Stroboscopic Backaction Evasion in a Dense Alkali-Metal Vapor

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We explore experimentally quantum nondemolition measurements of atomic spin in a hot potassium vapor in the presence of spin-exchange relaxation. We demonstrate a new technique for backaction evasion by stroboscopic modulation of the probe light. With this technique we study spin noise as a function of polarization for atoms with spin greater than $\frac{1}{2}$ and obtain good agreement with a simple theoretical model. We point that, in a system with fast spin exchange, where the spin-relaxation rate is changing with time, it is possible to improve the long-term sensitivity of atomic magnetometry by using quantum nondemolition measurements.

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Quantum nondemolition (QND) measurements form the basis of many quantum metrology schemes [1–3]. A QND measurement can drive the system into a squeezed state conditioned on the measurement result. In this state the uncertainty of the measured variable is reduced below the standard quantum limit (SQL) at the expense of an increase in the uncertainty of the conjugate variable. A key ingredient in QND measurements is a backaction evasion mechanism that decouples the measured variable from the quantum noise of the probe field.

Here we explore a new backaction evasion scheme in an alkali-metal vapor in a finite magnetic field. A QND measurement of an atomic spin component can be made by paramagnetic Faraday rotation of off-resonant probe light [4]. By stroboscopically pulsing the probe light at twice the frequency of Larmor spin precession, we achieve backaction evasion on one of the spin components in the rotating frame, while directing the quantum noise of the probe beam to the other rotating component. The stroboscopic modulation of the probe was first suggested in the context of mechanical oscillators [5]. In atomic systems with non-zero Larmor frequency, only more complicated schemes with two oppositely polarized vapor cells have been used to achieve backaction evasion [6].

The QND measurements in a dense alkali-metal vapor allow us to study atomic spin noise in the presence of various relaxation mechanisms. The behavior of collective spin in the presence of decoherence is not trivial [7–9]. We quantitatively measure spin noise as a function of atomic polarization for K atoms ($I = \frac{3}{2}$) with spin exchange, light scattering, and spatial diffusion as the dominant sources of relaxation and obtain good agreement with a simple model for quantum fluctuations.

Although QND measurements have been shown to increase the measurement bandwidth without loss of sensitivity [10,11], it has been known for some time that spin squeezing in the presence of a constant decoherence rate does not significantly improve long-term measurement sensitivity [12,13]. We point out that spin-exchange collisions, which are the dominant source of relaxation in a dense alkali vapor, cause nonlinear evolution of the atomic density matrix with a relaxation rate that changes in time. Under these conditions we show theoretically that QND measurements can, in fact, improve the long-term sensitivity of atomic magnetometers.

The experimental setup is shown in Fig. 1(a). The atomic vapor is contained in a 55 mm long, $D$-shaped cylindrical glass cell, with the probe beam going through the length of the cylinder. We use a mixture of potassium in...
natural abundance, 50 Torr of N\textsubscript{2} buffer gas for quenching and 400 Torr of 4He to slow down the diffusion of alkali atoms. The cell is heated in an oven with flowing hot air and is placed inside a double layer \(\mu\)-metal and a single-layer aluminum shield. A low noise current source generates a homogeneous dc magnetic field in the \(\hat{z}\) direction, corresponding to a Larmor frequency of 150 kHz for K atoms. The first-order gradient of this field along the direction of the probe beam is canceled by a gradient coil. Low pass filters are placed inside the shields on all cables to reduce high frequency noise. Narrow linewidth, amplified distributed-feedback (DFB) lasers are used for the pump and probe beams, and acousto-optic modulators provide fast amplitude modulation of the light. The circularly polarized pump beam creates atomic orientation in the \(\hat{z}\) direction. It is turned off after 10 msec of pumping before probe measurements. The profile of the pump beam is shaped by using spherical aberration effects so that the intensity is slightly higher at the edges of cell, where the spin-destruction rate is higher due to wall relaxation. Using gradient imaging we have measured and minimized the polarization nonuniformity of the vapor. A linearly polarized probe beam detuned from the D\textsubscript{1} line of K by 397 GHz to the red and propagating along the \(\hat{x}\) direction experiences Faraday rotation, which is measured with balanced polarimetry. A glass stress plate is used to compensate for any residual circular polarization of the probe beam inside the cell. The signal is digitized with a fast, low noise analog-to-digital card and recorded with a computer.

