A NEW LIMIT ON
LORENTZ- AND CPT-VIOLATING
NEUTRON SPIN INTERACTIONS
USING A K-\(^3\)He COMAGNETOMETER

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Abstract

Gravity and quantum mechanics are expected to unify at the Planck scale described by an exceedingly large energy of $10^{19}$ GeV. This regime is far from the reach of the highest energy colliders, but tests of fundamental symmetries provide an avenue to explore physics at this scale from the low energy world. Proposed theories of quantum gravity suggest possible spontaneous breaking of Lorentz and $CPT$ symmetry that have so far been unobserved. This thesis presents a test of a Lorentz- and $CPT$-violating background field constant across the solar system coupling to the neutron spin. A comagnetometer with polarized atomic vapors K and $^3$He suppresses magnetic fields but remains sensitive to non-magnetic spin couplings. In a tabletop setup, the comagnetometer measures the equatorial components to be $\tilde{b}_X = (0.1 \pm 1.6) \times 10^{-33}$ GeV and $\tilde{b}_Y = (2.5 \pm 1.6) \times 10^{-33}$ GeV, improving the previous neutron coupling limit by a factor of 30.

This work utilizes the CPT-II apparatus, a second generation comagnetometer installation which features a compact design and evacuated bell jar enclosing all optics for improved long term stability. A rotating platform provides frequent reversals of the apparatus orientation and represents a significant improvement in a Lorentz violation search. The comagnetometer is also a sensitive gyroscope, so reorientation with respect to Earth’s rotation contributes significantly to the signal. Reversals along both the north-south and east-west directions provide a separation of systematic effects from maximal and minimal gyroscope backgrounds. Systematic background effects associated with reversals of the apparatus are also identified and removed. Several novel features of comagnetometer behavior are also explored including the response to an ac magnetic field and magnetic field gradients.

Furthermore, a compact, nuclear spin source containing $9 \times 10^{21}$ hyper-polarized $^3$He spins is designed and constructed. The spin source is used in conjunction with the first generation apparatus CPT-I for a test of proposed long-range spin-dependent
forces on the scale of 50 cm. The spin source supports accurate real-time monitoring of the $^3$He polarization, an active magnetic field cancelation system, and efficient spin reversals with losses below $2.5 \times 10^{-6}$ per flip. This experiment leads to a new limit on neutron coupling to light pseudoscalar and vector particles, including torsion, along with constraints on possible couplings in recently proposed models involving unparticles and spontaneous breaking of Lorentz symmetry. This measurement improves the previous limit by a factor of 500 and reaches an energy resolution of $10^{-34}$ GeV, the highest energy resolution of any atomic experiment.
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4It may be closed but will live on in memory. For the record, The Princeton Gingerbread Museum is not a legitimate business nor is it affiliated with Princeton University in any way.

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Chapter 1

Introduction

The Standard Model of Particle Physics unifies three of the four fundamental forces, the electromagnetic, the weak, and strong into one model using continuous rotations of groups $SU(3) \times SU(2) \times U(1)$. General relativity describes gravity, space, and time on a grand scale through geometry. These two theories span the full scale of possible interactions observed in nature from quantum mechanics to the formation of the universe. Each theory has enormous predictive power, yet the two have not been reconciled. Attempts to unify these forces in a single theory of quantum gravity or Grand Unified Theory have led to widely varying models that have yet to be verified with any experimental signature.

It is expected that General Relativity and the Standard Model are low energy limits of a more fundamental theory. A simple scaling argument combines the fundamental constants from quantum mechanics $\hbar$, space-time $c$, and gravity $G$ into a complete set of characteristic length, time, and mass scales known as the Planck units. No combination of these constants can produce a dimensionless parameter, so these units are dimensionally independent [3]. The Planck mass is defined as

$$M_p = \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19} \text{ GeV}/c^2 \quad (1.1)$$
and is expected to be a boundary between standard theories of gravity and quantum mechanics.

Two mainstream theories compete to describe Planck scale physics, Loop Quantum Gravity [4] and String Theory [5]. String theory unifies gravity and gauge fields into a consistent quantum theory, but does not predict a 4-D full Standard Model phenomenology without unbroken supersymmetry. Loop Quantum Gravity provides an alternative unification of quantum mechanics and gravity but does not aim to describe all interactions as does String Theory. These theories distinctly differ in the treatment of spacetime as a fundamental or background field. A comprehensive comparison of these theories is beyond the scope of this work, but it suffices to point out that there have been no observed experimental signatures of either theory. Both are considered tentative and require experimental validation.

At present, physics at the highest energy scale is directly probed by the Large Hadron Collider (LHC). The LHC collides two proton beams at an energy of 7 TeV/beam. These events represent the highest energies ever attained in the laboratory under controlled conditions, but are still far from probing physics at or near the Planck scale. Astrophysical tests can explore these high energies. Hawking radiation is emitted from a black hole if it has a Hawking temperature above the temperature of the cosmic microwave background radiation. In the final stages of evaporation, the Hawking radiation should approach the Planck energy, but this radiation has yet to be observed [6]. Similarly, propagation of energy from distant gamma ray bursts can be affected by the Planck scale granularity of spacetime [7]. It is also expected that an imprint of primordial gravitational radiation can exist within the cosmic microwave background [8]. Conditions in these experiments are impossible to control and repeat under different parameters. If new physics is present at the level of the Planck scale, it is expected that experiments of exceptional sensitivity may provide a signature of these effects [9].
1.1 Lorentz and CPT Violation

As an alternative to high energy approaches, precision laboratory scale tests provide a unique opportunity to probe the highest energy scales within a controlled environment through tests of fundamental symmetries. Observable signals from a fundamental theory may also reveal themselves at a more accessible level, perhaps only suppressed by the ratio of the electroweak $m_W$ and Planck scales where $m_W/M_P \simeq 10^{-17}$ [9]. Lorentz and CPT symmetry are fundamental tenets of both the Standard Model and General Relativity, so evidence of a broken symmetry immediately provides a signature of new physics. Any realistic Lorentz-covariant fundamental theory with more than 4 dimensions is expected to produce spontaneous Lorentz violation [10], and it has already been shown that String Theory can lead to Lorentz and CPT violation [11]. It has also been shown that a feature of an interacting quantum field theory is that Lorentz violation will follow from CPT violation [12]. No precision measurement has yet to observe an experimental signature of quantum gravity.

1.1.1 Lorentz Symmetry

Lorentz symmetry states that the laws of physics should be the same in all inertial frames. This statement is expressed mathematically in terms of a Lorentz transformation. For a co-ordinate 4-vector $x_\mu = (ct, x, y, z)$, an example Lorentz transformation is the Lorentz boost where $x_\mu$ transforms to $x'_\mu$ along the $x$-axis with a velocity $v$ as

\begin{align*}
  t' &= \beta \gamma (t - vx/c^2) \\
  x' &= \beta \gamma (x - vt) \\
  y' &= y \\
  z' &= z
\end{align*}

(1.2)
where $\beta = 1/\sqrt{1 - (v/c)^2}$. This transformation preserves the metric of flat spacetime

$$ds^2 = -(c dt)^2 + dx^2 + dy^2 + dz^2$$

as an invariant. In general, a Lorentz transformation is any transformation that preserves the metric of flat spacetime (Eq. 1.3) and is called covariant [13].

Lorentz transformations can be broken into two types, observer Lorentz transformations and particle Lorentz transformations. An observer Lorentz transformation is described by

$$x'_\mu = \Lambda^\nu_\mu x_\nu + a_\mu$$

where $\Lambda^\nu_\mu$ describes rotations and boosts while $a_\mu$ describes translations. These transformations describe observations made in two inertial frames with differing orientations and velocities. A particle Lorentz transformation for a particle $\phi$ follows

$$U(a, \Lambda)\phi(x)U(a, \Lambda)^\dagger = \phi(\Lambda x + a)$$

where $U$ describes all of the boosts, rotations, and translations. These transformations relate the properties of two particles with different spin or momentum within a specific oriented inertial frame. In the absence of Lorentz violation, these transformations are equivalent, but in a Lorentz-violating theory, particles at different velocities experience different physics. Therefore, particle Lorentz transformations lead to experimentally observable deviations from the predicted theory.

### 1.1.2 CPT Symmetry

In contrast to Lorentz symmetry, CPT symmetry is a discrete symmetry under charge conjugation $C$, parity inversion $P$, and time reversal $T$. Effectively, charge conjugation casts a particle to its corresponding anti-particle [14], parity inversion considers a
“mirror” world, and time reversal switches a particle moving forward in time to one moving backwards in time (Fig. 1.1). Examples of these transformations on various physical parameters are listed in Table 1.1.

The CPT Theorem states that in a relativistic local quantum field theory the combination of charge, parity, and time will be good symmetries under the conditions of Lorentz invariance, hermiticity of the Hamiltonian, and locality of the theory [15, 16]. A violation of CPT symmetry implies that one or more of these assumptions is false. The Standard Model predicts CPT as a perfect symmetry, so any evidence of CPT violation is evidence of new physics.

Typical CPT tests involve comparison of the mass, lifetime, and the magnitude of both the charge and magnetic moment of matter and antimatter. In general, anti-matter experiments are especially challenging due to the difficult production and isolation of anti-matter particles. Recent theoretical work has demonstrated a connection between CPT and Lorentz violation and proposed tests of CPT without antiparticles [12, 17]. A constraint on Lorentz violation provides a constraint on CPT violation.

Many measurements as individual or pair symmetries of C, P, and T have been shown to be violated in the weak interaction [18, 19, 20, 21], but so far, no experimental violation of the CPT combination has been observed. CPT violation is possible in String Theory [11] and could lead to baryogenesis [22].

1.2 Lorentz Violation Searches

Tests of matter coupling to an anisotropy of space were first studied by Hughes [24] and Drever [25] in considering the orientation of the spin of 7Li to a preferred reference frame. This type of experiment is now referred to as a Hughes-Drever experiment. Further experiments have improved upon the sensitivity of these early mea-
Figure 1.1: Charge reversal changes a particle to its antiparticle. Parity reversal switches each of the coordinate axes. Note that the handedness has changed. Time reversal switches the direction of time. Each of these discrete symmetries is violated individually in the Standard Model, however, their product is preserved.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$C$</th>
<th>$P$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate</td>
<td>$\vec{x} \rightarrow \vec{x}$</td>
<td>$\vec{x} \rightarrow -\vec{x}$</td>
<td>$\vec{x} \rightarrow \vec{x}$</td>
</tr>
<tr>
<td>Time</td>
<td>$t \rightarrow -t$</td>
<td>$t \rightarrow t$</td>
<td>$t \rightarrow -t$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$\vec{p} \rightarrow \vec{p}$</td>
<td>$\vec{p} \rightarrow -\vec{p}$</td>
<td>$\vec{p} \rightarrow -\vec{p}$</td>
</tr>
<tr>
<td>Energy</td>
<td>$\epsilon \rightarrow \epsilon$</td>
<td>$\epsilon \rightarrow \epsilon$</td>
<td>$\epsilon \rightarrow \epsilon$</td>
</tr>
<tr>
<td>Angular Momentum</td>
<td>$\vec{L} \rightarrow \vec{L}$</td>
<td>$\vec{L} \rightarrow \vec{L}$</td>
<td>$\vec{L} \rightarrow -\vec{L}$</td>
</tr>
<tr>
<td>Spin</td>
<td>$\vec{S} \rightarrow \vec{S}$</td>
<td>$\vec{S} \rightarrow \vec{S}$</td>
<td>$\vec{S} \rightarrow -\vec{S}$</td>
</tr>
<tr>
<td>Charge</td>
<td>$q \rightarrow -q$</td>
<td>$q \rightarrow q$</td>
<td>$q \rightarrow q$</td>
</tr>
<tr>
<td>Electric Field</td>
<td>$\vec{E} \rightarrow -\vec{E}$</td>
<td>$\vec{E} \rightarrow -\vec{E}$</td>
<td>$\vec{E} \rightarrow \vec{E}$</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>$\vec{B} \rightarrow -\vec{B}$</td>
<td>$\vec{B} \rightarrow \vec{B}$</td>
<td>$\vec{B} \rightarrow -\vec{B}$</td>
</tr>
</tbody>
</table>

Table 1.1: Behavior of standard quantities under the influence of charge conjugation $C$, parity inversion $P$, and time reversal $T$.

A resurgence of interest in these searches has followed from the Lorentz- and $CPT$-violating formalism of the Standard Model Extension (SME) developed by Alan Kostelecký and colleagues [9, 10, 17]. This formalism provides an important framework for identifying Lorentz- and $CPT$-violating interactions and connecting theoretical predictions to experimental observations.

### 1.2.1 Standard Model Extension

The SME provides the most general framework for considering spontaneous Lorentz violation at the level of the Standard Model. It considers all possible Lorentz-violating
terms based on the $SU(3) \times SU(2) \times U(1)$ gauge structure by combining all Lorentz-violating operators and controlling coefficients to form observer-invariant terms. Consideration of all possible Lorentz-violating terms is challenging, so the minimal SME restricts operators to mass dimension less than or equal to 4 [26]. The SME is a viable framework in that it maintains energy-momentum conservation, observer Lorentz covariance, conventional quantization, hermiticity, microcausality, and power-counting renormalizability [17]. Here, it is particle Lorentz violation that is spontaneously broken rather than observer Lorentz violation (Section [1.1.1]). The advantage of the SME is that it considers all possible mechanisms based on the Standard Model, and it can then be used as a lexicon of possible Lorentz- and CPT-violating interactions where identification of a particular interaction can validate or disprove classes of theories.

The entire SME is large and much like the Standard Model rarely written out explicitly. A special case is often useful to consider, particularly in the context of an experiment. For a spin-1/2 fermion $\psi$ with mass $m$, the minimal SME includes the usual Standard Model lagrangian and the following dimension 3- and 4-operators that lead to Lorentz-violating terms [17]

$$L = \frac{1}{2} \bar{\psi} \Gamma_\nu \stackrel{\leftrightarrow}{\partial}^\nu \psi - \bar{\psi} M \psi$$

(1.6)

where\(^1\)

$$M = m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu}$$

(1.7)

and

$$\Gamma_\nu = \gamma_\nu + c_{\mu\nu} \gamma^\mu + d_{\mu\nu} \gamma_5 \gamma^\mu + e_\nu + i f_\nu \gamma_5 + \frac{1}{2} g_{\lambda\mu\nu} \sigma^{\lambda\mu}.$$  

(1.8)

The coefficients $a_\mu$, $b_\mu$, $c_{\mu\nu}$, $d_{\mu\nu}$, $e_\mu$, $f_\nu$, $g_{\lambda\mu\nu}$, and $H_{\mu\nu}$ determine the magnitude of the possible Lorentz-violating interactions and are real since $L$ is hermitian. The coefficients in $M$ have dimensions of mass and the coefficients in $\Gamma_\nu$ are dimensionless. In

\(^1\)The notation $A \stackrel{\leftrightarrow}{\partial}^\mu \equiv A \partial^\mu B - (\partial^\mu A)B$. 
addition, \(a_\mu, b_\mu, c_\mu, f_\mu,\) and \(g_{\lambda\mu}\) are CPT-odd while \(c_{\mu\nu}, d_{\mu\nu},\) and \(H_{\mu\nu}\) are CPT-even. Reference [17] discusses a number of suppression factors, and each of the coefficients have been shown to be small [27].

