Lorentz Invariance on Trial

Precision experiments and astrophysical observations provide complementary tests of Lorentz invariance and may soon open a window onto new physics. They have already constrained models of quantum gravity and cosmology.

Maxim Pospelov and Michael Romalis

The null result of the celebrated 1887 Michelson–Morley experiment was surprising and difficult to explain in terms of then prevalent physics concepts. It required a fundamental change in the notions of space and time and was finally explained, almost 20 years later, by Albert Einstein's special theory of relativity. (See the May 1987 special issue of PHYSICS TODAY devoted to the centennial of the experiment.) Special relativity postulates that all laws of physics are invariant under Lorentz transformations, which include ordinary rotations and changes in the velocity of a reference frame. Subsequently, quantum field theories all incorporated Lorentz invariance in their basic structure. General relativity includes the invariance through Einstein's equivalence principle, which implies that any experiment conducted in a small, freely falling laboratory is invariant under Lorentz transformations. That result is known as local Lorentz invariance.

Experimental techniques introduced throughout the 20th century led to continued improvements in tests of special relativity. For example, 25 years ago, Alain Brillet and John L. Hall used a helium–neon laser mounted on a rotary platform to improve the accuracy of the Michelson–Morley experiment by a factor of 4000. In addition to the Michelson–Morley experiments that look for an anisotropy in the speed of light, two other types of experiments have constrained deviations from special relativity. Kennedy–Thorndike experiments search for a dependence of the speed of light on the lab's velocity relative to a preferred frame, and Ives–Stilwell experiments test special relativistic time dilation.

In 1960, Vernon Hughes and coworkers and, independently, Ron Drever conducted a different kind of Lorentz invariance test.[1] They measured the nuclear spin precession frequency in lithium-7 and looked for changes in frequency or linewidth as the direction of the magnetic field rotated, together with Earth, relative to a galactic reference frame. Such measurements, known as Hughes–Drever experiments, have been interpreted, for example, in terms of a possible difference between the speed of light and the limiting velocity of massive particles.[2]

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Why bother?

Theorists and experimentalists in disciplines ranging from atomic physics to cosmology have been increasingly interested in tests of Lorentz invariance. The high sensitivity of experimental tests combined with recent advances in their theoretical interpretation allows one to probe ultrashort distance scales well beyond the reach of conventional particle-collider experiments. In fact, both the best experiments and astrophysical observations can indirectly probe distance scales as short as the Planck length $L_{\rm Pl} = (G\hbar/c^3)^{1/2} \sim 10^{-35}$ m. Experiments that probe such short scales can constrain quantum gravity scenarios.

The breaking of Lorentz symmetry enables the *CPT* symmetry, which combines charge conjugation (*C*), parity (*P*), and time-reversal (*T*) symmetries, to be violated. In conventional field theories, the Lorentz and *CPT* symmetries are automatically preserved. But in quantum gravity, certain restrictive conditions such as locality may no longer hold, and the symmetries may be broken. The breaking of *CPT*, combined with baryon-number violation, could be the source of the dynamically generated dominance of matter over antimatter in the universe. Unlike a more conventional scenario involving only *CP* violation, baryogenesis based on *CPT* violation would not require a departure from thermal equilibrium. (See the article by Helen Quinn, PHYSICS TODAY, February 2003, page 30.)

Cosmology provides an additional important impetus to look for violations of Lorentz symmetry. The recognition that the universe is dominated by dark energy suggests a new field—known as quintessence—that permeates all space. The interaction of that field with matter would manifest itself as an apparent breaking of Lorentz symmetry.

It could be argued on aesthetic grounds that the Lorentz and *CPT* symmetries should be preserved. Such arguments, however, do not find support in the history of physics. Nearly all known or proposed symmetries, such as parity and time reversal, electroweak symmetry, chiral symmetry, and supersymmetry, are spontaneously broken. Whatever the true origin of Lorentz or *CPT* breaking may be, the fact that it hasn't yet been observed means it must be small at the energy scales corresponding to known standard-model physics.

Effective field theory

How can one break Lorentz invariance in a controllable way? The least radical approach would be to assume that low-energy physics can be described by the Lorentz-invariant dynamics of the standard model plus a number of possible background fields. Those fields, taken to be constant or slowly varying, are vectors or tensors under Lorentz transformations and are coupled to ordinary particles in such a way that the whole Lagrangian remains invariant. In that framework, called an effective field the-

