in a semiclassical picture where the external drives \(\omega_\alpha/2\pi, \omega_\beta/2\pi, \omega_\gamma/2\pi\) with respective Rabi frequencies \(\Omega_\alpha, \Omega_\beta, \Omega_\gamma\) introduce coupling exclusively between neighboring energy levels. In the rotating-wave approximation, this situation is described by the interaction Hamiltonian

\[
H_{\text{int}} = \frac{1}{2} \hbar \Omega_\alpha \left( |f_0\rangle \langle g_0| e^{-i\omega_\alpha t} + |g_0\rangle \langle f_0| e^{i\omega_\alpha t} \right) + \\
\frac{1}{2} \hbar \Omega_\beta \left( |h_0\rangle \langle f_0| e^{-i\omega_\beta t} + |f_0\rangle \langle h_0| e^{i\omega_\beta t} \right) + \\
\frac{1}{2} \hbar \Omega_\gamma \left( |e_{-1}\rangle \langle h_0| e^{-i\omega_\gamma t} + |h_0\rangle \langle e_{-1}| e^{i\omega_\gamma t} \right). \tag{4.6}
\]

Since \(E_{h_0} > E_{e_{-1}}\), the time-dependent phase corresponding to the third term in eqn. 4.6 has the opposite sign. Combining the above expressions, the total Hamiltonian of the system is defined as \(H = H_0 + H_{\text{int}}\)
Figure 4.3: The multitone spectroscopy data, taken at $\Phi_{\text{ext}}/\varphi_0 = -0.46\pi$, demonstrating population transfer between $|g_0\rangle$ and $|h_0\rangle$ (a) with $\Omega_\gamma = 0$, and $|h_0\rangle$ to $|e_{-1}\rangle$ (b) with fixed $\omega_\alpha/2\pi = 7.78$ GHz. The white dashed lines indicate the maximum population from a multi-level master equation simulation eqn. 4.9. (c) A schematic diagram of the device 2 level structure in the presence of coherent external drives. The drives, with frequencies $\omega_i/2\pi$ and amplitudes $\Omega_i$ are detuned from the levels by $\Delta_i/2\pi$. (d) Three sequential $\pi$ pulses ($\sigma = 15$ ns) are applied at the transition frequencies to perform $T_1$ measurements of the $|g_{-1}\rangle$ state. The demodulated homodyne voltage from the readout resonator is measured as a function of $t_{\text{wait}}$. Figure adapted from [43].
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\[ H = \begin{bmatrix} 0 & \frac{\hbar \Omega_\alpha}{2} e^{i2\omega_\alpha t} & 0 & 0 \\ \frac{\hbar \Omega_\beta}{2} e^{i2\omega_\beta t} & E_{f0} & \frac{\hbar \Omega_\beta}{2} e^{i\omega_\beta t} & 0 \\ 0 & \frac{\hbar \Omega_\gamma}{2} e^{-i\omega_\gamma t} & E_{h0} & \frac{\hbar \Omega_\gamma}{2} e^{-i\omega_\gamma t} \\ 0 & 0 & 0 & E_{e-1} \end{bmatrix}. \] (4.7)

We now move to the rotating frame of the drives by applying the unitary
\[ U = |g_0\rangle\langle g_0| + e^{i2\omega_\alpha t} |f_0\rangle\langle f_0| + e^{i(2\omega_\alpha + \omega_\beta) t} |h_0\rangle\langle h_0| + e^{i(2\omega_\alpha + \omega_\beta - \omega_\gamma) t} |e-1\rangle\langle e-1|, \]
which results in
\[ \tilde{H} = \begin{bmatrix} 0 & \frac{1}{2} \hbar \Omega_\alpha & 0 & 0 \\ \frac{1}{2} \hbar \Omega_\alpha & h \Delta_\alpha & \frac{1}{2} \hbar \Omega_\beta & 0 \\ 0 & \frac{1}{2} \hbar \Omega_\beta & h (\Delta_\alpha + \Delta_\beta) & \frac{1}{2} \hbar \Omega_\gamma \\ 0 & 0 & \frac{1}{2} \hbar \Omega_\gamma & h (\Delta_\alpha + \Delta_\beta - \Delta_\gamma) \end{bmatrix}. \] (4.8)

Here, the detunings are \( h \Delta_\alpha = E_{f0} - 2\hbar \omega_\alpha \), \( h \Delta_\beta = E_{h0} - E_{f0} - \hbar \omega_\beta \) and \( h \Delta_\gamma = E_{h0} - E_{e-1} - \hbar \omega_\gamma \). We account for dissipation in the system with a Lindblad master equation of the form
\[ \dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \sum_j \left[ c_j \rho c_j^\dagger - \frac{1}{2} \{ c_j^\dagger c_j, \rho \} \right]. \] (4.9)

where the collapse operators \( c_j \) are defined as \( c_j = \sum \sqrt{\Gamma_{ij}} \sigma_{ij} \), for given energy states \(|i\rangle\) and \(|j\rangle\), where \( \Gamma_{ij} \) is the decay rate between them and \( \sigma_{ij} = |i\rangle \langle j| \). The steady-state solution of eqn. 4.9 is numerically obtained and the maximal excited state population, \( \max \left[ \rho_{f0f0} + \rho_{h0h0} + \rho_{e-1e-1} \right] \), is shown with dashed lines in Figure 4.3 (a) and (b).

With complete information regarding the energy of the fluxonium excited states, we determine the relaxation rate of the \(|g_{-1}\rangle\) state by performing time-resolved measurements [103]. We use the frequency values obtained from the Raman spectroscopy
and perform a pulse sequence which consists of three sequential $\pi$-pulses at the transition frequencies $(E_{f_0} - E_{g_0})/h$, $(E_{h_0} - E_{f_0})/h$ and $(E_{h_0} - E_{e_{-1}})/h$ to prepare the system in the $|e_{-1}\rangle$ state. At the end of this procedure, the system relaxes into the $|g_{-1}\rangle$ state, on the time scale of the plasmon $T_1$ ($\sim 600$ ns). On a longer timescale, the system relaxes back to $|g_0\rangle$. For $t_{\text{wait}} \gg T_{1e_0}$, the reduction in $|g_{-1}\rangle$ population follows an exponential decay with $T_{1g_{-1}} = 20 \mu$s.

### 4.4 Lifetime spectroscopy

Due to the high $E_J/E_C$ ratio, devices 1 and 2 lack flux insensitive sweet spots at zero and half flux. In order to fully characterize the coherence properties of the qubit and demonstrate coherent control between the fluxon states, we reduced the $E_J/E_C$ ratio in device 3. The overlap between the fluxon wavefunctions is made sufficiently large to directly observe the transition with a one-photon drive, which comes at the cost of increased sensitivity to different relaxation mechanisms. The low frequency, two-tone spectroscopy data for device 3 is shown in Figure 4.4. At $\Phi_{\text{ext}}/\varphi_0 = -\pi$, the spectrum shows a flux-insensitive fluxon transition, where we perform coherence measurements and find $T_1 = 220$ ns, $T_{2\text{Ramsey}} = 380$ ns and $T_{2\text{Echo}} \approx 2T_1$ indicating that the qubit dephasing is dominated by qubit relaxation.

By changing $\Phi_{\text{ext}}$, we measure $T_1$ of the fluxon transition as a function of qubit frequency. The data show an increase in $T_1$ as the qubit frequency is increased to a maximal value of $7 \mu$s for frequencies between 2-3 GHz. Upon further increasing the qubit frequency, $T_1$ decreases by an order of magnitude (Figure 4.4c).