Equation [1.6] leads to modifications to the equation of motion as described in Ref. [10]. Consideration of only nonzero \(a_\mu\) and \(b_\mu\) values produces the following lagrangian,

\[
\mathcal{L} = \frac{1}{2} i \overline{\psi} \gamma^\mu \overleftrightarrow{\partial_\mu} \psi - a_\mu \overline{\psi} \gamma^\mu \psi - b_\mu \overline{\psi} \gamma^5 \gamma^\mu \psi - m \overline{\psi} \psi.
\]  

(1.9)

The variational principle yields

\[
(i \gamma^\mu \partial_\mu - a_\mu \gamma^\mu - b_\mu \gamma^5 \gamma^\mu - m) \psi = 0
\]

(1.10)

which reduces to the Dirac Equation for a free particle when \(a_\mu = b_\mu = 0\). The constant background fields \(a_\mu\) and \(b_\mu\) transform as two 4-vectors under observer Lorentz transformations and as eight scalars under particle Lorentz transformations. Observer Lorentz violation remains invariant while particle Lorentz violation is partly broken and leads to observable deviations in the free particle trajectory. The SME assumes that the Lorentz-violating fields are constant across the scale of the solar system, so Earth-based searches rely on interactions that vary over the course of a sidereal day (Fig. 1.2).

### 1.2.2 Nuclear Spin Anisotropy Searches

Several Hughes-Drever Lorentz violation searches have been performed over the years using a variety of atomic species. These experiments search for an energy shift of a spin compared to the local celestial frame. The SME predicts several types of energy shifts that would appear as sidereal, semisidereal\(^2\), annual, and fixed-frame changes in the measured energy. Comparing the limits on the SME coefficients across

\(^2\)Second harmonic of sidereal oscillations.
different species is difficult due to the nuclear structure and geometrical factors involved [17]. However, it is relatively straightforward to compare the limits in terms of measured precession frequency. Figure 1.3 displays a timeline of improvements to Lorentz violation limits for nuclear spins in terms of the approximate precession frequency measured. The approximate measured values, species, and references are listed in Table 1.2. In general, the limits on an anomalous neutron spin coupling are consistently better than those for the proton, and a dramatic improvement in the precision of the measurement occurs from the use of differential measurements between two different spin species.

Prior to the work of this thesis, the most sensitive test was performed for the neutron using the Harvard-Smithsonian $^3$He-$^{129}$Xe dual noble-gas spin-maser [29]. This thesis uses a K-$^3$He comagnetometer to reach an energy resolution$^3$ of 0.7 nHz to improve the previous limit by a factor of 30. This result was first reported in Ref. [30] with further details described throughout this work.

$^3$The limit is closer to 1 nHz as listed in Table 1.2.
Figure 1.3: Summary of nuclear spin anisotropy searches in terms of the approximate spin frequency precision.

<table>
<thead>
<tr>
<th>Spin Species</th>
<th>Frequency</th>
<th>Author</th>
<th>Reference</th>
<th>Anisotropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7\text{Li}$</td>
<td>8 Hz</td>
<td>Hughes et al.</td>
<td>[24]</td>
<td>Sid.</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>40 mHz</td>
<td>Drever</td>
<td>[25]</td>
<td>Sid.</td>
</tr>
<tr>
<td>$^9\text{Be}^+$</td>
<td>100 $\mu$Hz</td>
<td>Prestage et al.</td>
<td>[31]</td>
<td>Sid./Semisid.</td>
</tr>
<tr>
<td>$^{199}\text{Hg}^{201}\text{Hg}$</td>
<td>1 $\mu$Hz</td>
<td>Lamoreaux et al.</td>
<td>[32]</td>
<td>Sid./Semisid.</td>
</tr>
<tr>
<td>$^{21}\text{Ne}^{3}\text{He}$</td>
<td>1 $\mu$Hz</td>
<td>Chupp et al.</td>
<td>[33]</td>
<td>Semisid.</td>
</tr>
<tr>
<td>Cs$^{199}\text{Hg}$</td>
<td>100 nHz</td>
<td>Berglund et al.</td>
<td>[34]</td>
<td>Sid.</td>
</tr>
<tr>
<td>H-maser</td>
<td>500 $\mu$Hz</td>
<td>Phillips et al.</td>
<td>[35]</td>
<td>Sid.</td>
</tr>
<tr>
<td>$^{129}\text{Xe}/^{3}\text{He}$-maser</td>
<td>50 nHz</td>
<td>Bear et al.</td>
<td>[29, 36]</td>
<td>Sid.</td>
</tr>
<tr>
<td>$^{129}\text{Xe}/^{3}\text{He}$-maser</td>
<td>150 nHz</td>
<td>Canè et al.</td>
<td>[37]</td>
<td>Annual</td>
</tr>
<tr>
<td>$^{3}\text{He}/^{129}\text{Xe}$</td>
<td>60 nHz</td>
<td>Kornack</td>
<td>[28]</td>
<td>Sid.</td>
</tr>
<tr>
<td>K$^{3}\text{He}$</td>
<td>100 $\mu$Hz</td>
<td>Wolf et al.</td>
<td>[38]</td>
<td>Sid./Semisid./Fixed</td>
</tr>
<tr>
<td>Ultracold neutron</td>
<td>10 $\mu$Hz</td>
<td>Altarev et al.</td>
<td>[39]</td>
<td>Sid.</td>
</tr>
<tr>
<td>K$^{3}\text{He}$</td>
<td>1 nHz</td>
<td>Brown et al.</td>
<td>[30]</td>
<td>Sid.</td>
</tr>
<tr>
<td>$^{3}\text{He}/^{129}\text{Xe}$</td>
<td>5 nHz</td>
<td>Gemmel et al.</td>
<td>[40]</td>
<td>Sid.</td>
</tr>
</tbody>
</table>

Table 1.2: Summary of nuclear spin anisotropy searches in terms of the approximate spin frequency precision. Entries are in order of publication date. Sidereal and semisidereal are abbreviated by Sid. and Semisid. respectively. In the case of multiple measurements, a representative value has been selected.
1.3 Dissertation

This dissertation describes the use of the K-$^3$He comagnetometer to perform a Hughes-Drever experiment to search for a Lorentz- and $CPT$-violating field coupling to the nuclear spin of $^3$He. The result is interpreted in terms of the SME and sets a new limit on $\tilde{b}_\perp$ for the neutron that improves upon the previous limit by a factor of 30. This second generation experiment supersedes its first generation predecessor by improving upon the long term stability of the apparatus using compact design and evacuated bell jar to enclose all of the lasers and optics. In addition, the entire apparatus is placed on a rotating platform for lock-in modulation of the proposed Lorentz-violating field every 22 s. Throughout this experiment, many new aspects of comagnetometer operation were also explored.

1.3.1 K-$^3$He Comagnetometer

This thesis refers to the atomic species K and $^3$He in a K-$^3$He comagnetometer. The comagnetometer consists of two overlapping spin ensembles, electrons in a polarized K vapor and nuclear spins in a hyper-polarized $^3$He buffer gas in a spherical vapor cell. Where appropriate, such as spin-exchange optical pumping, these species will be denoted (a)-alkali and (b)-noble. In the context of precision measurements and the comagnetometer, we will refer to the electrons (e) and nuclear (n) species.

The K-$^3$He comagnetometer suppresses magnetic interactions but remains sensitive to non-magnetic spin couplings, specifically, the difference in coupling between electron and nuclear spins. The behavior of these spin ensembles is unique in that the strong coupling between the electrons and nuclear spins quickly damps precession of the nuclear spins to the steady state within several seconds. The steady state response provides a measurement of anomalous spin couplings, and the fast damping to the steady state allows for frequent reversals of the apparatus. The comagnetometer
is also a sensitive gyroscope [41] which provides a large background response from the rotation of the Earth and a complication in a Lorentz violation search.

1.3.2 Unit Conventions

The natural choice of units in this thesis are fT. Presentation of results in GeV, eV, gauss, or Hz would also be acceptable, though historical development of the $K^3$He comagnetometer from high sensitivity magnetometers leads to the choice of fT. Readers dissatisfied with this selection can reference Sections 2.3.2 and 5.4.4 for conversions to their preferred unit.

A Lorentz-violating field is characterized by a sidereal variation in the energy shift of the spin. For convenience, long term measurements are recorded in sidereal days from January 1, 2000, as is convention in the field. One sidereal day is the length of time it takes the earth to rotate on its axis and precess in its orbit to reorient to the same celestial reference point. This time is roughly 23 h 56 min. It is also convention to distinguish between Local Sidereal Time (LST) and Greenwich Mean Sidereal Time (GMST). This distinction depends on the location of the measurement where LST differs from GMST in the relative longitude of the location of the measurement to Greenwich, England. Technical details on these conventions are included in Appendix A.

1.3.3 Structure

Chapter 2 provides a brief background of spin-exchange optical pumping, magnetometry, and coupled spin-dynamics. These insights allow for a complete description of the $K^3$He comagnetometer and the apparatus in Chapter 3. Two installations of the $K^3$He comagnetometer have been used for two related experiments in this thesis. Development of our modern comagnetometer apparatus extends over ten years with contributions from several students and post-doctoral researchers. The main
focus of this thesis is on the second generation apparatus to set a new limit on a Lorentz-violating field in Chapter 5. This measurement represents the highest energy resolution in any spin anisotropy measurement to date [30]. In performing this search, many new features of the K-3He comagnetometer were discovered and are described in Chapter 4. A search for a proposed long-range spin-dependant forces has been performed using a hyper-polarized 3He spin source. The main details of this experiment are in Refs. [42, 43] and will not be reported here. This measurement achieves a frequency resolution of 18 pHz which represents the highest energy resolution of any experiment. Specific features of the spin source including its construction and design not included in these references are discussed in Chapter 6.
Chapter 2

Background

The K-$^3$He comagnetometer described in Chapter 3 and the hyper-polarized $^3$He spin source described in Chapter 6 consist of glass vapor cells containing a hot K vapor and a high density $^3$He buffer gas. Circularly polarized light on resonance with the D1 transition in K polarizes the alkali-metal vapor through optical pumping. Spin-exchange collisions with the buffer gas transfer polarization to the nucleus of $^3$He through the hyperfine interaction. This chapter provides a brief summary of the relevant physics of these polarized spin ensembles. This is a well-developed field with many readily available references. The interested reader is encouraged to explore resources beyond this text for more details not included in the summary presented here.

2.1 Alkali-Metal Vapor

Before considering the interaction of K and $^3$He, it is useful to consider the properties of a K vapor in several amagats$^1$ of $^3$He buffer gas ($10^{20}$ atoms/cm$^3$). Atomic K has filled inner electron shells and a single valence electron. The hydrogen-like energy level

\footnote{An amagat is the a unit of number density equivalent to 1 atm of an ideal gas at 0°C. Confusion can often arise in defining this in terms of standard temperature and pressure. Ultimately, the number density of interest is $2.68 \times 10^{19}$/cm$^3$.}
structure simplifies treatment of the atomic structure. Collisions with the high density $^3$He buffer gas distort the electron wavefunction and further simplify discussion of the relevant atomic structure.

### 2.1.1 Energy Levels

In the ground state, the outermost electron in K resides in the $4S_{1/2}$ state. The atomic fine structure describes the splitting of the nearby $4P$ state into the $4P_{1/2}$ and $4P_{3/2}$ states from a combination of spin-orbit coupling and relativistic corrections (Fig. 2.1). The spin-orbit interaction $H_{so} = \alpha \vec{L} \cdot \vec{S}$ creates eigenstates of total angular momentum $\vec{J} = \vec{L} + \vec{S}$. The hyperfine interaction $H_{hf} = A \vec{I} \cdot \vec{S}$ further splits each of these states $\vec{F} = \vec{I} + \vec{J}$, but they are typically optically unresolved due to the pressure broadening in several amagats of $^3$He buffer gas. The hyperfine splitting will be relevant in Section 2.4.2 and 6.3.1 and addressed in context. In the absence of possible pressure shifts from the $^3$He buffer gas, the $D_1$ and $D_2$ transitions are at 770.1 nm and 766.8 nm respectively.

### 2.1.2 Light Absorption

Commercially available near-infrared lasers with linewidths less than 1 MHz provide a straightforward means to address the $D_1$ transition in K. In an atomic vapor, the
absorption of unpolarized photons near resonance as a function of propagation along
the z-direction follows Beer’s Law

\[ I(z) = I_0 e^{-n_a \sigma(\nu) z} \]  

(2.1)

where \( n_a \) is the alkali density, \( \sigma(\nu) \) is the frequency dependant cross section, and \( I_0 \) is the initial intensity. The total absorption over the entire propagation length \( L \) can be interpreted in terms of the optical depth \( OD \) and is given by

\[ OD = n_a \sigma(\nu) L \]  

(2.2)

where \( OD \gg 1 \) is referred to as “optically thick” and \( OD \ll 1 \) is referred to as “optically thin”. The frequency dependant cross section is given by

\[ \sigma(\nu) = cr_e f L(\nu) \]  

(2.3)

where \( c \) is the speed of light, \( r_e = 2.82 \times 10^{-15} \) m is the classical electron radius, and \( f \) is the oscillator strength of the transition\(^2\). The function \( L(\nu) \) describes the absorption lineshape

\[ L(\nu) = \frac{\Delta \nu/2}{(\nu - \nu_0)^2 + (\Delta \nu/2)^2} \]  

(2.4)

where \( \Delta \nu \) is the full-width at half maximum (FWHM) of the transition and \( \nu_0 \) is the central frequency. On resonance, the cross section reduces to

\[ \sigma(\nu_0) = \frac{2cr_e f}{\Delta \nu} \]  

(2.5)

---

\(^2\)The oscillator strength of the \( D_1 \) and \( D_2 \) transitions are 1/3 and 2/3 respectively.
and integrating the cross section over all frequencies produces

$$\sigma = \int_{-\infty}^{+\infty} \sigma(\nu)d\nu = \pi cr_e f. \quad (2.6)$$

A derivation of these statements is available in Ref. [44].

Several mechanisms determine the width of the resonance including the natural lifetime, pressure broadening, and doppler broadening. In the presence of several amagats of $^3$He, $\Delta \nu$ is dominated by the pressure broadening. The measured value\(^3\) is 13.2 GHz/amg for the FWHM \([45]\). The pressure of $^3$He is known at the time of cell filling following the procedure described in Ref. [43].