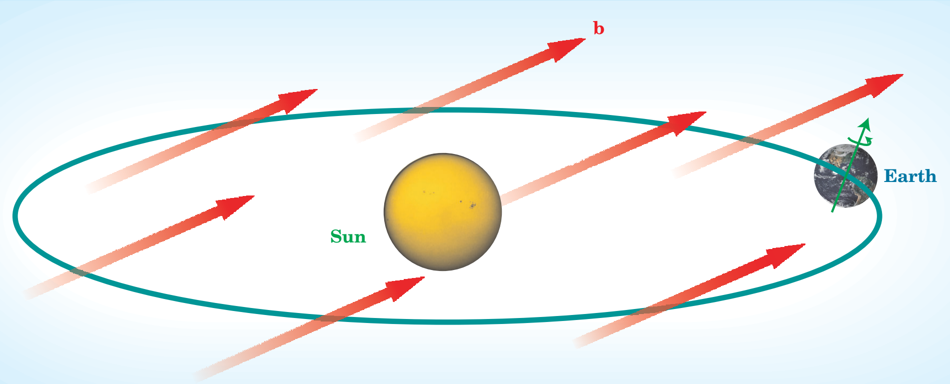


Figure 1. A uniform background vector b defines a preferred direction in space and so violates Lorentz invariance. In effective field theories like those discussed in the text, **b** is related to the three-vector part of a four-vector.

ory, the violation of Lorentz invariance is caused by nontrivial background fields such as the field illustrated in figure 1. The Lorentz violation therefore also appears as a spontaneous symmetry breaking.

One can classify all possible interaction operators by looking at their dimensions. In the natural units $\hbar = c = 1$, all interaction operators are energies raised to some positive power called the dimension of the operator. The lowest-dimension operators generally give the largest Lorentz-violating effects. The coefficient of a dimension-D operator in the Lagrangian has units of energy raised to the power $4-D$.

Effective-field-theory expansions have been widely applied in particle physics since Enrico Fermi, in 1933, parameterized the then-unknown Lagrangian of weak interactions by local operators. Effective theories can preserve all of a field theory's desirable features, including microcausality, unitarity, gauge invariance, renormalizability, spin statistics, and energy–momentum conservation. The effective theory approach to Lorentz violation was advocated by Alan Kostelecky (Indiana University) and coworkers,[3] who developed a systematic generalization of other theoretical descriptions of Lorentz-violating effects.[4–6] Applying the effective-field-theory approach to quantum electrodynamics, one finds that the lowest relevant operator dimension is three and that the QED Lagrangian may be expressed in terms of three dimension-three interactions:

$$\mathcal{L}^{(3)}_{\text{QED}} = -b_\mu \bar{\psi}\gamma^\mu\gamma_5\psi - \frac{1}{2} H_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi - k_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \frac{\partial}{\partial x^\alpha} A_\beta, \quad (1)$$

where ψ is the electron Dirac spinor, A_μ is the electromagnetic vector potential, the four-index ε is the totally antisymmetric Levi-Civita tensor, γ^μ are Dirac gamma matrices, and γ_5 and $\sigma^{\mu\nu}$ are standard combinations of those gamma matrices. The fields b_μ, k_μ, and $H_{\mu\nu}$ are external vector and antisymmetric tensor backgrounds that introduce a preferred frame and therefore break Lorentz invariance. The first and third terms change sign under the *CPT* operation and therefore violate *CPT* symmetry.

One can extend the QED Lagrangian by adding other standard-model fields or higher-dimension interactions. Operators of dimension five and higher can lead to significant modifications of the dispersion relations for particles at high energies.[7]

The effective-field-theory language allows scientists to systematically study violations of Lorentz invariance. They can assign to experiments a figure of merit that depends on the experimental sensitivity to the Lorentz-violating effects. The effective-field-theory framework also provides general guidance as to the types of effects one would observe. Among those effects are rotational dependences of spin-precession frequencies or of the speed of light (such effects can be measured in low-energy experiments), *CPT* violations in neutral meson systems, and effects that Lorentz violation would imprint on astrophysical observations.

Comagnetometers

Modern descendants of Hughes–Drever experiments provide very stringent constraints on many possible Lorentz-violating parameters. Consider the fermionic part (that is, the first two terms) of the Lagrangian given in equation 1. After adding the usual interaction between the magnetic field **B** and the magnetic dipole moment $\mu(\mathbf{S}/S)$ of a particle with spin **S**, and then expressing terms in a nonrela-

Comagnetometer Measurements

To understand more precisely the Lorentz-violating effects in comagnetometer experiments, which compare the precession of two particles, consider two nuclei with magnetic moments μ_1 and μ_2. Assume that they couple to the same Lorentz-violating field **b** but with different coupling coefficients β_1 and β_2. The precession frequencies of the two nuclei are given by

$$h\nu_1 = 2\mu_1 B + 2\beta_1(\mathbf{b}\cdot\mathbf{n_B})$$

and

$$h\nu_2 = 2\mu_2 B + 2\beta_2(\mathbf{b}\cdot\mathbf{n_B}),$$

where $\mathbf{n_B}$ is the unit vector in the direction of the spin quantization axis defined by the magnetic field **B**. One can readily obtain a combination of the frequencies that is independent of the external magnetic field:

$$\left(\frac{\nu_1}{\mu_1} - \frac{\nu_2}{\mu_2}\right) = \frac{2}{h}\left(\frac{\beta_1}{\mu_1} - \frac{\beta_2}{\mu_2}\right)(\mathbf{b}\cdot\mathbf{n_B}).$$

Thus, in comagnetometer measurements, one can detect a Lorentz-violating field only to the extent that it does not couple to particle spins in proportion to the particles' magnetic moments.