To understand the $T_1$ frequency dependence, we take into account inductive and capacitive loss mechanisms, which can be described with the following expressions,
Figure 4.4: (a) Low frequency spectroscopy data from device 3. (b) $T_1$ (red) and $T_{2\text{Ramsey}}$ (blue) data taken at $\Phi_{\text{ext}}/\varphi_0 = -\pi$. (c) $T_1$ as a function of qubit frequency. The lines represent the theory fits for total (red), inductive (blue) and capacitive (green) $T_1$. The $T_1$ values were obtained with both pulsed and mixed state driving. Measurements using both types of excited state preparation at the same flux gave the same value of $T_1$. Figure adapted from [43].

\[
\Gamma_{\text{ind}} = \frac{E_L}{\hbar Q_L} \left( \coth \left( \frac{\hbar \omega_q}{2k_B T} \right) + 1 \right) \left| \langle g_{-1} | \hat{\varphi} | g_0 \rangle \right|^2, \tag{4.10}
\]

\[
\Gamma_{\text{cap}} = \frac{\hbar \omega_q^2}{8E_C Q_C} \left( \coth \left( \frac{\hbar \omega_q}{2k_B T} \right) + 1 \right) \left| \langle g_{-1} | \hat{\varphi} | g_0 \rangle \right|^2, \tag{4.11}
\]
where \( |\langle g_{-1}| \hat{\phi} |g_0 \rangle|^2 \) is the transition matrix element between the fluxon states, \( Q_L \) and \( Q_C \) are the inductive and capacitive quality factors, respectively, \( k_B \) is the Boltzmann constant, \( T \) is the temperature and \( \omega_q \) is the fluxon transition frequency [65]. Based on previously reported measurements [44], the lifetime limitation from non-equilibrium quasiparticles is at least an order of magnitude larger than the observed relaxation times at all frequencies and is therefore not considered. Radiative loss due to the Purcell effect [45] is only significant when the qubit frequency is within \( \sim 50 \text{ MHz of } \omega_r/2\pi = 7.5 \text{ GHz}. \) Figure 4.4c shows the measured \( T_1 \) (blue markers) values along with the fitted \( T_1 = (\Gamma^{-1}_{\text{cap}} + \Gamma^{-1}_{\text{ind}})^{-1} \) (red line). The fit of \( T_1 \) vs \( \omega_q \) in Figure 4.4, gives \( Q_L = 39,000 \) and \( Q_C = 15,100 \), where the lifetime at low \( \omega_q \) is dominated by inductive loss and at high \( \omega_q \) by capacitive loss. The inductor can be modeled as a lossless inductor in series with a frequency dependent resistor, where \( R = \omega L/Q_{\text{ind}} \) corresponds to \( R = 27 \text{ m}\Omega \) at \( \omega/2\pi = 550 \text{ MHz.} \) The possible sources of the inductive loss can arise from a finite contact resistance between the NbTiN wire and the Al Josephson junction leads, loss from charge impurities on the surface of the wire, or some intrinsic loss from the bulk NbTiN material. In future devices, the geometry of the Al/NbTiN contact and nanowire dimensions could be modified to better determine what limits the inductive quality factor. Improvements to \( Q_C \) could be made by moving to a 3-D architecture, where the electric field participation at lossy interfaces is reduced [104].
Chapter 5

Protected qubits: The $0-\pi$ circuit

In Chapter 2, we provided a brief overview of several different two node superconducting qubits as well as introducing the $0-\pi$ qubit. The idea of constructing intrinsically protected qubits with disjoint wavefunctions has only been recently explored experimentally but has already yielded several exciting results including superconducting qubit lifetimes in excess of 8 ms [47, 48]. In this chapter, we discuss the experimental progress towards realizing a $0-\pi$ qubit. We also discuss the practical challenges of coupling to the qubit and different design trade-offs made in fabricating the circuit. We measure and present preliminary spectroscopy measurements as well as measurements of the qubit excited state lifetimes on a “soft” $0-\pi$ device.

5.1 $0-\pi$ readout coupling

Until very recently, no previous theoretical proposal [53, 56–58] has considered the effects of coupling to the $0-\pi$ before. Several factors make coupling to the $0-\pi$ non-trivial. The coupling method must be carefully chosen, as all four nodes need to be driven in order to only couple to a single mode of the circuit. Inductive coupling to the circuit presents two challenges. First, the addition of the coupling inductors breaks the compactness of the $\theta$ mode by introducing an inductive energy term with
CHAPTER 5. PROTECTED QUBITS: THE \(0-\pi\) CIRCUIT

a \(\theta\) dependence to the \(0-\pi\) Hamiltonian. Additionally, the inductive coupler acts as a shunt in parallel across the superinductors, renormalizing their values. In order to maintain a large effective superinductance value, the couplers themselves would need to be superinductors. Capacitively coupling allows us to circumvent these issues. In particular, we choose to couple to the \(\phi\) mode. Based on recent theoretical calculations [105], the \(\phi\) mode will have a much larger dispersive shift (as compared to \(\theta\) mode coupling), making it much better suited for readout.

Figure 5.1: The \(0-\pi\) circuit with capacitive coupling to the \(\phi\) mode.

As originally considered [53], no information was provided on how coupling modifies the \(0-\pi\) spectrum. If we include even a very modest gate capacitance, \(C_g = 3\) fF and recalculate the spectrum, shown in Figure 5.3, several distortions appear. The most concerning effects are, the breaking of the near degeneracy of the ground and first excited state levels, as well as the reduction in the second excited state frequency. This distortion can be understood as an increase in the effective mass along the \(\phi\) direction, resulting in a localization of the wavefunctions into the different potential wells, making the qubit spectrum more closely resemble that of a fluxonium qubit. Although the nominal qubit transition frequency would be comparable to \(k_B T\) in a true \(0-\pi\) qubit, sideband cooling [106] could be utilized to initialize the qubit in
its ground state, provided that the thermal occupation of the second excited state is negligibly small. The approximation of a unoccupied second excited state will break down as $C_g$ is increased and a new scheme for qubit initialization will have to be determined.

### 5.2 Soft 0−π regime

The original 0 − π target superinductance of $\sim 3 \, \mu\text{H}$ leads to an additional design constraint. If a JJ array is used to build the superinductor, the self-resonant modes of the array will set a hard limit on the number of junctions in the array. As was found in [39], the mode frequencies of a JJ array are given by

$$\omega_k = \omega_p \sqrt{\frac{1 - \cos \frac{\pi k}{N}}{\frac{C_0}{2C_J} + \left(1 - \cos \frac{\pi k}{N}\right)},}$$

(5.1)
CHAPTER 5. PROTECTED QUBITS: THE 0–π CIRCUIT

where \( \omega_p \) is the single junction plasma frequency, \( N \) is the number of junctions, \( C_J \) is the individual junction capacitance, \( C_0 \) is each junction’s parasitic ground capacitance, and \( k \) is the mode number. The array junction inductance needs to be kept low, about 3 nH/junction to avoid dephasing from quantum coherent phase slips [52], which implies an array length of \( \sim 1000 \) junctions. For carefully constructed junctions with a \( C_0/C_J = 50 \) and typical values of \( \omega_p/2\pi = 20 \) GHz, the first junction array mode would appear at 450 MHz, near and above which, the impedance of the superinductor is no longer linear. Construction of such a large superinductor with kinetic inductance materials involve fabricating wire lengths in excess of 5 mm (based on the \( L_k \) values from the NSFQ experiments). Due to these challenges, we have chosen a parameter regime dubbed the “soft” 0–π. The “soft” 0–π parameter set is chosen such that \( E_{J,E_{CJ}} > E_{C,E_L} \) by roughly one order of magnitude rather than the 2-3 orders in the original proposal. In this regime, the near degeneracy of the ground and first excited state has been traded away for a reduction in the number of junctions, and for a reduction in the size of the shunting capacitors between nodes 1-3 and 2-4. A near degeneracy of a few hundred MHz can still be achieved near the half-flux quantum sweet spot, which could be a promising spot to operate the qubit.

5.3 Fabrication

The design of the 0–π has gone through several iterations. In order to simultaneously maintain a low \( E_C \) and high \( E_{CJ} \), the distance between the positive and negative nodes of the circuit is spatially large, \( \sim 250 \) \( \mu \)m. This results in a very large flux loop, leaving the circuit more susceptible to dephasing from \( 1/f \) flux noise. An early design of the circuit had a gradiometric flux loop design [98] with wires crossing over a SiN dielectric bridge, in order to reduce the size of the flux loop and the susceptibility
to global flux noise. This design could be revisited in the future, however, presently, the additional fabrication steps for dielectric sputtering and etching seemed to have a detrimental effect on the device fabrication yield. Devices with both JJ arrays and NbTiN nanowires have been fabricated in the hope to find the best-performing design.