At room temperature, K is a solid metal with a small vapor pressure ($7 \times 10^8$/cm\(^3\)). Heating the cell to nearly 200°C can significantly increase the vapor density to $10^{14}$/cm\(^3\). The density follows the well-known empirical expression

$$n_a = \frac{10^{26.2682 - (4453 \text{ K})/T}}{(1 \text{ K}^{-1})T}/\text{cm}^3 \quad (2.7)$$

where $T$ is the temperature in Kelvin \([46]\). In practice, the temperature of a vapor cell indicates a 10-20% larger alkali density than is observed. The sensor is always external to the cell and measures the temperature at a single location. It is believed that the coldest region of the cell determines the alkali density. In addition, there are a number of surface effects that allow alkali atoms to be adsorbed into the glass, thereby lowering the effective density at a given temperature \([47]\). Nevertheless, Eq. (2.7) provides an reasonable estimate of the density at a given temperature.

\(^3\)This measurement is for the pressure broadening of K in a $^4$He environment. The value may be different for $^3$He, but it is expected to be within a few percent.
2.1.3 Lightshifts

The presence of an oscillating light field on the atoms induces an ac-Stark shift on the atomic vapor. This lightshift is equivalent to the effect of a dc magnetic field along the direction of light propagation and is commonly referred to as a lightshift. The energy shift is zero on resonance and dispersive in frequency

\[ D(\nu) = \frac{(\nu - \nu_0)}{(\nu - \nu_0)^2 + (\Delta\nu/2)^2} \]  

rather than absorptive as in Eq. 2.4. The lightshift also depends on the degree of circular polarization of light where linearly polarized light produces a zero lightshift. In units of equivalent magnetic field, the lightshift can be expressed as \[48\]

\[ \vec{L} = \frac{r_e c f \Phi \vec{s}}{\gamma_e A} D(\nu) \]  

where \( \Phi \) is the photon flux, \( A \) is the cross sectional area of the incident beam, \( \gamma_e \) is the electron gyromagnetic ratio (Section 2.3), and \( \vec{s} \) is the degree of circular polarization of the light \[4\]. Considering the dispersive nature of the lightshift, it is useful to consider the effects of both the closely spaced \( D_1 \) and \( D_2 \) transitions. They have opposite signs and distinct oscillator strengths, so the net effect is

\[ \vec{L} = \frac{r_e c f \Phi \vec{s}}{3\gamma_e A} \left( -D(\nu_{D1}) + 2D(\nu_{D2}) \right) \]  

where the signs and oscillator strengths have been explicitly inserted. A semi-classical derivation of this effect is available in Refs. \[49, 50\].

\[4\] Circularly polarized light provides \( |\vec{s}| = 1 \) and linearly polarized light is \( \vec{s} = 0 \)
2.1.4 Optical Pumping

A complete description of optical pumping is available in Ref. [51]. For simplicity, we ignore the nuclear spin and consider only pumping of the electron spin considering only the $4S_{1/2}$ and $4P_{1/2}$ states in the simplified energy level diagram in Fig. 2.2. Unpolarized atoms start with equal populations in the ground state sublevels. Circularly polarized light on resonant with the $4S_{1/2} \rightarrow 4P_{1/2}$ transition drives transitions with selection rules $\Delta m_J = \pm 1$. Left-hand, circularly polarized light $\sigma_+$ drives transitions from the $m_J = -1/2$ ground state to the $m_J = +1/2$ excited state. Collisions with buffer gas atoms mix the excited state populations. 50 torr of N$_2$ buffer gas provides quenching without reradiation of a photon in a random direction. In this way, angular momentum is transferred from the photons to the vapor as evidenced by a large fraction of the vapor pumped to the $m_J = +1/2$ ground state.
Optical pumping can be expressed quantitatively in terms of the polarization

\[
\frac{d\vec{P}}{dt} = R_p(\vec{s}_p - \vec{P}) - R_{sd}\vec{P}
\]

(2.11)

where \(|\vec{P}| = 1\) corresponds to 100% polarization of the vapor, \(R_p\) is the pumping rate, \(\vec{s}_p\) is the degree of circular polarization, and \(R_{sd}\) is the spin-destruction rate. The pumping rate is related to the intensity of the light as

\[
R_p = \frac{I\sigma(\nu)}{h\nu},
\]

(2.12)

and \(R_{sd}\) is a combination of several spin-destruction processes. For instance, collisions with \(^3\)He, \(\text{N}_2\), and other K atoms transfer angular momentum to an unpolarized spin of \(^3\)He to the rotational and translational degrees of freedom of the vapor in the formation of a van der Waals molecule. For pumping along the the z-axis, the equilibrium polarization becomes

\[
P_z = \frac{R_p}{R_p + R_{sd}}\vec{s}_p \cdot \hat{z}.
\]

(2.13)

In principle, the pumping rate decreases as a function of the polarization of the vapor. A circularly polarized pump beam propagating through an alkali vapor is described by

\[
\frac{dR_p}{dz} = -n_a\sigma(\nu)(1 - P_z)R_p
\]

(2.14)

where only unpolarized atoms are pumped, and \(\sigma(\nu)\) accounts for the detuning from resonance. In the absence of optical pumping, \(R_p(z)\) is simply a decaying exponential as in Eq. 2.1. In a fully polarized vapor, the pumping rate is constant. The general
solution to Eq. 2.14 can be written in terms of the product logarithm (ProductLog\footnote{The product logarithm is the inverse function of }f(w) = we^{w}. It is also known as the Lambert W-function or omega function.)

\[ R_p(z) = R_{sd} \text{ProductLog}\left[ e^{-n_a \sigma(\nu) z + R_p(0)/R_{sd} R_p(0)/R_{sd}} \right] \] (2.15)

The typical polarization lifetime of a polarized K vapor is 70 ms. This is determined by

\[ \frac{1}{T_1} = R_p + R_{sd} + R_{wall} \] (2.16)

where interaction with the wall of the vapor cell will also destroy the spin. The large buffer gas pressures in the cell limit the diffusion of atoms to the walls. The diffusion constant for K in a $^3$He buffer gas is

\[ D_K = 0.35 \text{ cm}^2/\text{s} \left( \frac{\sqrt{1 + T/(273.15 \text{ K})}}{n_b/(1 \text{ amagat})} \right) \] (2.17)

where $T$ is the temperature in Kelvin and $n_b$ is the density in amagats of $^3$He. For a cell of several cm, this timescale is on the order of seconds, so this relaxation can be safely ignored in this work. A thorough and instructive discussion of many of these issues is available in Ref. \cite{1}.

A common method to monitor the degree of polarization of the vapor is through optical rotation of linearly polarized light. The index of refraction for the left $\sigma_+$ and right $\sigma_-$ circularly polarized light in a polarized medium is different depending on the projection of the polarization along the direction of propagation. Linearly polarized light is an equal superposition of both $\sigma_+$ and $\sigma_-$ light, so the plane of linear polarization will rotate through a polarized medium. In a polarized alkali-metal vapor, a linearly polarized probe beam will rotate

\[ \phi = \frac{1}{2} l r e c f n_a P_x D(\nu) \] (2.18)
where \( l \) is the propagation length and \( P_x \) is the projection of the polarization along the propagation of the probe beam \([48]\). Notice that the optical rotation is zero if the light is on resonance. This is because the real part of the index of refraction is equal to 1 on resonance \([44]\). These ideas will be more fully explored in Section 2.4 in the context of an atomic magnetometer.

### 2.2 Spin-Exchange Optical Pumping

When spin-polarized K atoms collide with the \(^3\text{He}\) buffer gas, spin angular momentum transfers between the electron spin \( \vec{S} \) in K and the nuclear spin \( \vec{K} \) of \(^3\text{He}\) through the hyperfine interaction \( H_{hf} = -A \vec{K} \cdot \vec{S} \) in spin-exchange collisions. This is a well-developed field and a comprehensive review is available in Ref. \([53]\) and references therein. Large \(^3\text{He}\) polarizations of up to 81% has been achieved in optimized systems \([54]\). There are many details to consider, but this section serves to highlight aspects of this technique applicable to the K-\(^3\text{He}\) comagnetometer and the hyper-polarized \(^3\text{He}\) spin source.

#### 2.2.1 Spin-Exchange Collisions

Figure 2.3 illustrates polarization of the \(^3\text{He}\) buffer gas where polarized K atoms collide with unpolarized \(^3\text{He}\). The total angular momentum is conserved in the collision and spin polarization is transferred from the electron in K to the nucleus of \(^3\text{He}\). The closed electron shell in \(^3\text{He}\) isolates the nuclear spin from the environment such that this process occurs over many hours. Similarly, once polarized, the \(^3\text{He}\) polarization will relax over hours to days in a suitable environment. This is in stark contrast to the typical polarization lifetimes of 70 ms in a polarized K vapor.
Given the long polarization lifetime of $^3$He, it is important to consider $^3$He diffusion through the cell given by

$$D_{^3\text{He}} = 1.2 \text{ cm}^2/\text{s} \left( \frac{1 + T/(273.15 \text{ K})}{n_b/(1 \text{ amagat})} \right)$$

(2.19)

which is similar to Eq. 2.17 [52] and yields $D_{^3\text{He}} \simeq 0.2 \text{ cm}^2/\text{s}$ for 10 amagats of $^3$He at 180°C. This implies that polarized $^3$He will explore the entire cell over its lifetime over several hours. The $^3$He polarization $\vec{P}^b$ is described by

$$\frac{d\vec{P}^b}{dt} = R_{ne}^{\text{se}} (\langle \vec{P}^a \rangle - \vec{P}^b) - R_{sd}^{n} \vec{P}^b$$

(2.20)

where this expression holds for a spin-1/2 nuclear spin$^a$ and $\langle \vec{P}^a \rangle$ represents a spatial average of the K polarization. The nuclear spin-exchange rate $R_{se}^{ne}$ can be determined in terms of an effective cross section

$$R_{se}^{ne} = n_a \sigma_{se} \bar{v}$$

(2.21)

$^6$An extra factor is required for nuclei with higher moments [53].

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Figure 2.3: a) Circularly polarized photons polarize K spins through optical pumping. Collisions transfer angular momentum to the nucleus of $^3$He through the hyper-fine interaction. b) A spin-exchange collision preserves total angular momentum between K and $^3$He.
where $\bar{v} = \sqrt{8kT/(\pi\bar{m})}$ and $\bar{m}$ is the reduced mass of K and $^3$He. The nuclear spin-destruction rate $R_{sd}^n$ is a combination of a spin-destruction cross section with K where the angular momentum is lost to the vapor and some specific $^3$He relaxation mechanisms described in the next section.

### 2.2.2 Longitudinal Relaxation: $T_1$

Long $T_1$ times exceeding several days have been observed in hyper-polarized $^3$He systems. The dominant contributions to this relaxation is from $^3$He dipole-dipole collisions, relaxation along magnetic field gradients, and wall relaxation. The total relaxation rate observed is a contribution of each of these mechanisms

$$\frac{1}{T_1} = \frac{1}{T_{1dd}} + \frac{1}{T_{1\nabla}} + \frac{1}{T_{1\text{wall}}}. \quad (2.22)$$

The dominant relaxation in the spin source is the dipole-dipole relaxation given by

$$\frac{1}{T_{1dd}} = \frac{n_b}{744 \text{ amagat} \cdot \text{hour}} \quad (2.23)$$

where $n_b$ is the density in amagats [55]. The 12 amagat spin source exhibits relaxation times near the 60 hour limit determined by the dipole-dipole collision rate. In the K-$^3$He comagnetometer, magnetic field gradients are the dominant relaxation mechanism. This rate is given by

$$\frac{1}{T_{1\nabla}} = D_{^3\text{He}} \frac{|\nabla B_\perp|^2}{B_z^2} \quad (2.24)$$

where the notation $|\nabla B_\perp|^2 = |\nabla B_x|^2 + |\nabla B_y|^2$ and $B_z$ represents the magnitude of the magnetic field along the direction of polarization [56, 57]. Diffusion along transverse magnetic field gradients leads to relaxation of the spins. The gradients in the case of

---

7The notation for $R_{se}^{en}$ will clearer in the context two spin ensembles in Section 2.5.2 and the description of the K-$^3$He comagnetometer in Chapter 3. For instance, $R_{se}^{en} = n_b\sigma_{se}\bar{v}$. For instance, $R_{se}^{en} = n_b\sigma_{se}\bar{v}$. 24
the comagnetometer are limited by the nonuniform dipolar field created by uniformly polarized $^3$He with more details provided in Section 4.6.1.

Despite the isolation of the $^3$He nucleus by the closed shell of electrons, collisions with the wall can lead to further relaxation. This process is little understood, but the most comprehensive and recent $^3$He wall relaxation studies appear to be in Refs. 58, 59, 60 and references therein. There is some "black magic" in cell manufacture to avoid issues of wall-relaxation. In most cases in this work, significant wall relaxation has been avoided except as described in Section 4.6.2.

Including all of these effects, the equilibrium $^3$He polarization with a polarized K vapor along the z-direction is given simply by

$$P^b_z = \frac{R^{ne}_{se}}{R^{ne}_{se} + R^{n}_{sd}} \langle P^a_z \rangle$$

and reaches equilibrium in several time constants of $1/(R^{ne}_{se} + R^{n}_{sd})$.

### 2.3 Larmor Precession

The Lorentz violation search attempts to measure an energy shift in the spin precession frequency as an experimental signature of new physics. Both the K and $^3$He spins have an associated magnetic moment coupling to a magnetic field as does a classical magnetic dipole. Under the influence of a magnetic field, the spin will precess at the Larmor frequency. The following sections provide details on this phenomena and clearly define the sign convention of spin precession followed in this work.

#### 2.3.1 Isolated Spin

The interaction Hamiltonian for a magnetic moment in a magnetic field is

$$H = -\vec{\mu} \cdot \vec{B}.$$ (2.26)
Figure 2.4: Evolution of $\langle \vec{S} \rangle$ in a $B_z$ magnetic field as in Eq. (2.28). Here, the magnetic moment is anti-aligned with the spin.

For the case of the free electron, $\vec{\mu} = -g_S \mu_B \vec{S}$ where $g_S \approx 2$ and $\mu_B = 9.28476 \times 10^{-24} \text{J/T}$ is the Bohr magneton and yields

$$H = g_S \mu_B \vec{S} \cdot \vec{B}. \quad (2.27)$$

The time evolution of this spin in a magnetic field is known as Larmor precession. Using the time-dependant Schrödinger equation, the following equation of motion describes the expectation value of a spin in a magnetic field

$$\frac{d\langle \vec{S} \rangle}{dt} = \frac{g_S \mu_B}{\hbar} \vec{B} \times \langle \vec{S} \rangle = \gamma_e \vec{B} \times \langle \vec{S} \rangle \quad (2.28)$$

where $\gamma_e$ is the electron gyromagnetic ratio. This agrees with the classical result of the torque experienced by a magnetic moment in a magnetic field. The electron will precess about a magnetic field at frequency $\omega = \gamma_e |\vec{B}|$ (Fig. 2.4).