Precision measurements of such nonmagnetic spin interactions have been used over the years—for example, in searches for a permanent electric dipole moment or for spin-dependent forces mediated by a proposed particle called the axion. In single species experiments, one concern is the influence of Lorentz-violating effects in commonly used magnetic shielding; comagnetometer measurements, in contrast, are not sensitive to those effects.

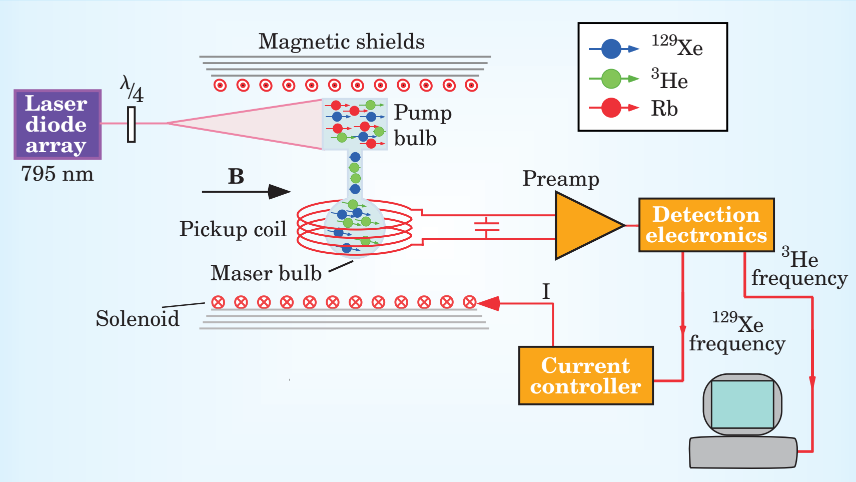


Figure 2. A spin maser experiment with helium-3 and xenon-129 atoms sets the best limit on Lorentz-violating effects for the neutron. A double-bulb glass cell contains ^{3}He and ^{129}Xe atoms in both chambers and additional rubidium atoms in the upper bulb. An optical pumping laser array spin-polarizes the Rb atoms, which transfer their polarization to the ^{3}He and ^{129}Xe via spin-exchange collisions. The lower bulb is located inside a pickup coil connected to an external resonator with resonances at the Zeeman frequencies for both ^{3}He and ^{129}Xe. Resonance currents in the pickup coil maintain a persistent precessing magnetization of both spin species while the current controller adjusts the magnetic field to maintain a constant spin-precession frequency for the ^{129}Xe. Any change in the ^{3}He spin-precession frequency thus indicates a Lorentz-violating spin interaction. (Figure courtesy of Ron Walsworth, Harvard–Smithsonian Center for Astrophysics.)

tivistic form, one finds that the Lorentz-violating background couples to spin much like a magnetic field does. Indeed, for $S = ½$, the interaction Hamiltonian may be expressed as

$$H_{\rm int} = -2\mu\mathbf{B}\cdot\mathbf{S} - 2\mathbf{b}\cdot\mathbf{S}. \qquad (2)$$

In this equation, **b** is a vector with components $b_i - \epsilon_{ijk} H_{jk}/2$. (The ε tensor is totally antisymmetric and we sum over repeated indices.) The spin precession frequency is given by

$$h\nu = 2\mu B + 2\mathbf{b}\cdot\mathbf{n_B}, \qquad (3)$$

where $\mathbf{n_B}$ is a unit vector in the direction of the spin quantization axis defined by **B**. Hence, the signal of Lorentz violation would be a change in the precession frequency, which would be caused by rotation of the magnetic field relative to the preferred direction defined by the vector **b**. Most Hughes–Drever experiments are sensitive to the dipole coupling between the spin and **b** vectors. Experiments using particles with spin greater than ½ or with nonzero angular momentum are also sensitive to higher-dimension operators that have a quadrupole character and induce a signal at twice the magnetic field's rotation frequency.

The precession frequency depends on the magnetic field, which generally is much greater than a possible **b** term. Thus, one must exclude drifts of the magnetic field as a possible source of frequency change. Typically, that's done with experiments that compare two frequencies proportional to the same magnetic field—either the spin precession frequencies of two different species or the spin and orbital precession frequencies of the same species. Because Lorentz-violating operators coupled to the spin cause only small additive energy shifts, experimentalists generally try to keep the precession frequency small to maximize the fractional shift in the frequency. The box on page 41 provides additional details about such measurements, usually called comagnetometer or clock-comparison experiments.