The circuit layout must be modified from Figure 5.1 in order to avoid crossing wires. The two nodes can be simultaneously coupled without wire crossings, as the electric field from the coupling capacitor is not completely screened by the first metallic wire as shown in Figure 5.2. The voltage pad dimensions are set to minimize the asymmetry in the gate capacitance $C_g$ between the lead and each of the circuit pads. A coplanar strip-line (CPS) $\lambda/2$ resonator is used to drive the $0 - \pi$ circuit. This requires the addition of a Krytar-4040124 180° hybrid coupler [78] to differentially drive the resonator. Using a CPS resonator obviates the need for an on chip ground
CHAPTER 5. PROTECTED QUBITS: THE 0−π CIRCUIT

plane, which helps reduce the stray capacitance, keeping $E_{CJ}$ large. The hybrid coupler has a limited frequency bandwidth, of 4-12 GHz, and so additional versions of the 0 − π circuit have been fabricated and tested with a CPW resonator that terminates in a CPW-CPS transition [107]. The circuit can be further twisted, see Figure 5.2 to eliminate the crossing of the shunt capacitors in the middle of the circuit.

5.4 Spectroscopy

Several 0 − π devices have been measured, and the data from one of the most recent devices is presented here. This device is $\phi$ mode coupled to a CPS resonator with JJ array superinductors. Using the measurement setup detailed in Chapter 3, we perform two-tone spectroscopy. The resulting data are shown in Figure 5.5, along with a corresponding theory fit overlay. The first plasmon and fluxon transitions (white circles) are accompanied by several additional transitions. There are several copies of the first fluxon transition (black circles), shifted higher in frequency by 2.2 and 4.4 GHz, corresponding exactly to the extracted frequency of the harmonic $\chi$ mode at $\chi = \sqrt{8E_L E_C} \rightarrow 2.2$ GHz. The visibility of these transitions indicates that there is enough disorder in the different circuit elements to couple the $\chi$ mode to the energy levels. In addition to the $\chi$ modes, there is a transition between the excited plasmon and fluxon states, indicating a non-zero thermal population of the fluxon excited state. The ratio of $E_J/E_{CJ}$ is sufficiently large that the sweet spot gap at zero and half a flux quantum is closed, indicating that the fluxon transition will not have a flux-insensitive sweet spot.
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Figure 5.4: The $0-\pi$ wavefunctions near $\Phi_{\text{ext}} = 0$. The color maps indicate the square of the magnitude of the wavefunction, $|\Psi|^2$, for the ground (left), first plasmon (middle), and fluxon (right) excited states, with darker colors indicating smaller wavefunction magnitudes.

5.5 $0-\pi$ lifetime measurements

Although the parameters of this implementation of the $0-\pi$ are not ideal for qubit operation, we can still characterize a relaxation time of the circuit. This particular $0-\pi$ device was recooled to characterize the excited state lifetimes. Using the measurement setup outlined in Chapter 3, we apply $X_{\pi}$ pulses, resonant with the first plasmon and fluxon transitions, and measure the excited state population as a function of wait time to determine $T_1$ as shown in Figure 5.6. As was the case in the NSFQ devices, there is a significant enhancement in the lifetime of the fluxon compared to the plasmon state which can be explained with the comparatively lower matrix element, $\langle 0|\hat{\varphi}|1\rangle > \langle 0|\hat{\varphi}|2\rangle$ (where the state $|1\rangle$ refers to the plasmon excited state and $|2\rangle$ refers to the fluxon). At this flux, $T_2 < 200$ ns and is expected to be limited by flux noise as the fluxon transition has a linear flux dispersion.

Although this $T_1$ is by no means a record value for superconducting qubits, there is certainly room for design improvements. Working at lower fluxon frequencies should help reduce capacitive loss, as well as increasing the qubit-measurement resonator detuning ($\Delta = 2$ GHz in this case). Future $0-\pi$ devices are currently being designed.
CHAPTER 5. PROTECTED QUBITS: THE 0−π CIRCUIT

![Figure 5.5: Two-tone spectroscopy data of a soft 0−π (transitions shown in blue). From fitting the data, we extract device parameters of $E_J = 10.8 \text{ GHz}$, $E_L = 0.79 \text{ GHz}$, $E_C = 0.65 \text{ GHz}$, $E_{CJ} = 1.75 \text{ GHz}$. The white circles mark the fluxon and plasmon transitions and the black circles mark the $\chi$ mode copies. The red circles are the transition between the fluxon and plasmon excited states.]

and tested with a reduced $E_J/E_{CJ}$ ratio in order to reopen the gap at zero and half flux to create sweet spots.
CHAPTER 5. PROTECTED QUBITS: THE $0-\pi$ CIRCUIT

Figure 5.6: The demodulated homodyne voltage from the measurement circuit as a function of wait time, $t$. $T_1$ is found from the exponential decay for the plasmon (left) and fluxon (right) states. These data were obtained near $\Phi_{\text{ext}} = \pi/4$, with a plasmon frequency of $\omega/2\pi = 6.5$ GHz and a fluxon frequency of $\omega/2\pi = 5.5$ GHz.
Chapter 6

Elimination of Static Crosstalk

In superconducting qubits, the leading approaches for implementing two qubit gates involve either fixed or tunable frequency qubits. Gates based on tunable qubits have the advantage of being fast and having high on/off ratios [108]. By keeping a large detuning between qubits when they are idle, the interaction strength can be greatly suppressed. However, these features come at the price of having an increased sensitivity to flux noise, as well as increasing the complexity of wiring and tuning up all single and two qubit gates.

An alternative scheme uses fixed frequency qubits with linear coupling to achieve a microwave activated two qubit gate [109]. This has the advantage of a simple microwave setup, requiring no additional bias lines for the fast flux pulses, as well as utilizing fixed frequency qubits which are insensitive to magnetic flux noise. A simplified Hamiltonian of two driven qubits coupled with a linear element can be written as

\[
H/\hbar = \varepsilon_1 (XI + (m_{12} - \nu_1)IX - \mu_1ZX - m_{12}\mu_2XZ) + \varepsilon_2 (IX + (m_{21} + \nu_2)XI + \mu_2XZ - m_{21}\mu_1ZX) + \omega_1 ZI/2 + \omega_2 IZ/2 - \zeta ZZ/4,
\]  

(6.1)
CHAPTER 6. ELIMINATION OF STATIC CROSSTALK

where \( AB \equiv \sigma_A \otimes \sigma_B \), \( \varepsilon_{1(2)} \) and \( \nu_{1(2)} \) are the strengths and frequencies of the two drives, \( m_{12(21)} \) are the spurious electromagnetic couplings between the qubits, and \( \mu_{1(2)} = J/\Delta \) is the interaction strength, \( J \), divided by the qubit frequency detuning \( \Delta \) \[110\]. This Hamiltonian describes the so called “cross-resonance” gate \[111\] in which the control qubit is driven at the target qubit’s frequency. The target qubit will experience an X rotation based on the state of the control qubit. The additional IX and XI terms represent the spurious electromagnetic crosstalk and the Stark shift on the control qubit from the off resonant drive at the target frequency. The final term in eqn. 6.1, \( \zeta_{ZZ}/4 \) describes a static, always on, quantum crosstalk between the qubits. This term describes the constant rate of phase accumulation between the two qubits which will lead to dephasing of the system. When multiple qubits become entangled, keeping track of the ZZ terms between all pairs of qubits quickly complicates any type of echo correction schemes. Additionally, this type of crosstalk limits the fidelity of XX parity measurements that would be used in a surface code quantum error correction scheme \[112, 113\]. Therefore eliminating the ZZ term in the Hamiltonian would be a major improvement in the system.