**Rotating Frame**

Classically, the time derivative of any time-dependent vector in the lab frame $d\vec{A}/dt$ and the time derivative of the same vector in the rotating frame $\partial \vec{A}/\partial t$ are related
by
\[
\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \vec{\Omega} \times \vec{A}
\]  
(2.29)
where \( \vec{\Omega} \) is the rotation vector. In the rotating frame, Eq. 2.28 transforms to
\[
\frac{\partial \langle \vec{S} \rangle}{\partial t} = \gamma_e \left( \vec{B} - \frac{\vec{\Omega}}{\gamma_e} \right) \times \langle \vec{S} \rangle.
\]  
(2.30)
Throughout this thesis, we use total derivatives to denote physics in the lab frame and partial derivatives to denote physics in the rotating frame. There are many roads to this result, both classical and quantum mechanical with a good summary provided in Ref. [61].

### 2.3.2 Bound Electron

In atomic K, the total angular momentum of the electron modifies the precession frequency depending on the atomic state and can be determined following the argument from Ref. [62]. The simplest case considers the electron magnetic moment as the sum of orbital and spin angular momenta
\[
\vec{\mu} = -\mu_B (g_S \vec{S} + g_L \vec{L}) = -\mu_B (\vec{J} + \vec{S})
\]  
(2.31)
where as usual \( g_S \approx 2 \) and \( g_L = 1 \). Here, \( \vec{J} = \vec{L} + \vec{S} \) is a “good” quantum number, so \( \vec{S} \) must be evaluated in the \( J \)-basis. It follows that
\[
\langle \vec{\mu} \rangle = -\mu_B \left( \langle \vec{J} \rangle + \langle \vec{S} \rangle \right) = -\mu_B \left( 1 + \frac{\langle \vec{S} \cdot \vec{J} \rangle}{J(J+1)} \right) \langle \vec{J} \rangle.
\]  
(2.32)
A specific case of the well-known Wigner-Eckhart Theorem known as the Projection Theorem \cite{14} gives

\[
\langle \alpha', jm' | V_q | \alpha, jm \rangle = \langle \alpha', jm' | \vec{J} \cdot \vec{V} | \alpha, jm \rangle \frac{j(j+1)}{2\hbar^2} \langle jm' | J_q | jm \rangle ,
\] (2.33)

to evaluate $\langle \vec{S} \rangle$ in the $J$-basis. The product $\langle \vec{S} \cdot \vec{J} \rangle$ is straightforward to evaluate to yield the g-factor for this state $\vec{\mu} = -g_J \mu_B \vec{J}$ where

\[
g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.
\] (2.34)

This result modifies Eq. 2.28 by replacing $g_S$ with $g_J$ in the gyromagnetic ratio and modifies the precession depending on the electronic state.

For $K$ in the $4S_{1/2}$ ground state, $J = 1/2$, $L = 0$, and $S = 1/2$ which yields $g_J = 2$, the same result as the g-factor for the free electron.

**Nuclear Spin Contribution**

The argument in the previous section neglects the contribution to the total angular momentum from the nucleus. For a complete treatment, the hyperfine interaction must be included in the magnetic moment

\[
\vec{\mu} = -g_F \mu_B \vec{F} = -\mu_B (g_J \vec{J} + g_I \vec{I})
\] (2.35)

where $\vec{F} = \vec{J} + \vec{I}$. Since the nuclear moment is several orders of magnitude smaller than the Bohr magneton, the approximation $g_I/g_J \ll 1$ neglects the second term in Eq. 2.35. Similar to Eq. 2.32, $\vec{J}$ must be expressed in terms of $\vec{F}$. Once again, the Projection Theorem proves useful with

\[
\langle \vec{J} \rangle = \frac{\langle \vec{J} \cdot \vec{F} \rangle}{F(F+1)} \langle \vec{F} \rangle
\] (2.36)
where \(\langle \vec{J} \cdot \vec{F} \rangle\) is simple to evaluate in the \(\vec{F}\)-basis. This produces

\[
g_F = g_J = \left( \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} \right)
\]

(2.37)

for easy replacement in Eq. 2.28 for any well-defined total angular momentum state.

Considering the general case of the outer shell electron in an alkali atom in the \(S_{1/2}\) ground state where \(J = 1/2\) and \(F = I \pm 1/2\), produces the well-known result

\[
\frac{\partial \langle \vec{S} \rangle}{\partial t} = \frac{g_F \mu_B}{\hbar} \vec{B} \times \langle \vec{S} \rangle = \pm \frac{\gamma_e}{2I+1} \vec{B} \times \langle \vec{S} \rangle.
\]

(2.38)

The rate of electronic precession appears to “slow down” in response to dragging the heavy nuclear spin. The slowing down increases for larger nuclear spins. The valence electron precesses in opposite directions depending on the hyperfine state, and the state with maximal angular momentum \(F = I + 1/2\) follows the same sign convention as the free electron. For \(^{39}\text{K}\), \(I = 3/2\), so \(\gamma_{\text{eff}} = \gamma_e/4\).

The precession of the electron in K is modified by its nonzero nuclear spin and the direction of precession depends on the hyperfine level. The spin precession of \(^{3}\text{He}\) is much less complex. In the ground state, the outer shell elections are paired so only the spin-1/2 nucleus is free to precess. The nuclear magnetic moment is a factor of \(10^3\) smaller than the electron magnetic moment and will precess at a lower frequency. The gyromagnetic ratios take the values

\[
\gamma_e = \frac{g_{\mu_B}}{\hbar} = 2\pi \times 2.8 \text{ MHz/gauss}
\]

(2.39)

\[
\gamma_n = \frac{\mu_{\text{He}}}{\hbar} = 2\pi \times 3.243 \text{ kHz/gauss}
\]

(2.40)

where we have selected the convention that the gyromagnetic ratio is a positive number. This explicit definition will help with clarity in the equations that follow. Both
the electron in K and nucleus in \(^{3}\)He have negative magnetic moments, so the precession in a magnetic field follows the sign of Eq. 2.28.

### 2.4 Alkali-Metal SERF Magnetometer

Atomic alkali-metal magnetometers use the fundamental precession of the outer shell electron spin in a polarized alkali-metal vapor to determine the local magnetic field. This idea was first proposed by Dehmelt [63] and achieved by Bell and Bloom [64]. The atomic magnetometer has since become a well-developed technology.

Over the decades, systematic improvements in this simple idea have increased sensitivity and reduced technical noise. Most recently, the development of the spin-exchange relaxation free (SERF) magnetometer has extended sensitivity below the fT level [65]. This magnetometer circumvents spin-exchange relaxation to be limited by the slower spin-destruction relaxation. It has achieved some of the best magnetic field measurements to date and has reached the level of 0.16 fT/√Hz in a gradiometer arrangement [66]. The comagnetometer described in this thesis inherits its sensitivity from the SERF, so it suffices to discuss the relevant details here.

#### 2.4.1 Magnetometry

The classic magnetometer scheme is to optically pump an alkali metal vapor with circularly polarized light on resonance (Fig. 2.5). The spins aligned along the z-direction precess under an applied magnetic field in the y-direction (Section 2.3). A linearly polarized off-resonant probe beam measures the projection of the spin along the x-direction (Eq. 2.18). A simple fixed polarizer translates the optical rotation into intensity of the light which is easily measured on a photodiode. This is referred to as the magnetometer signal.
Ignoring factors of 2 and the magnitude of the spin, the fundamental sensitivity of an atomic magnetometer is given by

\[ \delta B \simeq \frac{1}{\gamma \sqrt{NT_2 t}} \]  

(2.41)

where \( \gamma \) is the Larmor frequency of the atoms, \( N \) is the number of atoms in the measurement volume, and \( T_2 \) is the coherence time for the precession and the spins. The sensitivity improves as the square root of the measurement time \( t \). For many years it was believed that \( T_2 \) was always limited by spin-exchange collisions \( 1/T_2 \sim R_{se} \) constraining the best sensitivity to \( 1 \text{ fT cm}^{3/2}/\sqrt{\text{Hz}} \).

### 2.4.2 SERF Regime

The \( 4S_{1/2} \) ground state in \( ^{39}\text{K} \) splits into two hyperfine states \( F = 1 \) and \( F = 2 \). As shown earlier in Section 2.4, each of these states precess in the opposite direction. We can think of this precession as an effective vector (Fig. 2.6a). In an atomic vapor, these atoms undergo spin-exchange collisions in which the total angular momentum \( m_{F_1} + m_{F_2} \) is conserved. This implies that an individual atom has a probability
Figure 2.6: a) Precession of the $F = 1$ and $F = 2$ states in the $4S_{1/2}$ ground state in terms of an effective vector. Spin-exchange causes atoms to switch hyperfine levels and precess in opposite directions. b) Precession of the hyperfine levels in the SERF regime. Spin-exchange occurs more rapidly than precession. Due to the the higher statistical weight in the $F = 2$ state, the net precession is in this direction. c) Population of the hyperfine sublevels at $P^e \sim 50\%$.

of switching hyperfine levels and thus switches spin precession direction with every spin-exchange collision. The switching of precession direction leads to dephasing in the spin precession of the ensemble to limit $T_2$ and the magnetometer sensitivity (Eq. 2.41).

It was observed experimentally [67] and later explored theoretically [68] that a high-density, optically-pumped atomic vapor in a low magnetic field exhibits a longer $T_2$ than expected from consideration of the spin-exchange rate. In the limit where $R_{se}^{ee} \gg \omega_0$, the atoms switch hyperfine states faster than they can precess in the magnetic field. Under optical pumping conditions, the atoms are pumped towards the $|F = 2, m_F = 2\rangle$ end state which carries a greater statistical weight. In this way, the atoms lock to a net precession at a slower rate in the direction of the $F = 2$ state depending on the polarization (Fig. 2.6b).

A precise explanation for the precession of the atoms in the SERF regime is described in [69]. To determine the net precession at an arbitrary polarization, a spin-temperature distribution is assumed (Fig. 2.6c). For a spin-$3/2$ nucleus, we
arrive at the following expression

\[
\frac{\omega}{\omega_{\text{eff}}} = 2 - \frac{4}{3 + P^2}
\]  

(2.42)

where the \( \omega_{\text{eff}} = \gamma_{\text{eff}} |\vec{B}| \). This effect serves to further “slow down” the precession of the outer electron in an alkali atom. We define the following “slowing-down factor” for convenience

\[
Q(P) = 4/(2 - \frac{4}{3 + (P)^2})
\]

(2.43)

where the leading 4 comes from the argument in Section 2.3.2 which is further modified by the SERF regime. \( Q(P) \) varies between 4 for full polarization and 6 for zero polarization. Alkali-metal magnetometers achieve maximum sensitivity at \( P = 50\% \) where \( Q(0.5) = 5.2 \). This factor slows down all of the rates for the alkali atom in the SERF regime.

Despite the rapid switching of hyperfine states, it is interesting that the precession frequency is still on the same order of precession in the absence of spin-exchange collisions. However, this minor change in the precession frequency produces a dramatic change in the coherent precession of the spins.

The spin-exchange rate that contributes to the total relaxation of the spins becomes

\[
R_{se}^{ee} = \left( \frac{\gamma_e B}{Q(P)} \right)^2 \frac{[Q(P)]^2 - (2I + 1)^2}{2R_{se}/Q(P)}
\]

(2.44)

in the limit of low magnetic field \([70]\). The distinguishing feature of this regime is the dependance on the square of the magnetic field. The increase in \( T_2 \) improves the atom shot noise limit for magnetic sensitivity (Eq. 2.41). Spin-destruction collisions \( R_{sd}^{ee} \) have a cross section several orders of magnitude smaller than spin-exchange collisions and limit the fundamental sensitivity to 0.02 fT cm\(^{3/2}\)/\(\sqrt{\text{Hz}}\).
2.4.3 SERF Magnetometer

The SERF magnetometer is a vector magnetometer meaning that it is most sensitive to a particular direction of the magnetic field. In the SERF regime, spin precession is slow, so the spins precess in a small applied field by an angle

\[ \phi \propto T_2 \gamma_e B_y \]  

in the steady state.\textsuperscript{5} A 10 fT field and coherence time of 3 ms gives a spin precession angle of \(5 \times 10^{-6}\) rad. Detection of optical rotation of linearly polarized light near the atomic resonance provides a means to measure such small angles (Eq. 2.18).

The behavior of a magnetometer can be well-described by the phenomenological Bloch equation describing the precession of a spin-1/2 particle in a magnetic field

\[
\frac{\partial \vec{P}}{\partial t} = \frac{1}{Q(P)} \left[ \gamma_e \vec{B} \times \vec{P} + R_p (\hat{z} - \vec{P}) - R_{sd} \vec{P} \right]
\]  

(2.46)

where \(R_p\) is the pumping rate, \(R_{sd}\) is the spin-destruction rate, and \(Q(P)\) is defined in Eq. 2.43. Through optical pumping, the equilibrium polarization reaches

\[ P_z = \frac{R_p}{R_p + R_{sd}} = \frac{R_p}{R_{tot}}. \]  

(2.47)

The signal comes from the projection of the spins along the x-axis. The steady state polarization in this direction is

\[ P_x = P_z \gamma_e \frac{B_y R_{tot} + \gamma_e B_x B_z}{R_{tot}^2 + \gamma_e (B_x^2 + B_y^2 + B_z^2) / R_{tot}} \]  

(2.48)

\textsuperscript{8}There is no factor of \(Q\) here since it modifies both \(\gamma_e\) and \(T_2\).
where we have expressed the rates in terms of $R_{\text{tot}} = R_p + R_{sd}$. In the limit that the magnetic fields are small, $B_x$, $B_y$, and $B_z \ll \gamma_e/R_{\text{tot}}$, the leading behavior is

$$P_x = \frac{P_z \gamma_e}{R_{\text{tot}}} B_y$$

(2.49)

demonstrating the primary sensitivity of the SERF to a $B_y$ field. The coefficient $P_z \gamma_e/R_{\text{tot}}$ determines the sensitivity to magnetic fields. For fixed $R_{sd}$, the maximum sensitivity is reached when $R_p = R_{sd}$ at $P_z = 50\%$. The high sensitivity of the SERF comes from the dramatic reduction of $R_{ee}$ in $R_{\text{tot}}$ from Eq. 2.44.

### 2.5 Interacting Spin Ensembles

Spin-exchange optical pumping (Section 2.2) transfers angular momentum from polarized K to the nuclear spins in $^3$He. These co-located spin ensembles are coupled through spin-exchange and dipolar magnetic interactions. The spatial overlap enhances the magnetic interaction between spins compared to the case of an isolated spin. We observe the interaction of the spins by treating the K as a magnetometer (Section 2.4) and measuring the evolution of $P_z^e$ through optical rotation of a linearly polarized probe beam. The interactions described here will form the basis for the K-$^3$He comagnetometer described in Chapter 3.

#### 2.5.1 Contact Interaction

The magnetization of a polarized atomic species generates a dipolar magnetic field. It is well known that the magnetic field from a uniformly magnetized sphere (in cgs units) is given by

$$\vec{B} = \frac{8\pi}{3} \vec{M}$$

(2.50)
where $\vec{M}$ is the magnetization density vector. In the case of hyper-polarized K and $^3$He, the magnetic field of one species causes a shift in the precession frequency of the other.