Most experiments use a magnetic field fixed on Earth. They rely on Earth's rotation to change the direction of the field relative to the preferred frame defined by **b** and to produce a diurnal modulation of the precession frequency. But such a modulation is only sensitive to the components of **b** perpendicular to Earth's rotation axis. An experiment on a rotating platform or in orbit around Earth allows one to constrain other components of b_μ. Note that if the component b_0 is nonzero in the reference frame defined by the cosmic microwave background, then an experiment moving with respect to that frame will effectively be subject to a small **b** field given by $\mathbf{b} = (\mathbf{v}/c)b_0 \sim 10^{-3}\,\mathbf{n_v}\,b_0$, where $\mathbf{n_v}$ is the unit vector in the direction of the experiment's velocity.

Hughes–Drever experiments performed in recent years have probed a variety of systems, including singly charged positive beryllium ions, atom pairs such as mercury-199 and mercury-201 or ^{3}He and ^{21}Ne, proton–antiproton or electron–positron pairs in a Penning trap, and muons in a storage ring. With the help of simple atomic and nuclear models, experimenters can interpret their measurements in terms of limits on the Lorentz-violating interactions for the electron ($\mathbf{b}^e$), neutron ($\mathbf{b}^n$), and proton ($\mathbf{b}^p$). One needs a concrete model for the origin of Lorentz violation, though, to predict the strength of the background-field couplings to different elementary particles.

Laboratory bounds

The best limit on the Lorentz-violating $\mathbf{b}^n$ term—indeed, the highest overall energy sensitivity to Lorentz-violating frequency shifts—has been obtained in a ^{3}He–^{129}Xe maser developed at the Harvard–Smithsonian Center for Astrophysics (CfA).[8] Figure 2 illustrates a schematic of the experiment. The free spin precession frequencies of the ^{3}He and ^{129}Xe are simultaneously measured by maintaining persistent Zeeman maser oscillations for both species. The two atoms operate as a comagnetometer because, absent Lorentz violations, their precession frequencies are proportional to the same magnetic field. The frequency of the ^{129}Xe maser is held constant with the help of a magnetic field adjusted via feedback. Any change in the ^{3}He frequency with time would thus indicate an anomalous coupling to the nuclear spins.

In a simple nuclear shell model, both ^{3}He and ^{129}Xe have a single valence neutron, and thus their coupling coefficients, as defined in the box, are equal. But their magnetic moments differ by a factor of 2.75, and so one retains sensitivity to the neutron Lorentz-violating term. After approximately 90 days of integration, the CfA experiment observed, with an uncertainty of 45 nHz, no modulation of the ^{3}He precession frequency at Earth's rotation rate. Such precision means that the part of $\mathbf{b}^n$ perpendicular to Earth's

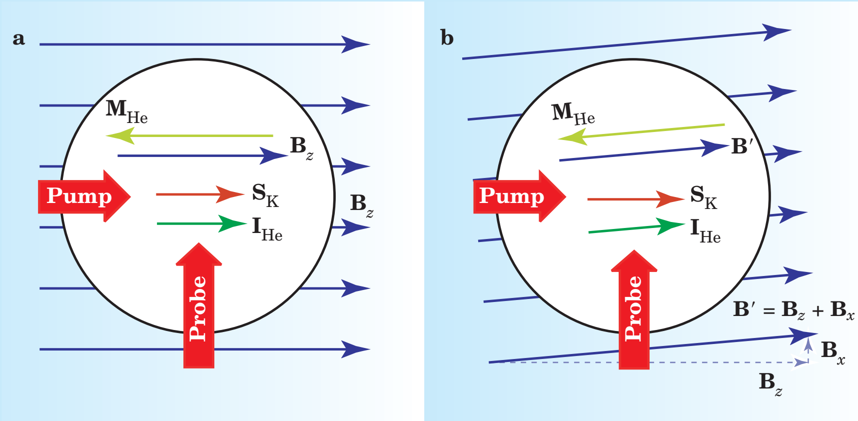


Figure 3. A new experiment being developed at Princeton University uses a self-compensating potassium–helium-3 comagnetometer. **(a)** Optical pumping polarizes K atoms, which transfer some of their polarization $\mathbf{S}_K$ to ^{3}He through spin-exchange collisions. The ^{3}He, whose nucleus has spin $\mathbf{I}_{He}$, develops a significant magnetization $\mathbf{M}_{He}$ directed opposite to the applied longitudinal magnetic field $\mathbf{B}_z$. The strength of the applied field is adjusted so that the total magnetic field seen by the K atoms vanishes. **(b)** If the external magnetic field is changing slowly in the transverse direction, the ^{3}He magnetization adiabatically follows it. As a result, the magnetic field seen by the K atoms remains zero. Absent Lorentz-violating effects, the polarization of K atoms, measured by the probe laser beam, is unchanged. However, a transverse **b** field that, for example, couples only to the electron spin of the K atoms will produce a torque that changes the K polarization without affecting ^{3}He. (Figure courtesy of Tom Kornack, Princeton University.)

rotation axis has a magnitude less than 5×10^{-32} GeV. That incredible sensitivity underlines how such bounds can constrain models of new physics.