6.1 ZZ crosstalk

The ZZ crosstalk arises from inclusion of the higher energy levels of the transmon qubits. By including all energy terms up to fourth order, the strength of the ZZ interaction for two transmons coupled by a bus cavity \[114, 115\] can be expressed as

\[
\zeta = 2g_1^2g_2^2 \left[ \frac{1}{(\Delta_{12} + \alpha_2)\Delta_1^2} + \frac{1}{(\Delta_{21} + \alpha_1)\Delta_2^2} + \frac{1}{\Delta_1\Delta_2^2} + \frac{1}{\Delta_1^2\Delta_2} \right], \quad (6.2)
\]

where \( g_i \) is the strength of the coupling between the \( i^{th} \) transmon and the bus cavity, \( \Delta_i \) is the detuning between the \( i^{th} \) transmon and the bus cavity, \( \Delta_{12} \) is the frequency
detuning between the qubits (with $\Delta_{21} = -\Delta_{12}$), and $\alpha_i$ is the anharmonicity of the $i^{th}$ transmon. More intuitively, the ZZ interaction arises from the frequency shift of the system when one of the qubits is in it’s excited state,

$$\zeta = \omega_{11} - \omega_{01} - \omega_{10}. \quad (6.3)$$

This intuitive picture is suggestive, as it alludes to a method of measuring the $\zeta$ with a cross-Ramsey experiment, described in Section 6.3.

A novel method for canceling $\zeta$ is to use a second coupling element to destructively interfere the $\zeta$ terms arising from each coupler. The Hamiltonian describing such a system is given by,

$$H/h = \sum_{i=1,2,\pm} \left( \omega_i a_i^\dagger a_i - \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i \right) + \sum_{j=\pm} \sum_{i=1,2} g_{ij} \left( a_i^\dagger a_j + a_i a_j^\dagger \right), \quad (6.4)$$

where the subscripts 1, 2, $\pm$ denote each of the transmons and couplers, with $\omega_i$, $\alpha_i$, and $g_{ij}$ as the frequencies, anharmonicities, and couplings of the $i^{th}$ object (only including $g_{1,\pm}$ and $g_{2,\pm}$). By numerically diagonalizing eqn. 6.4, we can extract the strength of $\zeta$ as a function of the qubit and resonator couplings, detunings, and anharmonicities [115]. In particular, the set of parameters for which $\zeta = 0$ occurs when $\Delta_{12} < \alpha_1, \alpha_2$ and $\omega_- < \omega_1, \omega_2 < \omega_+$. In theory, all components could be frequency tunable, but for coherence purposes, fixed frequency transmons are optimal, and only one coupler frequency needs to be tuned to achieve $\zeta = 0$. As fabrication processes are improved, reducing the spread in $E_J$ (and therefore $\omega_i$), one could imagine having both couplers fixed in frequency, eliminating the need for any on-chip flux bias lines.
6.2 Improved device design

The first generation of zero-ZZ devices [115] had several design parameters to be improved, first among them, the ratio of $E_J/E_C$. While working in the straddling regime, when the two transmon frequencies sitting between the two coupler frequencies, the appropriate choice of the remaining device parameters is key. In the first iteration of the zero ZZ device, one of the main design considerations was keeping the qubits and couplers well below the readout resonator frequency. Additionally, given the variations in $E_J$ at the time of fabrication of the first device, the designed $E_C$ was around -400 MHz, to increase the yield of devices without frequency collisions (e.g. $\omega_1 = \omega_2 - \alpha_2$). Having a large $E_C$ also has the added benefit of reducing leakage out of the computational subspace. The readout resonator frequency, generally speaking, should be as high as practically possible to eliminate thermal occupation of photons in the cavity which lead to qubit dephasing [75]. “Practical” has several different considerations including maintaining a reasonable dispersive shift, $\chi$, so that high fidelity readout can be achieved, and placing the resonators in the frequency band of low noise amplifiers (HEMTs, JPCs, TWPAs). This currently sets the upper limit at around 8 GHz. The first device was designed to have qubits detuned from the readout resonator by about 3 GHz which puts the qubits at 5 GHz. When calculating the $E_J$ for a fixed $E_C$ and $\omega/2\pi = 5$ GHz, we find

$$\omega_{01}/2\pi = \sqrt{8E_JE_C - E_C} \Rightarrow E_J = 9.1 \text{ GHz},$$  \hspace{1cm} (6.5)
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which gives a ratio of $E_J/E_C \approx 23$. When we consider the charge dispersion of such a system (eqn. 2.16)

$$\epsilon_1 = 2^9 E_C \sqrt{\frac{2}{\pi}} \left( \frac{E_J}{2E_C} \right)^{5/4} e^{-\sqrt{8E_J/E_C}} \Rightarrow \epsilon_1 \approx 1 \text{ MHz}. \quad (6.6)$$

With the given $E_J/E_C$ and $\epsilon_1$, the transmons are approaching the Cooper-pair box regime, and the first version of the zero ZZ devices suffered from short $T_\varphi$ due to charge noise. In addition to the large susceptibility to charge noise, the frequency for the lower coupler at the $\zeta = 0$ point was $\omega_\perp \approx 4$ GHz. With an effective cavity temperature of about 90 mK, this severely limited the iSWAP gate fidelity to $< 85\%$. As the lower coupler is formed with what amounts to a tunable transmon, a SQUID loop with large capacitance, this gives an even lower $E_J/E_C$ than the qubits. Although thermal occupation could potentially be addressed with better filtering and attenuation on the flux line, a better solution, solving both the charge dispersion and thermal issues, involves moving the qubits and couplers higher in frequency. Frequency crowding and relaxation could set an upper bound on how close the qubits can be moved to the readout resonators, however, future designs could incorporate a Purcell filter [79, 116] allowing for smaller detunings between the readout resonators and qubits.

With these considerations, we fabricate a second device with the parameters in Table 6.1. The primary considerations included both reaching $E_J/E_C = 50$ and increasing $g_{ij}$ between each of the couplers and the qubits. Traditionally, the coupling strength has been set as a tradeoff between maintaining large qubit-qubit interaction, $J$, and minimizing crosstalk between the qubits. With fixed linear couplers, there is a fundamental tension that comes from the fact that the strength of $J$ and $\zeta$ are both proportional to the coupling. With the two coupler design, we can now consider
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increasing $g_{ij}$ for each qubit, while $\zeta$ can be nulled out with the tunable coupler. Minor additional changes include decreasing $E_C$ and making asymmetric junctions in the coupler to reduce the flux sensitivity [117]. To have some estimate before fabricating the devices, we simulated the device design in Maxwell 3D Electrostatic solver, the details of which are in Appendix. B. This gives us a rough estimate for the capacitances of the device, from which, we can write a circuit model to calculate the $\beta$ for each coupler/resonator and $E_C$ for the qubits and couplers. Based on the two-tone spectroscopy measurements, the simulated $E_C$’s are within 5% of the experimentally measured values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q$_1$ frequency</td>
<td>$\omega_1/2\pi$</td>
<td>6.143 GHz</td>
</tr>
<tr>
<td>Q$_1$ anharmonicity</td>
<td>$\alpha_1/2\pi$</td>
<td>330 MHz</td>
</tr>
<tr>
<td>Q$_1$ relaxation time</td>
<td>$T_1^{(1)}$</td>
<td>12.5 $\mu$s</td>
</tr>
<tr>
<td>Q$_1$ coherence time</td>
<td>$T_{2E}^{(1)}$</td>
<td>22.5 $\mu$s</td>
</tr>
<tr>
<td>Q$_2$ frequency</td>
<td>$\omega_1/2\pi$</td>
<td>6.427 GHz</td>
</tr>
<tr>
<td>Q$_2$ anharmonicity</td>
<td>$\alpha_1/2\pi$</td>
<td>330 MHz</td>
</tr>
<tr>
<td>Q$_2$ relaxation time</td>
<td>$T_1^{(1)}$</td>
<td>7.0 $\mu$s</td>
</tr>
<tr>
<td>Q$_2$ coherence time</td>
<td>$T_{2E}^{(1)}$</td>
<td>9.3 $\mu$s</td>
</tr>
<tr>
<td>Bus cavity frequency</td>
<td>$\omega_+/2\pi$</td>
<td>7.073 GHz</td>
</tr>
<tr>
<td>Maximum coupler frequency</td>
<td>$\omega_{max}/2\pi$</td>
<td>7.19 GHz</td>
</tr>
<tr>
<td>Coupler anharmonicity</td>
<td>$\alpha_-/2\pi$</td>
<td>290 MHz</td>
</tr>
<tr>
<td>Q$_1$-bus coupling</td>
<td>$g_{1+}/2\pi$</td>
<td>102 MHz</td>
</tr>
<tr>
<td>Q$_2$-bus coupling</td>
<td>$g_{2+}/2\pi$</td>
<td>102 MHz</td>
</tr>
<tr>
<td>Q$_1$-tunable coupler coupling</td>
<td>$g_{1-}/2\pi$</td>
<td>85 MHz</td>
</tr>
<tr>
<td>Q$_2$-tunable coupler coupling</td>
<td>$g_{2-}/2\pi$</td>
<td>85 MHz</td>
</tr>
</tbody>
</table>

Table 6.1: Parameters for improved zero ZZ device.