Collisions within the vapor modify the magnetic interaction. Spatial overlap of the valence electron in K with the nucleus of $^3$He produces an enhancement in the effective magnetic field experienced by both the K and $^3$He. The Fermi-contact hyperfine interaction $\alpha(R) \vec{K} \cdot \vec{S}$ mediates the interaction between the noble-gas nuclear spin $\vec{K}$ and alkali-metal electron spin $\vec{S}$

$$\alpha(R) = \frac{8\pi}{3} g_s \mu_B \frac{\mu_K}{K} |\psi(R)|^2$$ (2.51)

where $|\psi(R)|^2$ is the square of the wavefunction for the valence electron in K at the $^3$He nucleus [71]. This interaction is well-described by effective magnetic fields with the appropriate enhancement factors [71],

$$\delta \vec{B}_a = \frac{8\pi}{3} \kappa_{ab} n_b \langle \vec{K} \rangle$$ (2.52)$$

$$\delta \vec{B}_b = -\frac{8\pi}{3} g_s \mu_B \kappa_{ba} n_a \langle \vec{S} \rangle$$ (2.53)

where $\delta \vec{B}_a$ is the shift for the alkali-metal spins and $\delta \vec{B}_b$ is the shift for the noble-gas spins. The enhancement factors $\kappa_{ab}$ and $\kappa_{ba}$ are defined as the ratio of field experienced by the alkali-metal/noble-gas spins to the macroscopic field produced by a spherical cell containing the magnetization of a noble-gas/alkali-metal at the corresponding density and polarization [72].

The enhancement factors $\kappa_{ab}$ and $\kappa_{ba}$ contain contributions from alkali-metal–noble-gas binary collisions $\kappa_0$ and van der Waals molecules $\kappa_1$. Noble-gas pressures above an atmosphere decrease the lifetime of van der Waals molecules such that both

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9The analogous expression in SI units is $\vec{B} = \frac{2}{3} \mu_0 \vec{M}$ where $\mu_0$ is the permeability of free space.
κ_{ab} and κ_{ba} converge to a single value of κ_0 \[71\]. This is the relevant regime of this work.

The interaction potential between atoms in collisions leads to a slight temperature dependence. This is well-measured over the range of temperatures used in spin-exchange optical pumping \[73\] and is given by

\[\kappa_0 = 5.99 + 0.0086[T - 200(\degree C)]/\degree C.\]  \hspace{1cm} (2.54)

For notational convenience, we use a simple expression to describe the field experienced by the electron and nuclear spins \(\vec{B}^e/n\). For the case of a uniformly magnetized sphere,

\[\vec{B}^e = \lambda M^e \vec{P}^e\]  \hspace{1cm} (2.55)
\[\vec{B}^n = \lambda M^n \vec{P}^n\]  \hspace{1cm} (2.56)

where \(\lambda = 8\pi \kappa_0/3\) is a geometrical constant containing the enhancement factor, \(M^e = -g_S \mu_B n_a\) and \(M^n = \mu_K n_b/K = \mu_{^3\text{He}} n_b\) give the total magnetization density of the cell, and \(\vec{P}^e/n\) is the electron/nuclear polarization vector. For reference, the relevant magneton values are

\[\mu_B = 9.274 \times 10^{-24} \text{ J/T}\]  \hspace{1cm} (2.57)
\[\mu_{^3\text{He}} = -1.074 \times 10^{-26} \text{ J/T}.\]  \hspace{1cm} (2.58)

Note that both the magnetic moments of optically pumped \(K\) and \(^3\text{He}\) are negative; this implies that \(M^e\) and \(M^n\) are negative.
2.5.2 Coupled K-$^3$He Dynamics

The dynamics of overlapping K and $^3$He spin ensembles can be modeled with a simplified set of coupled Bloch equations \cite{70}. The interaction between spins enters as the spin of one species precessing in the magnetization of the other (Section 2.5.1) and monitored through $P_{e,x}$ as in the SERF (Section 2.4). Following the spin evolution conventions in Section 2.3 the simplified Bloch equations are

$$\frac{\partial \vec{P}_e}{\partial t} = \frac{\gamma_e}{Q} (\vec{B} + \lambda M^n \vec{P}_n) \times \vec{P}_e + \frac{R_{tot}(P_{e,0} \hat{z} - \vec{P}_e)}{Q} \tag{2.59}$$

$$\frac{\partial \vec{P}_n}{\partial t} = \gamma_n (\vec{B} + \lambda M^e \vec{P}_e) \times \vec{P}_n + \frac{R_{tot}(P_{n,0} \hat{z} - \vec{P}_n)}{Q} \tag{2.60}$$

where the spins relax to the equilibrium polarizations of $P_{e,0}$ and $P_{n,0}$ for the electron and nuclear spins respectively. The relaxation rates $R_{tot}$ and $R_{tot}^{n}$ refer to generalized relaxation rates for the electron and nuclear spins. Also, the magnitude of the alkali polarization is assumed to remain constant such that the slowing down factor (Eq. 2.43) decouples from the polarization $Q(P_{e}) \rightarrow Q$. Here, we observe that the spins precess in different fields; the electron precesses in the sum of the applied field and magnetization of the nuclear spins while the nuclear spins precess in the sum of the applied field and the magnetization of the electron spins. The electron and nuclear spins experience the magnetic field from their own magnetization, but this effect is not listed in Eqs. 2.59 and 2.60. This field produces no torque ($\vec{P}_{e/n} \times \vec{P}_{e/n} = 0$) and is omitted.

To study small transverse excitations of the spins, these equations can be linearized. Shifting to complex form $P_{\perp} = P_{x}^e + iP_{y}^e$ and $P_{\perp} = P_{x}^n + iP_{y}^n$ with the assumption that $|P_{\perp}| \ll P_{e,0}^e$ and $|P_{\perp}| \ll P_{0}^n$ yields the linear coupled differential
equations

\[ P_{\perp}^e = (i\gamma_e (B_z^a + \lambda M^n P_z^n) P_{\perp}^e - R_{tot} P_{\perp}^e - i\gamma_e (\lambda M^n P_z^n) P_{\perp})/Q \]  \hspace{1cm} (2.61)

\[ P_{\perp}^n = i\gamma_n (B_z^a + \lambda M^n P_z^n) P_{\perp}^n - R_{tot} P_{\perp}^n - i\gamma_n (\lambda M^n P_z^n) P_{\perp} \]  \hspace{1cm} (2.62)

where \( B_z^a \) is an applied magnetic field along the z-direction. Linear algebra techniques efficiently solve for \( P_{\perp}^e \) and \( P_{\perp}^n \).

The exact solution takes the following form in terms of the eigenmodes of the system

\[ P_x^e(t) = Re[P_1 e^{(A_e + A_n - F)t/2} + P_2 e^{(A_e + A_n + F)t/2}], \text{ where} \]  \hspace{1cm} (2.63)

\[ A_e = (i\gamma_e (B_z^a + \lambda M^n P_z^n) - R_{tot})/Q, \]  \hspace{1cm} (2.64)

\[ A_n = i\gamma_n (B_z^a + \lambda M^n P_z^n) - R_{tot}, \text{ and} \]  \hspace{1cm} (2.65)

\[ F = \sqrt{(A_e - A_n)^2 - 4(\gamma_e \lambda M^n P_z^n) (\gamma_n \lambda M^n P_z^n)/Q}. \]  \hspace{1cm} (2.66)

Two distinct oscillation frequencies persist \((A_e + A_n + F)/2\) and \((A_e + A_n - F)/2\) corresponding to contributions from both nuclear and electron spins. For large magnetic fields, where \( B_z^a \) is much larger than the magnetization of either species, the response is the sum of both spins precessing independently in \( B_z^a \). The nuclear and electron spins are only loosely coupled through \( F \) and relax at their respective rates.

When the alkali atoms experience a small magnetic field, they enter the SERF regime (Section 2.4). We adjust \( B_z^a \) to the following condition

\[ B_z^c = -\lambda M^e P_z^e - \lambda M^n P_z^n \]  \hspace{1cm} (2.67)

to reduce the magnetic field experienced by the alkali from the magnetization of the \(^3\)He. We refer to \( B_z^c \) as the compensation point. Here, the spins precess in an effective
Figure 2.7: Decay of the spins from an initial small tipping angle. At the compensation point (solid), the damping time is dramatically shortened over a small detuning (dashed).

Field equal to their own magnetization

\[ A_e \rightarrow i\gamma_e (-\lambda M^e P^e_z)/Q - R_{tot}/Q \]  
\[ A_n \rightarrow i\gamma_n (-\lambda M^n P^n_z) - R^n_{tot} \]  

(2.68)  
(2.69)

And under typical comagnetometer conditions, these frequencies are on the order of tens of Hz and are nearly equal

\[ \gamma_n \lambda M^n P^n_z \approx \gamma_e \lambda M^e P^e_z / Q. \]  

(2.70)

In general, electron and nuclear precession frequencies are distinct by several orders of magnitude. Here, the spins experience different fields and precess together to form a hybrid resonance. The strong coupling in this resonance leads to some dramatic behavior in the evolution of the spins.

Analytic expressions for the oscillatory and damping response of the comagnetometer can be determined by separating the exponents in Eq. 2.63 into real and imaginary parts

\[ i\omega + \Gamma = (A_e + A_n \pm F)/2. \]  

(2.71)
Due to the square root in $F$, simple analytic expressions at the compensation point can be determined with the reasonable approximations $\gamma_e \lambda M^e P^e_z / R_{tot} \ll 1$ and $R_{tot}^n \ll R_{tot}$. The most dramatic behavior occurs for the decay of the the nuclear spins,

$$\Gamma_n = -\frac{(\gamma_e \lambda M^e P^e_z / Q)(\gamma_n \lambda M^n P^n_z) R_{tot} / Q}{R_{tot}^2 / Q^2 + (\gamma_n \lambda M^n P^n_z)^2} \quad (2.72)$$

where the damping rate is no longer determined by $R_{tot}^n$, but by the product of the magnetizations and the alkali relaxation rate. Under typical comagnetometer conditions, this is on the order of several seconds instead of the more usual hundreds of seconds. The nuclear spins precess in a field of the same magnitude as their magnetization

$$\omega_n = -\gamma_n \lambda M^n P^n_z \left(1 + \frac{(\gamma_e \lambda M^e P^e_z / Q)(\gamma_n \lambda M^n P^n_z)}{R_{tot}^2 / Q^2 + (\gamma_n \lambda M^n P^n_z)^2} \right) \quad (2.73)$$

with a correction for the presence of the alkali magnetization. The alkali damping rate continues to be dominated by $R_{tot} / Q$

$$\Gamma_e = -R_{tot} / Q + \frac{(\gamma_e \lambda M^e P^e_z / Q)(\gamma_n \lambda M^n P^n_z) R_{tot} / Q}{R_{tot}^2 / Q^2 + (\gamma_n \lambda M^n P^n_z)^2} \quad (2.74)$$

with a small reduction similar to Eq. 2.73. The precession of the spins

$$\omega_e = -\frac{(\gamma_e \lambda M^e P^e_z / Q)R_{tot}^2 / Q^2}{R_{tot}^2 / Q^2 + (\gamma_n \lambda M^n P^n_z)^2} \quad (2.75)$$

is mostly unaffected by the presence of the nuclear spin species. The slowest of the two frequencies is typically $\omega_n$ on the order of 10 Hz. We refer to $\omega_n$ as the compensation frequency.
2.5.3 Oscillatory Magnetic Field Suppression

It is interesting to consider the behavior of the coupled spins at the compensation point under the influence of a time-dependent magnetic field. Inserting an oscillatory field \( B_\perp(t) = (B_x + iB_y) \cos(\omega t) \) into Eqs. 2.61 and 2.62 yields the following set of linearized equations

\[
P^e_\perp = \left( i\gamma_e (B^e_x + \lambda M^n P^n_z) P^e_\perp - R_{tot} P^e_\perp - i\gamma_e (\lambda M^n P^n_z + B_\perp(t)) P^e_z \right) / Q \tag{2.76}
\]

\[
P^n_\perp = i\gamma_n (B^n_x + \lambda M^n P^n_z) P^n_\perp - R_{tot} P^n_\perp - i\gamma_n (\lambda M^n P^n_z + B_\perp(t)) P^n_z \tag{2.77}
\]

which can be solved in a straightforward manner for oscillatory solutions in complex form. It is important to consider both co-rotating \( e^{i\omega t} \) and counter-rotating \( e^{-i\omega t} \) solutions. Enforcing the compensation condition (Eq. 2.67) yields

\[
P^e_\perp(t) = -\frac{1}{2} \left[ \frac{P^e_\perp \gamma_e \omega (B_x + iB_y) e^{i\omega t} / Q}{\gamma_n \lambda M^n P^n_z + \omega} + \frac{P^n_\perp \gamma_n \omega (B^n_x + iB^n_y) e^{-i\omega t} / Q}{\gamma_n \lambda M^n P^n_z - \omega} \right] \tag{2.78}
\]

where it has been assumed that \( R^n_{tot} \simeq 0.25 \text{ hrs}^{-1} \) is much less than all the other rates.\(^{10}\) The two terms in Eq. 2.78 are equivalent for the replacement \( \omega \rightarrow -\omega \) as expected for the co-rotating and counter-rotating solutions.

It will be common for the terms \( \lambda M^n P^n_z \) and \( \lambda M^e P^e_z \) to appear repeatedly. Solutions become considerably simpler to interpret with the replacement

\[
B^e = \lambda M^e P^e_z \tag{2.79}
\]

\[
B^n = \lambda M^n P^n_z \tag{2.80}
\]

\(^{10}\)Equation 2.78 corrects the statement in Eq. 2.119 of Ref. [28] by including both the two rotating solutions and the inclusion of \( Q \).
where both $B^e$ and $B^n$ are negative following the convention described in Section 2.5.1.

In the limit that $\omega \ll |\gamma_n B^n|$

$$P_x^e(t) \approx \frac{P_z^e \gamma_e}{R_{tot}} \left( \frac{\omega B_x \sin(\omega t)}{\gamma_n B^n} + O(\omega^2) \right) \quad (2.81)$$

for a slowly modulated $B_x$ field where the $^3$He suppresses the effects of the applied magnetic field at low frequencies. The same is true for a slowly modulated $B_y$ field

$$P_x^e(t) \approx -\frac{P_y^e \gamma_e}{R_{tot}} \left( \frac{\omega^2 B_y \cos(\omega t)}{(\gamma_n B^n)^2} + O(\omega^3) \right) \quad (2.82)$$

where the low frequency behavior is further suppressed by an additional factor of $(\omega/\gamma_n B^n)$. We expect that a slowly modulated $B_z$ field will be further suppressed because it cannot provide a torque on spins oriented along the $z$-direction to precess into the $x$-direction where they can be detected.