The backaction of the probe originates from the ac Stark shift caused by quantum fluctuations of the circular polarization of the light. The tensor light shift for K atoms is less than 1% of the probe scattering rate and much less than all other rates \cite{14,15}. Therefore, the light-shift noise is described by a stochastic magnetic field along the direction of the probe beam. During a short measurement of \(F_x\) by the probe beam, this magnetic field rotates \(F_z\) polarization into the \(F_y\) direction, thus ensuring that the product \(\delta F_x\delta F_y\) satisfies the quantum uncertainty relationship. In the presence of a dc magnetic field in the \(\hat{z}\) direction, the \(x\) and \(y\) components of the collective spin undergo Larmor precession, so that over time scales longer than the Larmor period both \(F_x\) and \(F_y\) accumulate the backaction noise. The effect of backaction on the \(F_x\) measurement in the rotating frame can be suppressed by using a stroboscopic probe that turns on and off at twice the Larmor frequency. This way a measurement is performed only when the squeezed distribution is aligned with the probe direction; see Fig. 1(b).

The power spectral density (PSD) of a 3.6 msec recording of the polarimeter output is shown in Fig. 1(c) for both unpolarized and highly polarized atoms using stroboscopic probe modulation at twice the Larmor frequency. The longitudinal spin polarization does not change significantly on this time scale. The PSD can be described by a sum of a constant photon shot noise background and a Lorentzian-like atomic shot noise contribution \cite{10}. The deviation from the Lorentzian profile is notable in our experiment due to the effect of diffusion in and out of the probe beam (beam waist diameter \(2w_0 \approx 220 \mu\text{m}\)). As discussed in Ref. \cite{10}, the width and shape of the atomic noise peak do not affect the total atomic optical rotation variance \(\delta\phi_{\text{at}}^2\) given by the area under the noise peak, since \(\delta\phi_{\text{at}}^2\) can in principle be obtained from a series of infinitesimally short measurements.

For unpolarized atoms this noise area is a good measure of fundamental atomic shot noise (ASN) because it is not affected by light-shift or stray magnetic field noise, and the scattering of photons has an insignificant effect on the quantum noise properties \cite{11}. We find that the experimentally measured noise area is within 10% of the first-principles calculation \cite{10,16}. In the fully polarized ensemble the spin-exchange collisions between alkali atoms do not contribute to spin relaxation \cite{17}, and the spin noise linewidth is much smaller, as can be seen in Fig. 1(c). The area under the noise peak is also smaller for polarized atoms with spin \(>1/2\), as shown in Fig. 3.

The backaction evasion of the stroboscopic measurement is demonstrated in Fig. 2. The atomic noise is evaluated by numerical integration of the measured PSD after subtracting the constant photon shot noise background. For unpolarized atoms, there is no contribution of light shift to the total noise, and it remains independent of the stroboscopic modulation frequency. For polarized atoms, as the

![FIG. 2 (color online). Measurement of spin variance for unpolarized and polarized (\(P \approx 85\%\)) atomic ensembles as a function of stroboscopic frequency. The Larmor frequency is 150 kHz. While for the unpolarized case the noise does not depend on the frequency, for polarized atoms extra light-shift noise appears at detunings from the resonant condition. The shaded area of the noise dip can be compared with the theory. The data were taken by using a probe with a 10% duty cycle. Inset: Ratio of polarized to unpolarized noise (not including photon shot noise) as a function of the duty cycle of the strobe light. All data points were acquired with the same average intensity.](https://doi.org/10.1103/PhysRevLett.106.143601)
strobe frequency departs from the resonance condition of twice the Larmor frequency, light-shift backaction noise is added to the ASN, and the total noise increases until it reaches a maximum plateau. It can be shown that the total area of the noise dip (shaded in Fig. 2) is proportional to the intensity of the probe beam and is independent of the atomic spin-relaxation rate or the effects of diffusion [16]. The experimentally measured area of the noise dip is within 10% of the expected size for light shift due to quantum fluctuations of probe circular polarization. The backaction evasion is also observed when the noise is plotted as a function of the duty cycle of the stroboscopic probe. In the inset in Fig. 2, we normalize each point by the corresponding unpolarized ASN and show that the light-shift suppression is stronger for small duty cycle probe pulses.

In Fig. 3, the noise ratio for (partially) polarized to unpolarized atomic ensembles is plotted as a function of the longitudinal polarization for three different densities. The polarization is found from the optical rotation of the probe beam due to a known, small magnetic field in the probe direction ($B_x \ll B_z$). The largest uncertainty in this measurement originates from the determination of the atomic density $n$. To find $n$, we map the rf resonance curve at low pump intensity and associate the measured linewidth with the spin-exchange relaxation rate at operating conditions [17]. The two measurements give similar results for low atomic density but differ by 10% at the highest density. We believe this discrepancy is due to nonuniform polarization of the atomic ensemble, which becomes more pronounced at high densities due to limited pumping power.