6.2.1 Device fabrication

We provide a high level summary of the device fabrication, with the fabrication details listed in Appendix. A. A single photolithography step was performed to pat-
Figure 6.1: Theoretical plots for the strength of $\zeta$ (blue) and $J$ (orange) as a function of the detuning between the lower coupler and the lower frequency qubit. The left plot uses the device parameters found in [115], and the right plot uses the values from Table 6.1.

tern the readout resonators, coupler, and qubit pads followed by an SF$_6$:Ar etch and clean to remove extraneous Niobium. The JJ’s were fabricated using a bilayer of MMA/PMMA e-beam resist and exposed in the Elionix e-beam writer and the “Man-
hattan” style junctions were evaporated in the Plassys. A picture of a lithographically similar device is shown in Figure 6.2.

6.2.2 Spectrum characterization

Upon cooling the device, the first set of characterization measurements are two-tone spectroscopy to find the qubits. We repeat this measurement for different values of the coupler flux bias to find the straddling regime, when the tunable coupler drops below the qubits. This also allows a direct measurement of $g_i$ for the tunable coupler when it passes through and hybridizes with each of the two qubits.
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Figure 6.2: An optical image of the zero ZZ device. The two transmons are highlighted in blue, the tunable coupler in red, the fixed frequency coupler in green (connected outside image), and the coupler bias line in purple. The readout resonator leads for each qubit extend beyond the top and bottom of the image.

Figure 6.3: Two-tone spectroscopy data near the crossing of the qubit and the tunable coupler. From the hybridization of the qubit and coupler energy levels, we extract a $g_{1,2-}/2\pi = 85$ MHz.
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6.3 ZZ crosstalk map

Once the flux bias is set such that the tunable coupler is below the two qubits, we can begin to characterize the strength of the ZZ interaction. As previously alluded to, the magnitude of $\zeta$ is related to the frequency shift of one qubit based on the state of the second qubit. We can perform a Ramsey pulse sequence, consisting of two $X_{\pi/2}$ pulses separated by a wait time, to measure the frequency of one qubit (target), and then repeat the sequence with an additional $X_{\pi}$ pulse on the other qubit (control) before and after the Ramsey interferometric pulses to extract the difference frequency when the control is its ground or excited state. By repeating the measurement with the roles of the target and control qubits reversed, we can then extract the strength of the ZZ interaction. We repeat the cross Ramsey experiment as a function of the lower coupler frequency to find the point, or points, where $\zeta = 0$. We also note that having two solutions for $\zeta = 0$ is not universal, and for different parameter sets there can either be one or no solutions.

6.4 Single qubit gate tuneup

Once the zero ZZ points are found, the flux bias is fixed to the point with higher $J$ (see Figure 6.1). We perform several single qubit calibrations on each qubit, which include frequency calibration, amplitude for each of the pulses ($X_{\pi,\pm\pi/2}$ and $Y_{\pi,\pm\pi/2}$), and correcting the azimuthal angle between the two quadratures coming from the generator [118]. All pulses are generated with a vector signal generator set at the qubit frequency and gated with a Gaussian shape pulse with a width of $\sigma = 8$ ns generated with an arbitrary waveform generator (AWG). Calibration of the pulse amplitudes can be performed by applying a single $\pi/2$ pulse and then a sequence of $n$,
\[ \zeta \text{ plotted vs. coupler flux bias. } \zeta \text{ is obtained via a cross-Ramsey measurement, as described in the text. } \omega_\perp \text{ is decreasing with increasing flux bias. The } \zeta = 0 \text{ point at 1.11 V is used for all iSWAP and CR measurements.} \]

\[ \pi \text{ or } \pi/2 \text{ pulses around either the X or Y axis and then measuring the Z projection. If the pulses have the correct amplitude, then the qubit state will remain on the equator of the Bloch sphere and have equal probability of a Z projection of } |0\rangle \text{ or } |1\rangle. \text{ If there is a slight amplitude error causing under/over rotation, the state will move off the equator causing an imbalance in average measured state. This under/over rotation can then be fixed by adjusting the amplitude of the modulating Gaussian pulse from the AWG. A DRAG calibration [119] is also performed to reduce leakage into the second transmon level with the addition of a Gaussian derivative shaped pulse in the Q quadrature of the vector generator. Once the calibrations have been performed, measurements of all combinations of two sequential } I, X_\pi, X_{\pi/2}, Y_\pi, Y_{\pi/2} \text{ pulses are performed in an “AllXY“ [87] measurement (shown in Figure 6.5). This combination of gates ideally leaves the qubit at the north pole, equator, or south pole of the Bloch sphere. Deviations from these points generally indicate mis-calibrations of the single} \]
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Figure 6.5: Z projection for AllXY measurements of Q$_1$ at 1.11 V flux bias after performing amplitude and phase calibrations of all combinations of cardinal X and Y rotations. Q$_2$ is left idle during these measurements.

qubit gate pulses and the calibration procedures can be rerun to iteratively reduce the deviation from the ideal AllXY values.

6.5 Single qubit randomized benchmarking

Randomized benchmarking (RB) is an efficient tool to characterize qubit performance and is insensitive to so called state preparation and measurement (SPAM) errors making it an easy to implement in a wide variety of systems [120]. We take the set of 24 single qubit Clifford gates, choose 20 sequences of a random ordering of $n$ gates, and repeat each sequence $\sim 10^3$ times. At the end of the sequence a final “undo” Clifford [121] gate brings the system back into an eigenstate of Z, and the qubit is measured. The RB sequence acts as a depolarizing channel for the qubit [28] and for very long sequence lengths, a qubit should have $P|0\rangle = 0.5$ indicating that it is a fully mixed state. The curves are fit to a function of the form $P|0\rangle = A(1-2r)^n + B$ [122],

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where $A$ and $B$ are constants associated with the state preparation and measurement, $n$ is the number of gates, and $r$ is the error per gate which can further be divided by the average number of Cliffords per gate. In our decomposition, the average number of cliffords per gate is 1.875 [115]. To characterize our system, we can perform RB in two different ways, individually and simultaneously. For two qubit systems, the set of clifford gates grows to 11,520 [122, 123] which presents a challenge to efficiently detect correlated errors between the two qubits. Recent investigations have shown that the full set of two qubit cliffords does not have to be investigated and that useful information about the system can be extracted by performing single qubit RB on each subsystem (qubit) simultaneously [124]. Simultaneous RB will give us an additional measurement of the system crosstalk. The ZZ interaction has the effect of shifting one of the qubit frequencies based on the state of the other, and if there is crosstalk present between the two qubits, performing gates, such as $X_\pi$ will excite one qubit causing a Stark shifted energy of the second qubit. If this occurs while a pulse is applied to the second qubit, that pulse will now be off resonant causing a “misrotation”. When we compare the results of simultaneous and individual RB for the two qubits at the $\zeta = 0$ point in Figure 6.6, we see that the error per clifford is the same between the individual and simultaneous RB experiments, with a measured error that is consistent with the lifetime and coherence of the individual qubits. This measurement is an additional confirmation that the ZZ crosstalk has been removed from the system.