The fast damping of nuclear spins and suppression of slowly varying magnetic fields are fundamental features of these coupled spin ensembles. In the following chapter, we explore the consequences of their combined effect to describe a K-$^3$He comagnetometer.
Chapter 3

Next Generation K-\(^3\)He Comagnetometer

A comagnetometer is combination of two magnetometers using two different spin species. A magnetic field will interact with each species in a well-defined way such that a difference or a ratio of each response separates magnetic interactions from other effects. The K-\(^3\)He comagnetometer described in this chapter provides suppression of magnetic fields, but remains sensitive to non-magnetic spin couplings. It is well-suited for Lorentz violation or long-range spin-dependant force searches described in Chapters 5 and 6.

3.1 Comagnetometer Overview

The K-\(^3\)He comagnetometer contains two overlapping spin ensembles of K and \(^3\)He in a spherical glass vapor cell. We typically heat the cell to 180°C to produce a high-density alkali vapor of \(7 \times 10^{13}/\text{cm}^3\) from several mg of K metal in the cell. Optical pumping polarizes the electron spins in a K vapor. Spin-exchange collisions with several amagats of \(^3\)He buffer gas (\(1 \times 10^{20}/\text{cm}^3\)) hyper-polarize several percent of these nuclear spins. An off-resonant linearly polarized probe beam orthogonal to
the optical pumping direction continuously measures the projection of the K spins along the probe propagation axis through optical rotation of the linearly polarized light (Eq. 2.18).

This alkali-metal–noble-gas pair has been chosen for the small spin destruction cross section of K. These spins form a joint resonance at the compensation point (Section 2.5.2) where the comagnetometer also inherits the high sensitivity of the SERF magnetometer (Section 2.4.3). The coupled dynamics strongly damp excitations of the nuclear spins within a few seconds to quickly bring the system to the steady state. The response to slowly varying magnetic fields decreases for decreasing frequency as the $^3\text{He}$ buffer gas suppresses the effects of a magnetic field (Section 2.5.3).

This technique is unique in that it uses a pair of co-located electron and nuclear spins as a comagnetometer. Several experiments have used pairs of co-located nuclear spins as a comagnetometer [29, 33, 40]. A pair of electron and nuclear spins has been used previously [31], but these spins were located in separate vapor cells. No others systems have yet combined electron and nuclear spins in the same vapor cell as with the K-$^3\text{He}$ comagnetometer. The response of the K-$^3\text{He}$ comagnetometer will be described through a picture (Section 3.1.1), then through a more technical treatment of the Bloch equations (Section 3.1.3).

### 3.1.1 Comagnetometer Description

Figure 3.1a illustrates the configuration of the K-$^3\text{He}$ comagnetometer. K atoms are pumped along the z-direction and probed along the x-direction. At the compensation point, the applied magnetic field $B^a_c$ cancels the magnetization of the electron and nuclear spins to place the K in the SERF regime. In the case of a slowly $\omega \ll \omega_n$ applied $B_x \ll B^a_c$ perturbation, the K spins are continuously repolarized and have no long term memory while the $^3\text{He}$ atoms observe the changing field and precess in response (Fig. 3.1b). The $^3\text{He}$ spins quickly damp to align with the total applied field.
Since $^3$He has its spin oppositely oriented to its magnetic moment, $B^n$ cancels the $B_x$ perturbation to leading order, so the K atoms experience no change in magnetic field. The same holds for a small $B_y$ perturbation; however, a small change in $B_z$ is uncompensated. A small change in $B_z$ provides no torque on the spins, so no signal can be observed. An initial misalignment of the spins in the x- or y-direction will lead to sensitivity in $B_z$, but only as a product with the small misalignment (ie. $B_y B_z$). The comagnetometer suppresses the effects of magnetic fields to leading order, but imperfect cancelation can lead to such second order effects.

For the comagnetometer to be useful for anomalous spin searches such as for a Lorentz-violating field, it must preserve sensitivity to the anomalous field. Consider the case of a field that couples only to electron spins $\vec{\beta}^e$ and applied along the y-direction (Fig. 3.1c). The noble-gas does not precess to follow this field because there is no coupling. The K spins precess to project onto the probe direction. This gives a signal response proportional to the field. Now, consider an anomalous field that couples only to the nuclear spins $\vec{\beta}^n$ applied along the y-direction (Fig. 3.1d). The noble-gas spins follow the net projection of both magnetic and anomalous fields. From the point of view of the K atoms, the magnetization of $^3$He is uncompensated and provides a real magnetic field causing the K spins to precess. The magnetization of $^3$He is opposite to its spin, so the K responds in the opposite direction under $\vec{\beta}^n$ as it does under $\vec{\beta}^e$.

We expect the leading order behavior of the comagnetometer be $\beta_y^e - \beta_y^n$ with no first order magnetic field sensitivity. One way to think about this is to consider the case where the anomalous field couples to the spins in the same way as a magnetic field, that is, in proportion to the magnetic moment of the spin. In this case, $\beta_y^e = \beta_y^n$, so there is no leading order sensitivity. However, if the coupling differs from that of a magnetic field, then the K spins will precess in response to either the electron or
Figure 3.1: a) K-$^{3}$He comagnetometer at the compensation point. $B_{c}^{a}$ compensates for $B^{n}$ such that the K atoms see the low field condition of SERF. b) A small adiabatically applied $B_{x}$ field rotates the total applied field vector. The $^{3}$He rotates to follow the total field and its effective magnetic field cancels the change in magnetic field seen by the K. c) An anomalous field perpendicular to the pump and probe beams couples only to the K and rotates the spins into the probe beam. This leads to a signal response. d) An anomalous field couples only to the nuclear spins. The spins follow the combined magnetic and anomalous field and rotate out of the page. This leaves an uncompensated magnetic field projecting into the page. The K spins rotate under this field and project onto the probe beam in the opposite direction as in image (c).
nuclear spin coupling. In this way, the balance between the first order suppression of magnetic fields is broken.

In practice, the comagnetometer provides $10^3 - 10^4$ suppression of magnetic fields while remaining sensitive to the anomalous fields of interest. Further suppression of magnetic interactions can be introduced by enclosing the comagnetometer within many layers of magnetic shielding. The inherent magnetic field suppression of the comagnetometer provides shielding from thermal Johnson noise symptomatic to many magnetic shield materials.

This description of the comagnetometer relies on pictorial representation of the spin and associated magnetization vectors. A calculation of the imperfect compensation is provided in Appendix C. This will confirm the validity of this picture with the full solution in Section 3.2. The intuition gained through the picture provides a better understanding and interpretation of the full analytical solution.

### 3.1.2 Practical Anomalous Fields

We define an anomalous field as any that couples to the spin differently than in proportion to the magnetic moment. Such an interaction for the electron yields the Hamiltonian

$$H = \frac{g_s \mu_B}{\hbar} \vec{B} \cdot \vec{S} + \eta \vec{\beta}^e \cdot \vec{S}$$

(3.1)

where $\eta$ casts the anomalous field $\vec{\beta}^e$ into units and sign of an effective magnetic field. Lorentz-violating or other spin-dependant forces have been proposed, but not observed (Section 1.2 and 6.5.2). There are several well-known anomalous fields relevant to comagnetometer operation, namely lightshifts and rotations.

A lightshift comes from the ac Stark shift of the oscillating light field (Section 2.1.3). This effect shifts the energy levels much the same way as a magnetic field in the vicinity of an electronic transition. The on-resonant pump and near-
Figure 3.2: a) Rotation of the comagnetometer with respect to an inertial frame. The electron spins are quickly repolarized while the nuclear spins remain oriented with respect to an inertial frame. A small $B_x$ component of the compensation field deviates from the direction of the spins. b) The $^3$He responds to the $B_x$ field by precessing into the page. The magnetization extends out of the page and causes the K spins to project onto the probe beam. A positive rotation responds with the opposite sign as $\beta_n^y$ (Fig. 3.1d).

Resonant probe beams introduce such lightshifts to the K spins only. These fields are too far off-resonant to significantly influence nuclear spin transitions in $^3$He. The lightshift $\vec{L}$ presents itself as an anomalous field coupling only to electrons just as in Fig. 3.1c.

Furthermore, a rotation about an inertial frame at a rate slower than the compensation frequency introduces an anomalous field. Spins carry angular momentum and will maintain the same direction with respect to an inertial frame under a rotation of the comagnetometer. The case where the comagnetometer is rotated about the y-direction is shown in Fig. 3.2a. The K spins are quickly repolarized along the new direction by the pump laser; however, the $^3$He spins remain in the inertial frame. The $^3$He experiences a torque from the reoriented compensation field and precesses into the page (Fig. 3.2b). The magnetization projects out of the page and causes the K atoms to precess into the probe beam. The response of a positive rotation around the y-direction results in the opposite response as the the response from an anomalous
nuclear field in Fig. 3.1d and defines an equivalent anomalous field due to rotations as $-\vec{\Omega}/\gamma_n$. In fact, precession of both spin species contributes to the rotation response; however, the response is dominated by the nuclear spins because $\gamma_e/Q(P^e) \gg \gamma_n$.

### 3.1.3 Complete Bloch Equations

A more technical description of the comagnetometer follows from a complete set of Bloch equations. The evolution of the K electron spin polarization $P^e$ and the $^3$He nuclear spin polarization $P^n$ follow

\[ \frac{\partial \vec{P}^e}{\partial t} = \frac{\gamma_e}{Q(P^e)} \left( \vec{B}^a + \lambda M^n \vec{P}^n + \vec{L} + \vec{\beta}^e \right) \times \vec{P}^e - \vec{\Omega} \times \vec{P}^e \\
+ \left( R_p(\vec{s}_p - \vec{P}^e) + R_m(\vec{s}_m - \vec{P}^e) + R_{se}^n(\vec{P}^n - \vec{P}^e) - R_{sd}^e \right) / Q(P^e) \tag{3.2} \]

\[ \frac{\partial \vec{P}^n}{\partial t} = \gamma_n \left( \vec{B}^a + \lambda M^e \vec{P}^e + \vec{\beta}^n \right) \times \vec{P}^n - \vec{\Omega} \times \vec{P}^n + R_{se}^e(\vec{P}^e - \vec{P}^n) - R_{sd}^n \vec{P}^n \tag{3.3} \]

where this result is explicitly written in the rotating frame (Eq. 2.30). The electron spins precess in the applied magnetic field $\vec{B}^a$, the magnetization of the nuclear spins $\lambda M^n \vec{P}^n$, lightshifts $\vec{L}$, and anomalous fields coupling to electron spins $\vec{\beta}^e$. The polarization is also influenced by a number of rates (Table 3.1). In order, we introduce pumping from the pump beam $R_p$, pumping from the probe beam $R_m$, spin-exchange with nuclear spins $R_{se}^n$, and spin-destruction of the alkali $R_{sd}$. The terms $\vec{s}_p$ and $\vec{s}_m$ explicitly account for the degree of circular polarization and orientation of the pump and probe beams with $|\vec{s}| = 1$ for circularly polarized light. The slowing down factor $Q(P^e)$ affects all the rates for the electron spins. Similarly, the nuclear spins precess in the applied magnetic field $\vec{B}^a$, the magnetization of the electron spins $\lambda M^e \vec{P}^e$, and the anomalous field coupling to nuclear spins $\vec{\beta}^n$. The relaxation of the nuclear polarization is simpler and influenced by the spin-exchange rate between nuclear and electron spins $R_{se}^e$, and the nuclear spin-destruction rate $R_{sd}^n$. 

50
The terms in Eqs. 3.2 and 3.3 lead to a lot of algebra, so the appearance is simplified with a few tasteful definitions. Cross products are grouped and rates are collected into a more compact form to produce

\[
\frac{\partial \vec{P}_e}{\partial t} = \gamma_e \left( \vec{B}_a + \lambda M^n \vec{P}_n + \vec{L} + \vec{\beta}_e - \frac{\vec{\Omega} Q(P_e)}{\gamma_e} \right) \times \vec{P}_e
\]

\[
+ \left( R_p \vec{s}_p + R_m \vec{s}_m + R_{en} \vec{P}_n - R_{tot} \vec{P}_e \right) / Q(P_e)
\]

\[
\frac{\partial \vec{P}_n}{\partial t} = \gamma_n \left( \vec{B}_a + \lambda M^n \vec{P}_e + \vec{\beta}_n - \frac{\vec{\Omega} \gamma_e}{\gamma_n} \right) \times \vec{P}_n + R_{se} \vec{P}_e - R_{tot} \vec{P}_n
\]

introducing the definitions \( R_{tot} \equiv R_p + R_m + R_{en} + R_{se} \) and \( R_{tot}^n \equiv R_{se} + R_{sd}^n \).

This format highlights the similarity between the complete Bloch equations and the simplified model presented in Section 2.5.2 that motivated the comagnetometer and justifies the approximations made in Ref. [70]. It also highlights the similar footing for each of the anomalous fields, proposed and practical. For reference, a summary of the notation in the Bloch equations is available in Table 3.1.

As written, \( \vec{B}_a \) refers to the applied magnetic fields. The behavior of interest occurs at the compensation point (Eq. 2.67) and it is simplest to consider fields

\[1\] The work in Refs. [43, 74] choose to group and rescale these equations further. This leads to a mathematically more elegant solution, but is more difficult to interpret in terms of the physically relevant parameters. This author chooses to carry around the extra factors for ease of interpretation within the context of doing experiments related to physical parameters in the lab.
relative to this point. The definition,

\[
B^a_2 = B^c_2 + B_2 = -B^n - B^e + B_2
\]  

(3.6)

along with \(B^a_2 = B_x\) and \(B^a_2 = B_y\) achieves this goal. From now on, we refer to all magnetic fields with respect to the compensation point.

### 3.2 Steady State Response

Searches for anomalous spin dependent forces rely on a correlated measurement with the comagnetometer signal and the source of the anomalous field. The leading order comagnetometer response is the difference in anomalous field coupling between electron and nuclear spins. It is expected that the anomalous fields of interest are small, so drifts from higher order corrections to the comagnetometer response become systematic effects.

#### 3.2.1 Leading Response

As written, this author believes the full Bloch equations to be impossible to solve analytically. An approximate solution can be recovered by assuming that the equilibrium polarization for \(\vec{P}^e\) and \(\vec{P}^n\) is oriented in the z-direction. In this way, equilibrium polarization values for \(P^e_z\) and \(P^n_z\) can be determined

\[
P^e_z = \frac{R^{en}_{se} P^n_z + R_m \vec{s}_m \cdot \hat{z} + R_p \vec{s}_p \cdot \hat{z}}{R^{en}_{se} + R_m + R_p + R_{sd}} \approx \frac{R_p \vec{s}_p \cdot \hat{z}}{R_{tot}}
\]  

\(3.7\)

\[
P^n_z = \frac{R^{ne}_{se} P^e_z}{R^{ne}_{se} + R^{en}_{tot}}
\]  

\(3.8\)

similar to the results obtained for spin-exchange optical pumping (Section 2.2). One can treat the equilibrium polarizations as constants and consider small deviations in the transverse direction as in the linearized Bloch equations (Section 2.5.2). Here, we
forgo the complex notation. The resulting coupled equations are linear and can be solved exactly.