The best limit on Lorentz violation for electrons comes from a torsion pendulum experiment developed by the Eöt–Wash group at the University of Washington. It uses a toroidal pendulum consisting of two different kinds of permanent magnets, an aluminum-nickel-cobalt-iron (Alnico) alloy whose magnetization is mostly produced by electron spin alignment, and a samarium–cobalt magnet whose magnetization has a significant contribution from the orbital angular momentum of the Sm electrons. By adjusting the magnetization of the Alnico magnets, the Eöt–Wash experimenters can balance the magnetization in the toroidal ring and almost perfectly cancel the net magnetic moment of the pendulum. However, because part of the contribution to the magnetization comes from orbital angular momentum, the pendulum still has a large unbalanced electron spin.

The pendulum hangs from a fiber and rests inside a set of magnetic shields. The whole apparatus sits on a precision turntable surrounded by Helmholtz coils and suitably positioned masses that reduce magnetic and gravitational field gradients. Any Lorentz-violating spin coupling would cause a torque on the pendulum; that torque would oscillate at the approximately 1-hour rotation period of the turntable. A laser beam reflected from a mirror mounted on the pendulum allows the Eöt–Wash researchers to measure the pendulum's rotation with a sensitivity of 4 nanoradians.

The pendulum experiment set limits on the $\mathbf{b}^e$ components of less than 2×10^{-29} GeV. Note that the turntable-mounted experiment constrains the component of $\mathbf{b}^e$ parallel to Earth's rotation axis. A similar experiment, recently completed at Tsing Hua University in Taiwan, has achieved comparable sensitivity to that of the Eöt–Wash group.[9]

Effective field theories can also include interactions that allow Lorentz violation in the photon sector: The last term of equation 1 is an example. It and similar higher-dimension terms lead to light speeds that depend on the propagation direction or to rotations in the polarization of light propagating in a vacuum.

Modern descendants of the Michelson–Morley experiment furnish the best limits on light-speed anisotropies. Stimulated by renewed interest in tests of Lorentz invariance, groups at Stanford University, Humboldt University in Berlin, Germany, and the Observatoire de Paris have recently completed such experiments, in which one measures the resonance frequency of an optical or microwave cavity cooled to liquid He temperatures. As Earth rotates, the orientation of the cavity changes relative to a fixed reference frame. If the velocity of light were to depend on direction, the changing orientation would cause a shift in the resonance frequency. The most sensitive present limits have been set by the Humboldt group, which compared the resonant frequencies of two optical cavities made from crystalline sapphire that were oriented at 90° to each other.[10] They collected data for about a year and established a limit for variations in the speed of light of $\Delta c/c \lesssim 2 \times 10^{-15}$. Because they collected data for a long time and measured signals at various combinations of Earth's daily and yearly rotation frequencies, the Humboldt researchers were able to place independent constraints on nearly all Lorentz-violating terms that cause anisotropy in the speed of light.

The next generation of experiments is already being developed. The CfA group is presently working to improve the long-term stability of their ^{3}He–^{129}Xe maser. They are also working on improving a hydrogen maser experiment that has set the best limit on the Lorentz-violating proton interactions. A group at Amherst College is improving a 1995 Hughes–Drever-type experiment[11] that compared the spin precession of mercury-199 and cesium-133 atoms and achieved a high short-term sensitivity: Their new experiment will sit on a magnetically shielded rotary table. The Eöt–Wash group is also implementing significant improvements, including an active tilt-stabilization system for their turntable and a pendulum with a higher spin moment. A new experiment being developed at Princeton University will use potassium and ^{3}He atoms that together operate as a self-compensating atomic comagnetometer[12] that is sensitive to a particular combination of neutron and electron violating coefficients, $\mathbf{b}^n/\mu_{He} - \mathbf{b}^e/\mu_e$. The experiment's principle of operation is illustrated in figure 3.

The experiments now being developed should improve existing limits by about two orders of magnitude. One challenge faced in experiments designed to look for Lorentz violations is that the slow frequency of signal modulation makes them very susceptible to $1/f$ noise. Experimenters can achieve superior improvements in sensitivity by increasing the signal modulation frequency—either by placing an experiment on a rotating platform or by placing it on a satellite, as is already planned for future Michelson–Morley experiments.