6.6 iSWAP gate

A parametric two qubit drive can be activated when the coupling strength between two qubits is modulated at the frequency difference between the qubits [125–129].
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Figure 6.6: Individual and simultaneous randomized benchmarking at the $\zeta = 0$ point. The extracted error per Clifford gate is $4.75 \times 10^{-3}$ for $Q_1$ and $1.25 \times 10^{-2}$ for $Q_2$ where the lower fidelity in $Q_2$ is due to the shorter coherence time. The error per gate for simultaneous RB is within the measurement uncertainty of the individual RB error for each qubit.

Following the description in [130], the two qubit interaction is given by

$$
U_{int} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(gt/2 + \phi) & -i \sin (gt/2 + \phi) & 0 \\
0 & -i \sin (gt/2 + \phi) & \cos (gt/2 + \phi) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

(6.7)

where $g$ is the interaction strength and $\phi$ is the azimuthal angle of the microwave generator controlling the flux pulse. When the interaction is left on for a time $gt = \pi$, the input state $|01\rangle$ is changed to $i|10\rangle$ known as an iSWAP gate, which, with the addition of single qubit gates, can be used to perform a CNOT gate. Although each of the qubits in the device are single junction transmons and should nominally have a fixed frequency, interaction with the tunable coupler effectively dresses the qubits and will cause a small shift in qubit frequency as the tunable coupler is adjusted, allowing
us to perform a two qubit iSWAP gate. We can probe this interaction by applying a sinusoidal drive near $\Delta_{12}$ for varying lengths of time. A $X_{\pi}$ pulse is first applied to $Q_1$, putting the system into the state $|10\rangle$ before the coupler drive is turned on. The coupler pulse consists of a sinusoidal drive modulated with a rectangular pulse having $3\sigma$ (with $\sigma = 5$ ns) rising and falling Gaussian edges. Figure 6.7 shows the Z projection of the qubit states. There is a slight difference in the maximum Chevron swap frequency and $\Delta_{12}$ due to the ac Stark shift arising from the coupler drive pulse [131].

6.6.1 Quantum state tomography

To characterize the fidelity of our iSWAP gate we use quantum state tomography (QST) to prepare a specific quantum state and determine the distance to the ideal state [28]. Before performing QST, the system is calibrated to individually readout the state of each qubit after a single pulse sequence. To do this, the phase of the local oscillator (LO) for each measurement resonator is rotated to move the demodulated
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homodyne readout signal from each qubit into a single field quadrature. For each qubit, we apply either an $X_\pi$ or $I$ pulse before measurement for $\sim 10^4$ trials and histogram the results to generate a readout matrix, $C_i$, which characterizes how often the correct state is identified [129] for the $i^{th}$ qubit. We also perform a “crosstalk calibration” measurement, which prepares the states $|00\rangle$, $|10\rangle$, $|01\rangle$, and $|11\rangle$ and measure the 4x4 matrix, $C_{ct}$, corresponding to the prepared vs. measured two qubit state. The corrected state is given by

$$p_{corr} = R^{-1}p_{meas}, \quad (6.8)$$

with

$$R = C_{ct} \cdot (C_I \otimes C_{II}). \quad (6.9)$$

An example of the readout corrected single shot data for a coupler pulse is shown in Figure 6.8.

Written in the Pauli basis form [28], the density matrix can be expressed as

$$\rho = \frac{1}{2^n} \sum_i^{16} \text{tr} (P_i \rho) P_i, \quad (6.10)$$

where $n = 2$ for a two qubit system and the sum runs over 15 independent measurement correlators $P_i \in \{I, X, Y, Z\}^{\otimes 2} - \{II\}$. In this set, six correlators are “single qubit like”, that is are equivalent to $\langle M_1 I \rangle$ or $\langle IM_2 \rangle$, and the remaining nine are the two qubit correlators, $\langle M_1 M_2 \rangle$. Experimentally, we measure three independent probabilities, $P_{00}$, $P_{10}$, and $P_{01}$ (with $P_{11}=1-P_{00}-P_{10}-P_{01}$), and we can express these
correlators as the following

$$\langle M_1 I \rangle = P_{11} + P_{10} - P_{01} - P_{00}$$

$$\langle IM_2 \rangle = P_{11} + P_{01} - P_{10} - P_{00}$$

$$\langle M_1 M_2 \rangle = P_{11} + P_{00} - P_{01} - P_{10}.$$ (6.11)

To perform tomography, we apply certain gates prior to readout to rotate the measurement basis, explicitly, $Y_{-\pi/2}$ for Pauli X and $X_{\pi/2}$ for Pauli Y, and $I$ for Pauli Z. Once the readout correction matrices (eqn. 6.9) have been measured, we can prepare a specific state, in this case an $X_{\pi}$ is applied to $Q_1$ followed by 105 ns long coupler drive, which should give a $\sqrt{iSWAP}$ gate based on the measurements in Figure 6.8. This sequence produces the maximally entangled state $|\Psi\rangle = 1/\sqrt{2} (|10\rangle + i|01\rangle)$. The
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tomographic readout of this state is shown in the top panel of Figure 6.9 compared to the theoretically predicted state. Due to noise and other systematic readout errors, the measured density matrix may be unphysical, e.g. have negative eigenvalues, and the raw density matrix is fit to obtain the closest physical density matrix [132]. This is then compared to the ideal theoretical density matrix to obtain a state fidelity of 99.47%. The concurrence [28], is defined as

\[ C(\rho) \equiv \max (0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \]

where \( \lambda_i \) are the sorted eigenvalues of the density matrix. From the fit density matrix, we extract maximum concurrence of \( C = 0.985 \) for the prepared state.

6.7 Cross-resonance gate

As an alternative to modulating the coupler bias voltage, one of the qubits can be driven at the second qubit’s frequency to perform a cross-resonance (CR) two qubit gate. As recently explored theoretically [133], the strength of the two qubit interaction, \( \mu \), in the Hamiltonian eqn. 6.1, to third order, can be expressed as

\[ \mu = -\frac{J\Omega}{\Delta_{12}} \left( \frac{\alpha_1}{\alpha_1 + \Delta_{12}} \right) + \frac{J\Omega^3\alpha_1^2}{4\Delta_{12}^3 (\alpha_1 + \Delta_{12})^3} \left( 3\alpha_1^3 + 11\alpha_1^2\Delta_{12} + 15\alpha_1\Delta_{12}^2 + 9\Delta_{12}^3 \right), \]

where \( J \) is the interaction strength, \( \Omega \) the drive strength, \( \alpha_1 \) the anharmonicity of the control qubit, and \( \Delta_{12} \) the qubit frequency detuning. Working at the \( \zeta = 0 \) point, we use \( Q_1 \) as the control qubit and \( Q_2 \) as the target qubit and apply the CR drive. As the ZX rotation depends on the state of \( Q_1 \), we perform the experiment twice, once with an \( X_\pi \) pulse on \( Q_1 \) before the CR drive, and once without the pulse (with \( Q_1 \...
CHAPTER 6. ELIMINATION OF STATIC CROSSTALK

Figure 6.9: Quantum state tomography for the state $|\Psi\rangle = \frac{1}{\sqrt{2}} (|10\rangle + i|01\rangle)$. The closest fit experimental density matrix (bottom) is compared to the theoretical matrices (middle) to obtain $F=99.47\%$. The black bounding rectangles on the state tomography plot (top) indicate the ideal values of each measurement operator.
Figure 6.10: The population of $Q_2$ with the CR pulse applied to $Q_1$ (left), where the blue (red) curves corresponding to the control $Q_1$ in the ground (excited state). A CNOT gate can be realized when the two curves are $180^\circ$ out of phase at 155 ns. Although the fit to eqn. 6.13 is rather poor (right), this is likely due to the approximations in [133] and the fact that the theoretical model only considers one coupler. The second coupler will need to be included to fully capture the dynamics of the system.

assumed to be in $|0\rangle$) and compare the rotations, as shown in Figure 6.10. This can repeated for different strengths of the CR drive pulse.