The resulting expression for $P^e_x$ is similar to Eq. 2.48, but with many more terms. With the compensation point definition enforced (Eq. 3.6), the leading terms are

$$P^e_x = \frac{P^e_{z,e}}{R_{tot}} \left( \frac{\beta^e_y - \beta^n_y + L_y + \Omega_y \gamma_n - \Omega_y Q}{\gamma_e} \right) \quad (3.9)$$

with magnetic field, pumping, and spin-exchange effects forming higher order terms.

These effects are difficult to observe directly from the analytic expression. The overall expression can be separated into a numerator and denominator with terms expanded and ranked by size. A strategy of asserting numerical values for each term and sorting terms in descending order has been developed in Ref. [28]. Typical values for all of the terms in the Bloch equations are given in Table 3.2. As in previous work, we consider the comagnetometer under normal operating conditions. The resulting expressions are much simpler if $L_y$ is ignored as well as $\beta^e/n$ and $\beta^n/e$; however, these dependencies are revisited in Section 3.2.9.

### 3.2.2 Numerator

From now on, we will adopt the convention of referring to $P^e_x$ as the signal $S$. This will be especially useful considering the dependance of $S(X)$ on variable $X$. The large number of terms in the equilibrium solution at the compensation point are broken into a numerator $N$ and denominator $D$. For simplicity, we consider

$$S(X) = \frac{P^e_{z,e}}{R_{tot}} \left( \frac{N(X)}{D} \right) \quad (3.10)$$

\[2\]This involves a tour de force of algebraic trickery to coax a program such as Mathematica to collect terms in a meaningful way.
Table 3.2: Estimated values for the comagnetometer under normal operating conditions. The cell is heated to 180° and filled with 9.4 amagat of 3He buffer gas. Values measured experimentally are within reasonable agreement of these estimates. We ignore $L_y$ and propagate $\beta^e_y$ and $\beta^n_y$ as the leading anomalous fields. The most relevant rotations are those of the Earth.

where we consider the coefficients of $X$ in $N(X)$ and the full denominator. A simpler expression can be found by expanding the denominator for small values $1/(1 + \epsilon) \simeq (1 - \epsilon)$, but care must be taken here because $(1 - \epsilon)$ must be multiplied by the full $N(X,Y,Z,XY,YZ...)$ to catch all of the relevant cross terms.

We will present most of our expressions with the leading $P_z^e \gamma^e_e / R_{tot}$ since this term serves as an overall measure of the scaling of the signal or sensitivity. We will group terms in descending order of dimensionless corrections to the variable of interest.

A direct comparison of the results that follow and those presented in Ref. [28] shows a distinct difference in the sign of the interactions. This comes from a difference in the definition of sign of the gyromagnetic ratios in the Bloch equations and an unaccounted for arbitrary sign inserted into the previous calculation. This can cause confusion, so we have been explicit about sign conventions and definitions in this work.

\[ \gamma^e_n = 2\pi \times 3.243 \text{ kHz/gauss} \]
\[ \gamma^e_e = 2\pi \times 2.8 \text{ MHz/gauss} \]
\[ Q(P^e_e) = 5.2 \]
\[ n_a = 7 \times 10^{13}/\text{cm}^3 \]
\[ n_b = 2.5 \times 10^{20}/\text{cm}^3 \]
\[ \lambda M^e = -28 \mu\text{gauss} \]
\[ \lambda M^n = -130 \text{ mgauss} \]
\[ \vec{s}_p = \{10^{-4}, 10^{-4}, 1\} \]
\[ \vec{s}_m = \{10^{-4}, 0, 0\} \]
\[ \vec{B} = \{10^{-6}, 10^{-6}, 10^{-6}\} \text{ gauss} \]
\[ \vec{L} = \{10^{-7}, 0, 10^{-7}\} \text{ gauss} \]
\[ P_z^e = 50\% \]
\[ P_n^z = 2\% \]
\[ |\vec{\Omega}^e| = 7 \times 10^{-5} \text{ rad/s} \]
\[ \beta^e_e = \{0, 10^{-8}, 0\} \text{ gauss} \]
\[ \beta^n_e = \{0, 10^{-8}, 0\} \text{ gauss} \]
### 3.2.3 Denominator

The overall scaling of \( P_{z}^e e / R_{tot} \) has already been removed from the signal, so we can consider a dimensionless denominator

\[
D_S = \left(1 + \frac{\gamma_e^2}{R_{tot}^2} (B_z + L_z)^2 + 2C_{se}^m + 2 \frac{B_z}{B_c} + 2D_{se}^e + 2D_{se}^e \frac{\gamma_e^2}{R_{tot}^2} (B_z + L_z)^2 + O(10^{-6}) \right)
\]

(3.11)

where we define \( C_{se}^e \) and \( D_{se}^e \) to simplify terms involving spin-exchange and magnetizations (Table 3.3). The terms in Eq. 3.11 are loosely in decreasing order except where an algebraic regrouping simplifies the expression. The second term is roughly \( 10^{-2} \), and we include terms up to \( 10^{-5} \). The leading terms in the denominator depend on \( B_z \), \( L_z \), and spin-exchange expressions which will simplify combining the numerator and denominator. The main dependencies of other terms will reside simply in the numerator, and the presence of \( D_S \) signifies that the denominator has not been expanded. Products of fields with parameters \( B_c \simeq 3 \) mgauss and \( \gamma_e / R_{tot} \simeq 1/(20 \ \mu\text{gauss}) \) that are much smaller than unity lead to expansions of the denominator. Expanded expressions will not be valid if these conditions are exceeded. For the most part, contributions from the denominator need not be expanded until terms involving \( B_z \) or \( L_z \) are involved.

### 3.2.4 Anomalous Fields

The leading terms to the comagnetometer dependance involve the difference in anomalous fields (Eq. 3.9). The higher order corrections to this response are

\[
S(\beta_y^e) = \frac{P_{z}^e e}{R_{tot} D_S} \beta_y^e \left(1 + 2 \frac{B_z}{B_c} + C_{se}^m (1 + F_r) + D_{se}^e - 2 \frac{\Omega_z}{\gamma_n B_c} + O(10^{-7}) \right),
\]

\[
S(\beta_y^n) = -\frac{P_{z}^e e}{R_{tot} D_S} \beta_y^n \left(1 + C_{se}^m (1 + F_r - \frac{B_z}{B_c}) + \frac{B_z}{B_c} + D_{se}^e + O(10^{-7}) \right)
\]

(3.12)
Table 3.3: Dimensionless terms found in the steady state expansion containing spin-exchange and magnetization expressions. These coefficients are evaluated using the estimates in Table 3.2.

where we introduce $F_r$ (Table 3.3) to represent the magnetization ratios and

$$B_c = -B^n \simeq -B^n - B^e$$

(3.13)

to represent the compensation of magnetic fields by the $^3$He magnetization. This choice of definition will be useful from an experimental point of view because the applied $B_c$ field points in the same direction as $B_z$. There are subtle differences between the expressions for $S(\beta^e_y)$ and $S(\beta^n_y)$, most notably the factor of 2 on the $B_z/B_c$ term and the appearance of $\Omega_z$ in $S(\beta^e_y)$.

Expanding the denominator in Eq. (3.11) these expressions become

$$S(\beta^e_y) = \frac{P^e R^e}{P^e L^e} \beta^e_y \left( 1 - \frac{\gamma^2_e}{R^2_{tot}}(B_z + L_z)^2 - C^e_{se} - D^e_{se} + O(10^{-5}) \right),$$

$$S(\beta^n_y) = -\frac{P^n R^n}{P^n L^n} \beta^n_y \left( 1 - \frac{\gamma^2_n}{R^2_{tot}}(B_z + L_z)^2 - C^n_{se} - D^n_{se} + \frac{B_z}{B_c} - D^e_{se} + O(10^{-5}) \right)$$

(3.14)

where we keep terms up $10^{-4}$. The leading correction is $6 \times 10^{-3}$ and depends on $B_z$ and $L_z$. The dependence of both expressions is similar except for the cancelation of $B_z/B_c$ in $S(\beta^e_y)$. If the contribution of $B_z$ and $L_z$ can be reduced, the response to
anomalous fields is simply

\[ S(\beta^e_y, \beta^n_y) = \frac{P^e \gamma_c}{R_{tot}} (\beta^e_y - \beta^n_y) \left( 1 - C_{se}^m - D_{se}^e + O(10^{-6}) \right). \] (3.15)

### 3.2.5 Pumping

Before launching into several higher order terms, it is important to identify another first order contribution. Pumping from either the pump or probe beams along the x-direction produces

\[ S((\vec{s}_p + \vec{s}_m) \cdot \hat{x}) = \frac{R_p}{R_{tot}} \vec{s}_p \cdot \hat{x} + \frac{R_m}{R_{tot}} \vec{s}_m \cdot \hat{x} \] (3.16)

as an experimental imperfection. This contribution can be re-expressed in terms of the orthogonality of the pump and probe. For an alignment angle \( \pi/2 + \alpha \), the signal is

\[ S(\alpha) = \alpha \frac{R_p}{R_{tot}} \] (3.17)

and generates a fixed offset in the response. Any mechanical drift through time of the pump and probe alignment introduces a time dependant response to the signal.

Since the linearly polarized probe beam is nominally oriented along the x-direction, we can consider its contribution in terms of the remnant circular component,

\[ S(s_m) = s_m \frac{R_m}{R_{tot}}. \] (3.18)

Once again, a contribution from this term contributes an offset and any change through time leads directly to drift. The probe beam is nominally linearly polarized, so \( |\vec{s}_m| \ll 1 \). We have implemented a scheme to reduce the remaining circular component of the probe beam (Section 4.2.7) and to stabilize the relative orientation of the pump and probe (Section 4.2.9).
3.2.6 Magnetic Fields

In a magnetometer, the leading sensitivity is to the $B_y$ field. In the comagnetometer, the leading $B_y$ dependence is suppressed by a product with $B_z$,

$$S(B_y) = \frac{P_z \gamma_e}{R_{tot} D_S} B_y \left( \frac{B_z}{B_c} - \frac{\Omega_z}{\gamma_n B_c} - \frac{B_z + L_z}{B_c} C_{se} + \frac{B_z^2}{B_c^2} - \frac{L_z}{B_c} C_{se} + \mathcal{O}(10^{-6}) \right).$$  \hspace{1cm} (3.19)

The leading corrections come from fields along the z-direction including lightshifts and rotations. Each of these is suppressed by $B_c$ (Eq. 3.13). Larger $^3$He polarizations reduce the influence of these terms on the signal.

The dependance on $B_z$ contains a dependance on $B_y$ as expected from Eq. 3.19,

$$S(B_z) = \frac{P_z \gamma_e}{R_{tot} D_S} B_z \left( \frac{\gamma_e L_x (1+F_r)}{R_{tot}} + \frac{B_y + \beta_y}{B_c} - \frac{R_p \tilde{s}_p \cdot \hat{y}}{P_z R_{tot}} + \frac{\gamma_e \Omega_x}{R_{tot} \gamma_n} + \frac{\beta_y - \beta_n}{B_c} + \mathcal{O}(10^{-6}) \right).$$  \hspace{1cm} (3.20)

The leading term in this case comes from $L_x$. Also, there is an $\Omega_x/\gamma_n$ contribution that is also modified by $\gamma_e/R_{tot}$. It is interesting to also observe the appearance of anomalous fields $\beta_y$ and $\beta_n$. A measurement of these effects is most direct from Eq. 3.9. In the case that $B_y$ is small, these effects are already suppressed. The appearance of a pump beam projection along the y-direction also leads to a $B_z$ dependance. These will be particularly important since the $B_z$ field is continuously changing due to drifts in the polarization of $^3$He.

Because of the dependance of $B_z$ in the denominator, expansion of Eq. 3.20 becomes

$$S(B_z) = \frac{P_z \gamma_e}{R_{tot} D_S} B_z \left( \frac{\gamma_e L_x (1+F_r)}{R_{tot}} + \frac{B_y + \beta_y}{B_c} - \frac{R_p \tilde{s}_p \cdot \hat{y}}{P_z R_{tot}} + \frac{\gamma_e \Omega_x}{R_{tot} \gamma_n} + \mathcal{O}(10^{-5}) \right)$$  \hspace{1cm} (3.21)

where the $-2B_z/B_c$ term in the denominator in combination with the leading $(\beta_y - \beta_n)$ response cancels much of the anomalous field response to leave a lonely $\beta_y$ term. This is in agreement with Eq. 3.14. It is worth pointing out that the same cancelation
provides a $-\Omega_y/(\gamma_n B_c)$ term buried in the higher order part of the expression. This is negligible unless large rotations about the z-axis are introduced.

Both the $B_y$ and $B_z$ fields are suppressed by the product with another small field; however, $B_x$ introduces a first order dependance to the signal,

$$S(B_x) = \frac{P_{ze}^e}{R_{tot} D_S} B_x \left( \frac{\gamma_e B_x (B_z + L_z)}{R_{tot} B_c} + \frac{R_{tot}}{\gamma_e B_c} (C_{se} + C_{ne}) + O(10^{-6}) \right). \quad (3.22)$$

where we consider the terms in the same way as $S(B_y)$ and $S(B_z)$. This can be rearranged into the following more illuminating format

$$S(B_x) = \frac{P_{ze}^e B_x}{B_c D_S} \left( \frac{\gamma_e B_x (B_z + L_z)}{R_{tot}^2} + \frac{C_{se} + C_{ne}}{R_{tot} B_c} \right). \quad (3.23)$$

The symmetry of the magnetic field cancellation is broken by spin-exchange terms on the order of $10^{-2}$. The leading field dependance is the product of three magnetic fields $B_x B_z^2$ with no binary contributions. It is also interesting to note that there is no $L_z^2$ present as there was in the denominator (Eq. 3.11).

It is useful to consider the $B_z^2$ dependance since this can lead to an evaluation of the contribution to $B_x$. As noted earlier, it is not simple to directly access this term, since the denominator also carries a $B_z^2$ dependance. This evaluation determines

$$S(B_z^2) = \frac{P_{ze}^e \gamma_e^2}{R_{tot}^2 D_S} B_z^2 \left( \frac{(B_x + L_x)(1 + F_x) + L_x}{B_c} + \frac{B_y R_{tot}}{B_c^2 \gamma_e} - \frac{2 R_y \hat{s}_p \cdot \hat{y}}{B_c \gamma_e P_{ze}^e} - \frac{\Omega_z}{\gamma_n B_c} + O(10^{-8}) \right) \quad (3.24)$$

which contains a $B_x$ term as expected from Eq. 3.23. It is also influenced by $L_x$ at $10^{-4}$. The effects of pumping along the y-direction and $B_y$ are suppressed at $10^{-6}$. These terms are superseded by those from the expansion of the denominator. We can include the leading affects of the numerator by considering the product with the first
order terms in Eq. 3.9 to provide

\[ S(B_z^2) = \frac{P_z^e \gamma_e}{R_{tot}} \left[ \frac{\gamma_e}{B_c} B_z^2 \left( B_x \frac{B_z}{B_c} - \frac{\gamma_e \Omega_y}{R_{tot} \gamma_n} + \mathcal{O}(10^{-5}) \right) \right] \]  

(3.25)

which picks up an important \( \Omega_y B_z^2 \) term at \( 10^{-4} \). Also, the \( L_x \) dependance is removed during expansion from the product of the \(-2B_z/B_c\) term in the denominator with the leading \( L_x \) dependance in the numerator (Eq. 3.34).