CPT

Particle physicists have a long history of testing *CPT*. Since the discovery of antiparticles, increasingly precise constraints have been placed on the equality of masses, decay rates, magnetic moments, and other properties of particles and antiparticles.

The most stringent constraints on *CPT*-violating mass splitting arise from measurements of K^0–$\bar{K}^0$ mixing and decays. They imply that the *CPT*-odd parameter $|m(K^0) - m(\bar{K}^0)| < 5 \times 10^{-19}$ GeV. Measurements with neutral mesons have the unique feature that they are sensitive to interference effects between particles and antiparticles. Thus, they can constrain model parameters that cannot be accessed via separate measurements on matter and antimatter.

Lorentz-symmetry and *CPT* violations described by effective field theories should cause many *CPT*-violating signatures in particle physics experiments to acquire sidereal time dependence that could be exploited to improve sensitivity. In principle, low-energy Hughes–Drever-type experiments can achieve good sensitivity to the Lorentz-violating parameters for stable first-generation particles, but experiments with neutral mesons and other unstable particles are the ones that sensitively probe Lorentz violation for the other two generations.

The coupling of the *CPT*-odd b_μ field to particle spin allows one to test *CPT* without the need for antiparticles: By flipping a particle's spin, one can measure the same energy shift as would be obtained by comparing a particle and an antiparticle. Thus, $g - 2$ experiments and other measurements performed with trapped positrons and antiprotons can be directly compared with spin-precession measurements on ordinary particles. If *CPT* violation is ever found, then a comparison of results for particles and antiparticles will allow physicists to separate *CPT*-odd from *CPT*-even Lorentz-violating effects. They could, for example, distinguish the separate contributions of b_i and $\epsilon_{ijk} H_{jk}$ to the $\mathbf{b}$ field.

Astrophysical and cosmological tests

Astronomical observations can place extremely tight constraints on certain types of Lorentz violation, such as changes in the polarization of light propagating through space. A rotation in linear polarization is implied, for example, by the last term of equation 1. For that particular interaction, the effect is independent of wavelength. To probe the effect, one can observe the polarization of synchrotron radiation emitted from distant radio galaxies. The initial polarization depends on the emitting galaxy's magnetic field and can be correlated with the elongation of the galaxy. An analysis of observed synchrotron radiation[5] constrains the Lorentz-violating coefficient k_μ to have a four-dimensional magnitude less than 10^{-42} GeV. That energy corresponds to a frequency somewhat smaller than the Hubble expansion rate.

Other Lorentz-violating terms cause polarization rotations that depend on wavelength. They can be measured by observing visible polarized light emitted by distant galaxies. Scattering from dust or electrons is typically responsible for polarizing the light, and so the initial polarization is independent of wavelength. By measuring the polarization of scattered light over a range of wavelengths, one can set a limit on wavelength-dependent rotation due to Lorentz-violating effects.[13]

Another manifestation of Lorentz violation, which can arise when higher-dimension interactions are included in an effective theory, is a modification of the usual dispersion relation $E^2 = (pc)^2 + (mc^2)^2$. Such modifications lead to a wide variety of effects for high-energy particles—effects that include a spread in the arrival time of photons from gamma-ray bursts;[14] limits on the maximum possible velocity of a massive particle; and new types of particle processes, such as the decay of a high-energy photon into a positron and electron.[4]

Observed very high-energy astrophysical processes have significantly constrained possible modifications to the standard dispersion relation. For example, synchrotron radiation from the Crab Nebula (figure 4) has been observed with energies of up to 100 MeV, and several methods estimate the nebula's magnetic field to be in the range of 20–50 nanotesla. Together, those two values imply that electrons are accelerated up to energies of at least 1500 TeV, corresponding to velocities that differ from the speed of light by less than one part in 10^{19}. The electrons accelerated to such energies must be stable against the vacuum Čerenkov radiation process $e \to \gamma e$. Thus, the mere existence of such high-energy electrons places strong constraints on how dispersion relationships for electrons and photons may be modified.[15]