Looking to future work, the second coupling element needs be included to expand on existing theory to better match the measured $\mu$ vs. drive amplitude, as well as implementing full tomography to characterize the fidelity of this gate and the tuneup of an active cancellation pulse [134], to eliminate additional unwanted drive terms.
Chapter 7

Conclusion and Outlook

In this thesis, we have explored new materials for constructing superconducting qubits, new circuit designs for qubits, and a method for elimination of crosstalk in two qubit gates.

We have fabricated and measured a NbTiN based nanowire superinductance fluxonium qubit, with a qubit lifetime of up to $T_1 = 20 \ \mu$s. The qubit lifetime is limited by inductive losses at qubit frequencies below 3 GHz. The presence of standing wave modes in the nanowire modify the fluxonium spectrum and are visible above 16 GHz. As the modes of the nanowire strongly depend on the parasitic and stray capacitances of the wire, using a shorter wire with higher sheet inductance (for example high quality granular aluminum films with one hundred times larger $L_k = 2 \ \text{nH/□}$ [69, 90]), or integrating the fluxonium into a 3D cavity or waveguide [98], could reduce unwanted capacitances and help to push the nanowire self-resonant modes to higher frequencies. Understanding the origin of the inductive loss is next on the list of short term experiments to be performed on the nanowire fluxonium. Several potential sources of loss are the contact between the Al Josephson junction layers, residue or debris on the surface of the nanowire, non-equilibrium quasiparticles breaking the superconductivity in small localized regions of the wire, and some intrinsic loss source.
CHAPTER 7. CONCLUSION AND OUTLOOK

associated with the NbTiN wire itself. Although previously observed in granular Al nanowires [42], the large superconducting gap in NbTiN makes it unlikely that quasiparticle poisoning is the main source of loss in the wire. The most likely culprit is the NbTiN/Al contact. This could be checked by changing the contact area between the two layers, as well as changing the fabrication recipe related to making this contact. Additionally, different wires could be fabricated with different aspect ratios, in particular making a very thin and wide wire for maximal surface area, to determine if the nanowire loss is related to this interface. An alternative set of investigations could be performed making fluxonium devices with different $L_K$ materials as well. If an appropriate material is found, with a sufficiently large coherence length, $\xi$, it is conceivable to make an entire fluxonium device from a single fabrication layer, making the long inductive nanowire thinner than $\xi$ for a short length to form a weak-link junction in series with the inductive wire.

We have started the experimental exploration of the Kitaev $0 - \pi$ [53] qubit by fabricating and measuring devices which are in the so called soft $0 - \pi$ regime, finding a qubit lifetime of $T_1 = 80 \mu s$. Although the soft $0 - \pi$ regime is currently accessible, there are several non-ideal features, which need to be addressed in future experiments. Maintaining a small $E_{C\theta}$ simultaneous to a large $E_{C\phi}$ pushes the limits of what can be reliably fabricated. Maintaining a large $E_{C\phi}$ is critical to having a flux intensive sweet spot, however this may be moving us into a regime where the qubit has a large charge dispersion and is no longer insensitive to charge noise. The only solution may be a continued efforts and improvements in the device fabrication to reach the true $0 - \pi$ limit which supports disjoint, nearly degenerate states which are insensitive to charge noise. One additional challenge that needs further exploration is the implementation of gates between the disjoint levels of the qubit to build up a universal gate set, although recent theoretical work along this front has been recently
CHAPTER 7. CONCLUSION AND OUTLOOK

pursued [105] with promising results. While its not clear that the $0 - \pi$ will become the eventual replacement to the transmon, the concept of disjoint wavefunctions and near degenerate eigenstates are features of multi-node circuits that will very likely lead to a few order of magnitude increase in the lifetime and coherence of superconducting qubits.

We have additionally fabricated, measured, and demonstrated a two qubit coupling architecture which eliminates the ZZ type of parasitic qubit coupling. The immediate follow up experiment would be to measure a three qubit device and verify that the ZZ crosstalk can be simultaneously eliminated between all qubits. For completeness, this would be a demonstration that “spectator-qubit” errors [112] can be eliminated, opening the door for near term quantum error correction. Improvements to the device design include adding Purcell filters [79, 116] to the readout resonators as well as the flux bias line on the coupler. As fabrication improvements continue to reduce the fabrication spread in $E_J$, it also is conceivable to make devices with two fixed frequency couplers and perform all two qubit gates with a cross-resonance gate, eliminating the need for on chip flux bias.

In the past two decades, we have seen tremendous progress in superconducting qubit lifetimes and coherences as well as an increased understanding in the physics of cQED. Although it’s not clear which system will ultimately realize a significant advance over conventional computation, the steady progress in superconducting qubits continues to keep the field rich with many new avenues of research to explore for the future generation of Quantum Engineers.
Appendix A

Fabrication Recipes

A.1 NSFQ and nanowire 0 – $\pi$ process flow

- NbTiN sputtering on Sapphire (performed by StarCryo).

- Photolithography to protect NbTiN in select regions of device for nanowire patterning and etch NbTiN in unprotected regions.

- Photolithography to cover NbTiN and sputter Nb.

- Photolithography for readout resonators and etch Nb.

- E-beam lithography for nanowire and etch NbTiN.

- Anti-charge layer for Josephson junctions and e-beam lithography for junctions.

A.2 Zero ZZ process flow

- Sputter Nb.

- Photolithography for readout resonators and etch Nb.

- Anti-charge layer for Josephson junctions and e-beam lithography for junctions.
APPENDIX A. FABRICATION RECIPES

- Dicing into 7 x 7 mm chips is usually done after anti-charge layer and before e-beaming the junctions but could be performed before anti-charge if there is a concern of resist aging.

A.3 Niobium preparation

This is the new recipe using TAMI (Toluene, Acetone, Methanol, Isopropanol) cleaning before sputtering Nb in the AJA tool. As of the time of writing this recipe gives consistent $T_1$ times of $\approx 35 \, \mu$s for single junction planar transmons, as well as single photon Q’s above 1 million. C-plane (0001) sapphire substrates with 500 $\mu$m thickness are purchased from CrysTec GmbH.

Substrate TAMI Cleaning

- Clean glass beakers (in order) with Acetone, Methanol and Isopropanol by spraying each beaker and dumping waste rinse twice.

- Sonicate wafer in Toluene for 2 minutes. Remove from Toluene while spraying wafer with Acetone to prevent drying and place into beaker with Acetone.

- Sonicate wafer in Acetone for 2 minutes. Remove from Acetone while spraying wafer with Methanol to prevent drying and place into beaker with Methanol.

- Sonicate wafer in Methanol for 2 minutes. Remove from Methanol while spraying wafer with IPA to prevent drying and place into beaker with Isopropanol.

- Sonicate wafer in Isopropanol for 2 minutes. Remove from Isopropanol with one final spray rinse over beaker and blow dry with nitrogen.
APPENDIX A. FABRICATION RECIPES

Sputtering

- For 100 mm (4”) wafers, clamp the edges of the wafer onto the aluminum chuck (or Nb sputtering chuck for smaller substrates) from AJA tool, blow off dust from the wafer and load into AJA loadlock, and transfer wafer to main chamber when the appropriate vacuum has been reached.

- Run recipe TH_sputter_Nb_200nm_12mTorr_90s_delay, which heats substrate to 200 °C for 10 minutes, allows the substrate to cool for 10 minutes, and sputters 200 nm of Nb onto the substrate. Non-heated recipe is JS_sputter_Nb_200nm_12mTorr_90s_delay.

Dicing

- Prep the wafer for dicing by spinning S1818 or equivalent as a protection layer. Add resist and spin at speed of 4000 RPM for 60 seconds with a ramp rate of 10,000 RPM/s. Bake on center of hotplate at 115 °C for 1 minute.

- Place a tape ring on tape dispenser and place substrate with protective resist face down. Carefully roll dicing tape over ring and substrate to avoid trapped air bubbles, cut away excess tape and load into ADT dicing saw.