The measured noise ratio is well described by a simple theoretical model. For our conditions, the density matrix can be approximated for arbitrary longitudinal polarization $P$ by the spin temperature distribution [19]: $\rho = e^{\beta S_z}/Z$, where $Z$ is the partition function and $\beta = \ln((1 + P)/(1 - P))$. Then, by taking into account the two hyperfine manifolds of the alkali-metal atoms [10], the ASN variance of the collective spin composed of $N_a$ atoms can be written:

$$\langle F_z^2 \rangle = \frac{N_a}{2Z} \sum_{m=-a}^{a} e^{\beta m}[a(a + 1) - m^2]$$

$$+ \sum_{m=-b}^{b} e^{\beta m}[b(b + 1) - m^2].$$  (1)

Here, $a = I + 1/2$ and $b = I - 1/2$, with $I$ being the nuclear spin. In contrast to a spin-$1/2$ system, for $I = 3/2$ the ASN power is smaller for polarized atoms by a factor of 2/3 compared with unpolarized atoms, in agreement with the experiment. These data represent the first systematic study of collective spin measurements on partially polarized atomic states, discussed theoretically in Ref. [8]. They also show that spin relaxation due to pairwise correlated spin-exchange collisions leads to the same spin noise as for uncorrelated collisions, contrary to the conclusion in Ref. [7].

As can be seen in Fig. 1, the resonance linewidth is significantly reduced for high spin polarization due to suppression of the spin-exchange relaxation. In the time domain this is manifested by a nonexponential decay of the transverse spin polarization, shown in Fig. 4(a). In a highly polarized vapor the initial spin-relaxation rate is suppressed. This allows one to improve the overall long-term measurement sensitivity by using QND measurements.

To model this behavior quantitatively we consider a measurement scheme using two short pulses of probe light [20]. The first pulse is applied immediately after turn-off of the pump beam and the second after a measurement time $t_m$. The best measurement of the magnetic field is obtained by using an estimate $S_x(t_m) - S_x(0)\text{cov}[S_x(0), S_x(t_m)]/\text{var}[S_x(0)]$, where $S_x(0)$ and $S_x(t_m)$ are measurements of spin projection from the two probe pulses. For simplicity we consider spin-$1/2$ atoms here. One can show that $\text{var}[S_x] = (1 + 1/\epsilon OD)N_a/4$, where $\epsilon$ is the strength of a far-detuned probe pulse, given by the product of pulse duration and photon scattering rate, OD is the optical density on resonance, and $N_a$ is the number of atoms. The covariance of the two measurements is given by [21]

$$\text{cov}[S_x(0), S_x(t_m)] = \langle N_a/4 \rangle \exp \left[ -\int_0^{t_m} R(t')dt' \right].$$  (2)

where $R(t)$ is a time-dependent transverse spin-relaxation rate. In the presence of spin-exchange collisions the relaxation rate can be approximated by $R(t) = R_{sd} + (1 - P_s)R_{re}$ [17]. Using this model we optimize the measurement procedure with respect to the strength of the first and the second probe pulses and $t_m$. We assume that the

FIG. 3 (color online). Ratio of polarized to unpolarized ASN (variance) as a function of the mean longitudinal polarization of the ensemble for three different densities. The duty cycle of the probe was 10%.
initial state preparation time is negligible and the measurement repetition time is equal to $t_m$. The results of the model are plotted in Fig. 4 for varying spin-exchange rates. For comparison, we also plot in Fig. 4(b) the variance of a single-pulse measurement after time $t_m$, which does not rely on spin squeezing. The results are scaled relative to the SQL for $N_A$ atoms with spin-relaxation rate $R_{sd}$, $\delta B_{SQL}^2 = 2R_{sd}/(N_A t^2)$, where $t$ is the total measurement time and $\gamma$ is the gyromagnetic ratio.

It is instructive to compare our results with those of Ref. [12]. In the absence of spin squeezing and spin-exchange relaxation, the smallest possible magnetic field variance is given by $\epsilon \delta B_{SQL}^2$, in agreement with Ref. [12]. Using the two-pulse measurement one can reduce the variance by a factor of $\epsilon$, the same factor as obtained in Ref. [12] with partially entangled states. In the presence of spin-exchange relaxation, the sensitivity is degraded for the one-pulse scheme but asymptotically reaches the same exchange relaxation, the smallest possible magnetic field variance corresponding to a constant relaxation rate. These results also apply to hyperfine transitions which are broadened by spin exchange [22], and, generally, to other relaxation effects due to nonlinear interactions, such as dipolar spin couplings [23].

In summary, we have explored quantum nondemolition measurements of collective spin in a dense alkali-metal vapor. We demonstrated a new stroboscopic technique for backaction evasion and used it to measure atomic spin noise as a function of spin polarization in the presence of several spin-relaxation mechanisms. We considered QND measurements in a system with nonlinear spin relaxation and showed theoretically that they can improve the long-term sensitivity in atomic spectroscopy.

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