Challenges for quantum gravity

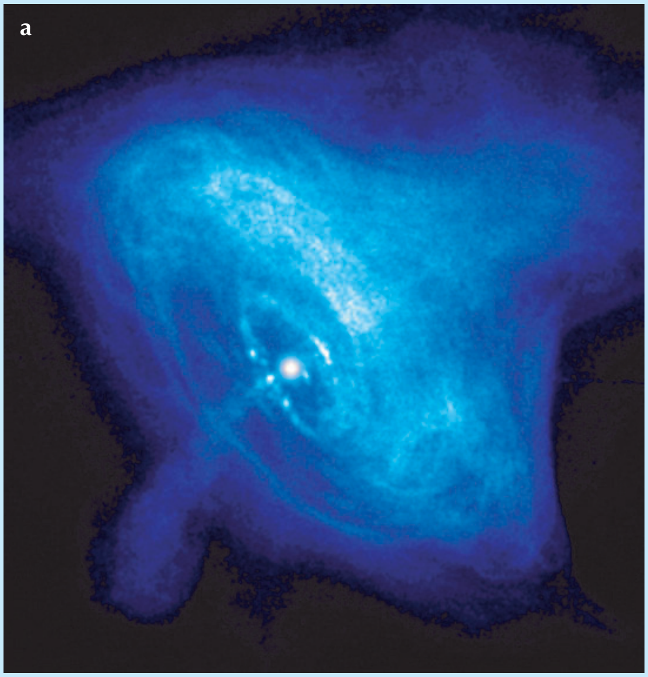
The Planck mass $M_{\rm Pl} = (\hbar c/G)^{1/2} \sim 10^{19}$ GeV/c^2 signals a conceptual problem for quantum field theories. When the momentum transfer in particle collisions is comparable to $M_{\rm Pl}c$, graviton exchange becomes strong and the standard perturbative field-theory description breaks down. Is it possible that unknown dynamics at the Planck scale could lead to Lorentz violation? In string theory, a leading candidate as the theory of quantum gravity, Lorentz- and *CPT*-violating effects are allowed but not required. In other approaches to quantum gravity, such as loop quantum gravity, it is often argued that the discrete nature of spacetime at short distances will induce violations of Lorentz invariance and *CPT*. Such violations lead to dispersion relations of the form[14]

$$E^2 = (pc)^2 + (mc^2)^2 + \xi \left(\frac{E^3}{M_{\rm Pl}c^2}\right) + \ldots, \qquad (4)$$

where the ellipses denotes further additions with the general form $E^{n+2}/(M_{\rm Pl}c^2)^n$.

For modified dispersion relations arising from dimension-5 operators, left- and right-handed fermions may both be described by the three displayed terms in equation 4, but the ξ coefficients for the two species are independent; they are denoted η_L and η_R, respectively. A single ξ describes the dispersion modification for photons, but with an opposite sign for left- and right-circularly polarized photons.

Astrophysical observations, such as those from the Crab Nebula, constrain various combinations of the electron's and photon's ξ and η parameters with unprecedented accuracy,[15] sometimes to better than 10^{-7}. Perhaps more surprising is that spin precession experiments also provide comparable constraints.[7] If one assumes the dispersion relationships are written in the cosmic microwave background rest frame, then any difference between η_L and η_R for quarks would create an effective spin coupling on the order of $10^{-3}\,\mathbf{n}_{\rm v}\,(\eta_L - \eta_R)\,(m_{\rm n}^2/M_{\rm Pl})c^2$, where $m_{\rm n}$ is the nucleon mass. When combined with experimental limits from Hughes–Drever experiments, that result yields a



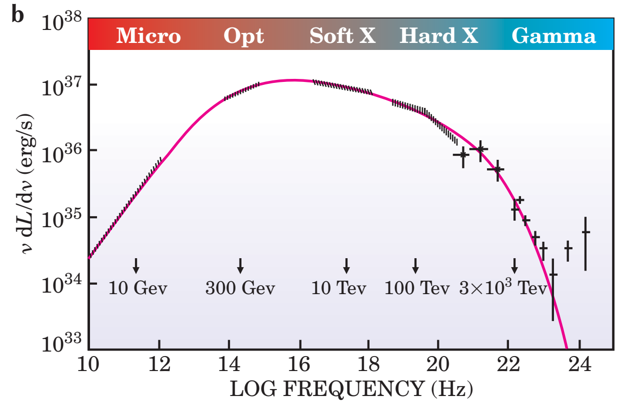


Figure 4. (a) X rays in the Crab Nebula are the synchrotron radiation of electrons accelerated in the shock waves created by the supernova explosion of 1054 AD. This image was taken by the *Chandra X-ray Observatory*. **(b)** The luminosity L of the synchrotron radiation emitted by the Crab Nebula is a function of frequency ν. Ground-based observations provide the data for the microwave (Micro) and optical (Opt) parts of the spectrum. X-ray and gamma-ray measurements were performed with detectors aboard the NASA satellites *HEAO-1* and the *Compton Gamma-Ray Observatory*. The curve through the data is a theoretical fit for which the magnetic field in the nebula was taken to be 20 nanotesla. Arrows indicate the energy of electrons needed to produce the various parts of the spectrum: The highest-energy electrons severely constrain possible modifications of dispersion relations. (Panel b adapted from A. M. Atoyan, F. A. Aharonian, *Mon. Not. R. Astron. Soc.* **278**, 525, 1996.)

bound on $|\eta_L - \eta_R|$ of about 10^{-8}. The electromagnetic interactions inside nucleons and nuclei lead to an additional dependence of spin precession on the ξ parameter, whose magnitude can be constrained to be less than 10^{-5}. Thus, astrophysical and terrestrial bounds on Lorentz violation firmly rule out $E^3/M_{\rm Pl}$ modifications of the dispersion relations for photons, quarks, and electrons—a serious challenge for theories of quantum gravity that predict such modifications.