- Dice wafer with appropriate recipe.

- Place diced substrate into UV lamp to reduce tackiness of dicing tape.
A.4 Photolithography

Clean and spin coat

- If the chip has photoresist from previous lithography/etching/dicing steps, clean chip in N-Methyl-pyrrolidone (different trade names Nano\textsuperscript{TM} Remover PG, 1165 or MICROPOSIT\textsuperscript{TM} REMOVER 1165) heated to 80 °C for > 30 minutes.

- Spray clean with Isopropanol and blow dry with nitrogen

- Repeat TAMI cleaning steps.

- Spin S1811 at speed of 5500 RPM for 60 seconds, with a ramp rate of 10,000 RPM/s. Bake on center of hotplate at 115 °C for 1 minute.

- Optional: Briefly inspect substrate for resist bubbles or debris with UV filtered microscope.

Exposure and development

Use Heidelberg DWL 66+ (Jadwin) for direct writing on chip. For sapphire substrates, there must be metal on the substrate to use optical focus. If some metal has been etched in previous lithography steps, start with pneumatic focus before finding a region of the substrate with Nb and using optical focus. Standard exposure time for 25 mm chip is \(~17\) minutes.

- 5 mm write head: Focus = \(-76\), filter 10 \%, intensity 90 \% - 95 \%.

- Acetone, Methanol, Isopropanol clean development beakers. Turn on gentle stream of Deionized (DI) water from faucet.
APPENDIX A. FABRICATION RECIPES

- Place chip in Shipley MF-319 for 60 seconds. Either do not move chip during development or gently swirl the chip by hand. In general, agitation during development leads to cleaner features but has the potential for some variation in feature size if the agitation is inconsistent between runs.

- Remove the chip and hold at a 45° angle in the stream of DI water for 30 seconds to rinse off developer. Blow dry with nitrogen.

A.5 Niobium etching

Preform a trial etch run in the South Bay PE2000 plasma etcher. Tool parameters are fixed with gas flow rates of Ar 400 sccm, SF$_6$ 200 sccm, and rf power 50 W. Turn on the gases and let the pressure stabilize for 10 seconds before turning on rf power. During the etch, the chamber pressure should be 95-115 mTorr, the plasma should be a fabulous shade of magenta, and the reflected power should be tuned to 0 W. The approximate etch rate is 0.45 nm/s for Nb and 0.15 nm/s for NbTiN. The standard etch times are 8:45 minutes for 200 nm of Nb and 2:30 minutes for 15 nm of NbTiN.

A.6 Electron beam lithography

The Josephson junction and NbTiN nanowire patterns are exposed with the Elionix ELS-125. The lab bottles of resist are labeled and stored in MNFL cleanroom.

E-beam resist

- Clean chip in NMP for > 30 minutes. Spray clean with Isopropanol and blow dry with nitrogen

- Repeat TAMI cleaning steps.
APPENDIX A. FABRICATION RECIPES

- Spin MMA(8.5) MAA EL13 at speed of 5000 RPM, with a ramp rate of 500 RPM/sec for total of 70 seconds.

- Bake at 175 °C for 2 minutes on the center of the hotplate.

- Spin PMMA950 A3 at speed of 4000 RPM, with a ramp rate of 500 RPM/sec for total of 68 seconds.

- Bake at 175 °C for 30 minutes on the center of the hotplate.

Anti-charging evaporation

After spinning resist, mount chips on a cleaned glass slide or aluminum holder using Kapton tape, blow off the chips with the nitrogen in the C421 lab hood and quickly load into the Plassys evaporator. Follow instructions posted on the tool and evaporate aluminum for the e-beam anti-charging layer. Standard evaporation is 40 nm at a rate of 0.4 nm/s.

Exposure

For Josephson Junctions, load a previous “.fxx” file in Beamer software to perform Proximity Error Correction (PEC). PEC accounts for forward and back scattering of electrons in resist stack, generally reducing the dose in the center of large polygons and increasing the dose near the edges.

- Standard beam parameters are 1.15 µs/dot, 1 nA beam current, 500 µm write field, 50,000 dots per field, 60 µm beam aperture, 1 x 1 shot pitch. Full clear (both PMMA and MMA) dose factor is 1, undercut (MMA only) dose is 0.32.
APPENDIX A. FABRICATION RECIPES

Development

- Remove the aluminum anti-charging layer with MF-319 developer for 4 minutes, no agitation.
- Rinse 1 minute with DI water and dry with nitrogen.
- Develop in 1:3 Methyl Isobutyl Ketone (MIBK):IPA for 50 seconds, with gentle swirling of the sample.
- Place sample in IPA for 10 seconds. Remove sample and spray with IPA from squeeze bottle and dry with nitrogen.
- Visually inspect in microscope, checking for residue or other issues with development/exposure.
- Optional: Run an oxygen descum in Tepla, 2 minutes, 200 W, 500 mTorr. This will etch a few nm of resist making $E_J$ larger, but leaves the chip surface cleaner than not performing a descum.

A.7 Aluminum evaporation

Descum

Load chips into Plassys, following the diagram on the tool which indicates the correct undercut orientation. This descum recipe is one of the most aggressive used among common lab recipes in order to make low loss contacts between Al, Nb and NbTiN.

- Recipe name on the tool is “Manhattan Descum, 45 sec, 400 V”. Parameters: Acceleration voltage $-50$ V and anode voltage 400 V. The sample chuck is
APPENDIX A. FABRICATION RECIPES

rotated and held for 45 seconds at both orientations used during the junction evaporation.

Evaporation

Follow prompts for automated evaporation recipe “Manhattan Andrei Al” or “Manhattan Andrei Al4” depending on which aluminum crucible is used.

- Fill cold trap with liquid nitrogen.
- Evaporate titanium at 0.3 nm/s for 1 minute and wait to reduce chamber pressure to $\sim 2 \times 10^{-8}$ mbar.
- Evaporate 30 nm of aluminum at rate 0.4 nm/s with tilt angle 40° and rotation angle 0°.
- Oxidize in 15%/85% O$_2$/Ar mixture at 200 mbar for 15 minutes.
- Evaporate 60 nm of aluminum at rate 0.4 nm/s with tilt angle 40° and rotation angle 90°.
- Oxidize for 10 minutes to passivate the aluminum surface.

Liftoff

- Spray clean a beaker with Acetone, Methanol and Isopropanol, blow dry with nitrogen and fill with NMP.
- Liftoff in NMP at 80 °C for 1 hour.
- Spray clean a second beaker and fill with NMP. Grab chip in first beaker and without letting it dry, spray with Isopropanol to remove excess metal.
APPENDIX A. FABRICATION RECIPES

- Once most of the metal is removed, place into beaker with fresh NMP and sonicate for 3 seconds.

- Spray with Isopropanol while removing from NMP and blow dry with nitrogen.
Appendix B

Electrostatic simulations for design of zero ZZ device

Import the device CAD file into Maxwell3D, and set solution type to electrostatic. Thicken all metal sheets to 200 nm and set material to perfect conductor. Add 500 µm thick sapphire substrate below, with \( \varepsilon_r = 10.5 \), and a bounding box of vacuum 500 µm above and below the sapphire. Assign voltages, in 10 mV increments to all conductors and then solve capacitance matrix. Depending on the desired accuracy and mesh refinement, the simulation should finish in about 10 minutes.

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Table B.1: Capacitance matrix exported from Maxwell3D (HFSS). The self-capacitance of each island is listed in blue.
Through several trials, we found that the inclusion of a back ground plane or top ground plane 1 mm from the chip has a negligible contribution on the capacitance matrix and for faster simulation speeds, we forgo including it in these models. The transmon pads (6,7 and 4,5) and the ground plane (3) are shown in red, the bus coupler leads (1,2) in blue, the readout resonator leads (8,9) in black and the tunable coupler pads (10,11) in green.
Appendix C

Publications and Presentations


• T. M. Hazard, A. Gyenis, and A. A. Houck. High Kinetic Inductance Based Fluxonium Circuit. APS March Meeting, Los Angeles, California, March 2018.


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