An important and different kind of small-length-scale physics that leads to the violation of Lorentz invariance is a feature of noncommutative field theories. In such theories, which naturally arise in the context of certain string theories,[16] spacetime coordinates are noncommuting operators: $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. The tensor $\theta^{\mu\nu}$ has the dimensions of an inverse energy squared and defines a so-called energy scale of noncommutativity $\Lambda_{\rm NC} \sim \theta^{-1/2}$. At energies below $\Lambda_{\rm NC}$, noncommuting coordinates would be manifest as interactions with standard-model operators of dimension 6 such as $(\theta^{\mu\nu}F_{\mu\nu})(F^{\alpha\beta}F_{\alpha\beta})$, where $F^{\mu\nu}$ is the electromagnetic field tensor.

An antisymmetric tensor, $\theta^{\mu\nu}$ has the same Lorentz transformation properties as the electromagnetic field tensor, and so it may be no surprise that its "magnetic" component $\theta_i = \epsilon_{ijk}\theta_{jk}$ couples to nuclear spin. The noncommutative extension of quantum chromodynamics leads[17] to a modulation of the nucleon spin frequency corresponding to an energy of order 0.1 GeV^3 $\Lambda_{\rm NC}^{-2}$. That result, combined with Hughes–Drever experiments, establishes that the energy scale of noncommutativity is greater than about 10^{14} GeV.

Considerations of modified dispersion relations and noncommutative field theories underline the importance of continued experiments designed to detect Lorentz invariance. In both examples, the scale of the high-energy physics responsible for Lorentz breaking can be probed with unprecedented reach.

Cosmological models

Our discussion thus far has been in the framework of special relativity, for which constant Lorentz-violating backgrounds can be justified. In general relativity, such constant fields necessarily become a function of coordinates: Backgrounds such as b_μ and $H_{\mu\nu}$ acquire kinetic terms and thus participate in dynamics as additional low-energy degrees of freedom. Although gravitational theories with additional scalar degrees of freedom have been studied for many years,[2] the recent discovery of dark energy has intensified research efforts that explore such theories.

A possible candidate for dark energy that avoids some of the fine-tuning problems associated with the cosmological constant is quintessence, a very low-energy field with a wavelength comparable to the size of the observable universe. In addition to its effect on the expansion of the universe, quintessence might also manifest itself through its possible interactions with matter and radiation.[2,18]

One would expect that a quintessence field ϕ related to dark energy would still be evolving and that its continuing evolution in time and space would lead to Lorentz-violating effects. As an example, consider the simplest form of scalar and pseudoscalar (that is, containing γ_5) couplings between a massive Dirac fermion—say, an electron or quark—and the scalar quintessence field ϕ. The Lagrangian

$$\mathcal{L} = -m_\psi \bar{\psi} \left(\frac{\phi}{F_{\rm s}} - \frac{\phi}{F_{\rm a}} i\gamma_5 \right) \psi \qquad (5)$$

yields the interaction Hamiltonian

$$H_{\rm int} = \frac{1}{F_{\rm s}} m_\psi (\nabla\phi \cdot \mathbf{r}) + \frac{1}{F_{\rm a}} (\nabla\phi \cdot \mathbf{S}). \qquad (6)$$

The scalar interaction leads to a modification of the fermion mass as a function of coordinates and violates the equivalence principle: The particle feels an extra force in the direction of $-\nabla\phi$. The pseudoscalar interaction couples the spin to $\nabla\phi$ and thus creates in the spin precession a Lorentz-violating signature that is identical to the interaction parameterized by **b**. When a similar derivation is repeated for a photon, one finds that the scalar coupling leads to a fine-structure constant that varies in space and time and that the pseudoscalar coupling takes essentially the same form as the last term in equation 1.

The time and space derivatives of ϕ should be somewhat smaller than the square root of the present energy density in the universe. Assuming that the spatial derivative $\nabla\phi$ has a magnitude on the order of $(0.01\ \text{eV})^2$, Hughes–Drever experiments imply that the pseudoscalar (F_a) couplings to electrons and nuclei are greater than about 10^9 GeV. Astrophysical constraints on k_μ imply[5,18] that the corresponding pseudoscalar coupling to photons is greater than the Planck energy.

If future searches for Lorentz invariance and time-dependent fundamental "constants" bring positive results, they may prove that the universe contains a long-wavelength degree of freedom and point toward the nature of quintessence. Then the variation of fundamental constants and apparently Lorentz-violating spin precessions might be completely demystified: They could both follow from the conventional physics of an interacting scalar field.

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