Similarly, the product \( B_y B_z \) is worth considering as this will later provide a means of calibrating the magnetometer (Section 4.3.1). Before expanding the denominator,

\[ S(B_y B_z) = \frac{P_z^e \gamma_e}{R_{tot} B_c D_S} B_y B_z \left[ 1 - C_{se}^e - \frac{2 \Omega_z}{\gamma_n B_c} - \frac{L_z}{B_c} C_{se}^e + \left( \frac{\Omega_z}{\gamma_n B_c} + \frac{\Omega_z Q_e}{\gamma_n B_c} \right) C_{se}^e + \mathcal{O}(10^{-11}) \right] \]  

(3.26)

where two \( C_{se}^m \) terms have already canceled and the sum of effects from rotations and from electron and nuclear spins appears. The contribution from the denominator must be evaluated carefully by also considering the numerator with \( B_y \) (Eq. 3.19)

\[ S(B_y B_z) = \frac{P_z^e \gamma_e}{R_{tot} B_c} B_y B_z \left( 1 - C_{se}^e - 2 C_{se}^m + 2 \frac{B_z}{B_c} + L_z C_{se}^e - 2 \frac{B_z}{B_c} + L_z C_{se}^m + \mathcal{O}(10^{-5}) \right) \]  

(3.27)

which simplifies to

\[ S(B_y B_z) = \frac{P_z^e \gamma_e}{R_{tot} B_c} B_y B_z \left( 1 - C_{se}^e - 2 C_{se}^m + \mathcal{O}(10^{-5}) \right) \approx \frac{P_z^e \gamma_e}{R_{tot} B_c} \]  

(3.28)

if the \( B_z \) and \( L_z \) fields are reduced. As defined, \( B_c \) follows the same sign convention as \( B_z \), so the slope reverses if the comagnetometer axes are reversed. The \( \Omega_z \) term remains as a \( 10^{-5} \) correction.
3.2.7 Rotations

Sensitivity to rotations enters at the first order since rotations behave as anomalous fields. The dominant sensitivity is along the y-direction,

$$S(\Omega_y) = \frac{P_e^e \gamma_e}{R_{\text{tot}} D_S} \frac{\Omega_y}{\gamma_n} \left( 1 - \frac{\gamma_n Q}{\gamma_e} + \frac{B_z}{B_e} D_{se} + \mathcal{O}(10^{-6}) \right).$$  \hspace{1em} (3.29)

The terms in the numerator are similar to those in the denominator such that

$$S(\Omega_y) = \frac{P_e^e \gamma_e \Omega_y}{R_{\text{tot}} \gamma_n} \left( 1 - \frac{\gamma_n Q}{\gamma_e} - \frac{\gamma_e^2}{R_{\text{tot}}^2} (B_z + L_z)^2 - C_{se} - \frac{B_z}{B_e} D_{se} + \mathcal{O}(10^{-6}) \right)$$  \hspace{1em} (3.30)

with the denominator fully expanded. Rotations are dominated by the $\Omega_y/\gamma_n$ term with a 0.5% adjustment for the electron spin contribution. As with the anomalous fields in Section 3.2.4,

$$S(\Omega_y) = \frac{P_e^e \gamma_e \Omega_y}{R_{\text{tot}} \gamma_n} \left( 1 - \frac{\gamma_n Q}{\gamma_e} - C_{se} - \mathcal{O}(10^{-6}) \right)$$  \hspace{1em} (3.31)

for sufficiently small $B_z$ and $L_z$.

Rotations about the x- and z-axes are further suppressed, but still provide some rotational sensitivity,

$$S(\Omega_x) = \frac{P_e^e \gamma_e \Omega_x}{R_{\text{tot}} D_S \gamma_n} \left( \gamma_e (B_z + L_z) R_{\text{tot}} + \gamma_e B_z F_r - \frac{\gamma_n (B_z + L_z)}{R_{\text{tot}}/Q} + \mathcal{O}(10^{-5}) \right)$$  \hspace{1em} (3.32)

$$S(\Omega_z) = \frac{P_e^e \gamma_e \Omega_z}{R_{\text{tot}} D_S \gamma_n} \left( - \frac{B_y}{B_e} - \frac{\gamma_e}{R_{\text{tot}} L_x} (F_r + \frac{\gamma_n Q}{\gamma_e}) - \gamma_e B_z R_{\text{tot}} B_e + \mathcal{O}(10^{-5}) \right)$$  \hspace{1em} (3.33)

where sensitivity to $\Omega_z$ is suppressed by $10^{-4}$ whereas sensitivity $\Omega_z$ is only suppressed by $10^{-2}$. This is from the presence of $\gamma_e/R_{\text{tot}}$ on terms in $\Omega_x$ and $B_e$ on terms in $\Omega_z$. 4This leads to the behavior of the comagnetometer as a sensitive gyroscope [41].
3.2.8 Lightshifts

Contributions to the signal from lightshifts from the probe and pump beams lead to second order terms,

\[ S(L_x) = \frac{P_z \gamma_e}{R_{tot} D_S} L_x \left( \frac{\gamma_e}{R_{tot}} (B_z + L_x) + \frac{\gamma_e}{R_{tot}} B_z F_r + \frac{2\gamma_e B_z (B_z + L_x)}{R_{tot} B_c} + \mathcal{O}(10^{-6}) \right) \]

\[ S(L_z) = \frac{P_z \gamma_e}{R_{tot} D_S} L_z \left( \frac{\gamma_e}{R_{tot}} L_x - \frac{R_p \hat{s}_p \cdot \hat{y}}{P_z R_{tot}} + \frac{\gamma_e \Omega_x}{R_{tot} \gamma_n} + \frac{\gamma_e B_z (B_z + 2L_z)}{R_{tot} B_c} + \mathcal{O}(10^{-6}) \right) \]

(3.34)

(3.35)

where a common \( L_x L_z \) term is one of the leading effects. The numerator of \( L_x \) terms has a strong \( B_z \) dependance whereas the \( L_z \) term has the product \( B_z (B_x + 2L_x) \). Expansion of the denominator will completely cancel the last term in Eq. 3.34 and the \( 2L_x \) term in Eq. 3.35.

3.2.9 Refinements

The preceding expressions are in good agreement with those presented in Ref. [28] with the main differences including choice of sign conventions and some corrected sign errors in this presentation. These parameters are useful for considering normal comagnetometer operation. In this work, we introduce a third laser to introduce additional lightshifts and pumping along the x- and y-direction, so it is useful to include such terms in the analysis. We extend the \( L_y \) lightshift to a similar magnitude as \( L_x \) and \( L_z \) (Table 3.2). The signal dependance becomes

\[ S(L_y) = \frac{P_z \gamma_e}{R_{tot} D_S} L_y \left( 1 + 2 \frac{B_z}{B_c} + C_{se}^m (1 + F_r) + D_{se}^e - 2 \frac{\Omega_z}{\gamma_n B_c} + \mathcal{O}(10^{-7}) \right). \]

(3.36)
The denominator is unaffected by this change, so the full dependence can be simply expressed as

$$S(L_y) = \frac{P_z e_y}{R_{tot}} L_y \left( 1 - \frac{\gamma_e^2}{R_{tot}^2} (B_z + L_z)^2 - C_{se}^e - D_{se}^e + O(10^{-5}) \right)$$

(3.37)

where these expressions are precisely the same as those for $S(\beta^e_y)$ with the replacement $\beta^e_y \rightarrow L_y$.

Furthermore, it is important to consider pumping effects from the pumping rate $R_l$ along the y-direction from a third beam

$$S(\vec{s}_l \cdot \hat{y}) = -R_l \vec{s}_l \cdot \hat{y} \frac{\gamma_e}{R_{tot}} (B_z + L_z).$$

(3.38)

If there is a misalignment into the x-direction, the signal is affected simply by Eq. 3.16.

Additional pumping along the z-direction provides no change in the signal other than contributing to the value of the equilibrium polarization.

It is also reasonable to assume that $\beta^{e/n}_x$ and $\beta^{e/n}_z$ also exist; the leading terms are

$$S(\beta^e_x) = \frac{P_z e_y}{R_{tot} D_s} \beta^e_x \left( \frac{\gamma_e R_{tot}}{L_x} (B_z + L_z) + \frac{\gamma_e}{R_{tot}} B_z F_r + \frac{2\gamma_e B_z (B_z + L_z)}{R_{tot} B_c} + O(10^{-6}) \right)$$

(3.39)

$$S(\beta^n_x) = \frac{P_z e_y}{R_{tot} D_s} \beta^n_x \left( -\frac{\gamma_e (B_z + L_z)}{R_{tot}} - \frac{\gamma_e B_z F_r}{R_{tot}} + O(10^{-5}) \right)$$

(3.40)

$$S(\beta^e_z) = \frac{P_z e_y}{R_{tot} D_s} \beta^e_z \left( \frac{\gamma_e R_{tot}}{L_x} \frac{\vec{s}_p \cdot \hat{y}}{P_z} + \frac{\gamma_e \Omega_x}{R_{tot} R_{tot} \gamma_{ln}} + \frac{\gamma_e B_z (B_z + 2L_x)}{R_{tot} B_c} + O(10^{-6}) \right)$$

(3.41)

$$S(\beta^n_z) = \frac{P_z e_y}{R_{tot} D_s} \beta^n_z \left( \frac{B_y}{B_c} + \frac{\gamma_e L_x F_r}{R_{tot}} + \frac{\gamma_e B_z B_z}{R_{tot} B_c} + O(10^{-5}) \right)$$

(3.42)

with no first order dependence. Since these fields are already well-constrained and all the other fields are made small, we can neglect these effects. In the event that an anomalous field is measured, these terms can play a role in confirming that it is real.
3.2.10 Typical Leading Terms

The expressions in the preceding sections cannot be blindly added together to form one large dependence as this would double count a large number of terms. By carefully selecting independent terms, a compact expression that incorporates the most relevant terms relevant to experiments is

\[
S_L = \frac{P_e \gamma_e}{R_{tot}} \left[ \beta_y - \beta_n^y + \frac{\Omega_y}{\gamma_n} - \frac{\Omega_y Q_e}{\gamma_e} + L_y + \frac{\alpha R_p + s_m R_m}{\gamma_e P_z} \right.
\]

\[
+ B_z \left( \frac{B_y + \beta_n^y}{B_e} + L_x \frac{\gamma_e}{R_{tot} \gamma_n} + \frac{\gamma_e \Omega_x}{R_{tot} \gamma_n} \right)
\]

\[
+ \frac{\gamma_e}{R_{tot}} \left( B_x B_z \left( B_x + L_z \right) - L_y \frac{\gamma_e}{R_{tot}} B_z^2 - \frac{\gamma_e \Omega_y B_z^2}{R_{tot} \gamma_n} \right) \right].
\]

(3.43)

where these have been selected to include the leading effects of each field. This expression can inform almost all of the measurements in Chapter 4. This is a simple enough expression to remember the pattern after some experience with comagnetometer operation. Only the \( \Omega_z \) term has been neglected, but this sensitivity has been suppressed by \( 10^{-5} \) and will be difficult to measure unless a large rotation about the z-axis is performed.

3.3 Experimental Implementation

The measurements in this thesis bridge two generations of comagnetometer development. We will refer to the first generation as CPT-I and the second as CPT-II. The comagnetometer suppresses magnetic field changes at frequencies below the compensation frequency of several Hz. Therefore, long term drifts from \( 1/f \) noise can dominate the signal in the frequency range of interest. Much of the development of a robust apparatus addresses the issue of long term drift.
3.3.1 General Features

The comagnetometer cell contains several amagats of $^3$He buffer gas, several Torr of N$_2$ for quenching, and a few mg of K metal. Corning 1720 or GE 180 glass is chosen for its chemical resistance to the alkali and its high density to prevent the escape of $^3$He by diffusion through the walls. The spherical cell has a diameter of 2.5 cm with a 2 mm wide stem from where the cell is pulled off the vacuum manifold during the filling procedure. The cell is mounted by the stem in an oven and heated to achieve a K vapor density of $\sim 10^{13}$/cm$^3$. A temperature between 160°-180° is maintained using a temperature sensor and simple feedback loop on the heating elements. Nesting the oven within several layers of magnetic shielding further suppresses external magnetic effects. Magnetic field coils within the shields provide control of each of the independent magnetic fields and gradients. All materials inside the shields are non-magnetic to preserve the uniformity of the local magnetic fields (Fig. 3.3).

Two semiconductor diode lasers provide the light for independent pump and probe beams. A simple arrangement of spherical lenses shape the beam to illuminate the cell. A polarizer and quarter waveplate circularly polarizes the pump beam, and a polarizer sets the initial linear polarization of the probe beam. At an alkali density of $10^{13}$ cm$^3$ in the a 2.5 cm diameter cell, only tens of mW of pump power is required to
reach an alkali polarization near 50%. A final polarizer on the probe beam converts optical rotation to intensity which is monitored on a photodiode. Achieving long term stability for anomalous spin coupling searches is more difficult and requires a more sophisticated design.

Magnetic shields do not reduce the sensitivity to anomalous fields. A magnetic field exterior to the shields causes the electrons in the shield to align with the field. The magnetic moment of the electrons anti-aligned with the spin contributes a real magnetic field in the opposite direction. The sum of these two contributions decreases the net effect of the applied field inside the shields. In the case of an applied $\vec{\beta}^e$, electrons in the shield again align to the anomalous field and create a real magnetic field in response. The comagnetometer experiences contributions from both the anomalous field and the real magnetic field. Since the comagnetometer is insensitive to magnetic fields, the magnetic field is canceled, but the anomalous field remains to generate a response. The case of an applied $\vec{\beta}^n$ is much simpler; the electrons in the shields ignore the nuclear spin coupling, so an anomalous nuclear coupling passes through the shields unimpeded. Applying a lightshift bypasses the effects of the magnetic shields and is therefore not a useful demonstration of this argument. Similarly, the dominant response of the comagnetometer gyroscope comes from the nuclear spin coupling [41]. A careful measurement of the electron contribution to the gyroscope response would experimentally confirm this argument. In any event, the measurements in this thesis present new limits on nuclear spin couplings, so the leading gyroscope response provides sufficient experimental evidence to confirm the comagnetometer’s sensitivity to an anomalous nuclear spin coupling.

3.3.2 Historical Development

K-3He comagnetometer development began at the University of Washington under Michael Romalis for spin-dependant tests of Lorentz violation. Specifically, interest
in $\tilde{b}_\perp$ in the SME prompted a search for an anomalous spin coupling where the measurement timescale of interest is one sidereal day. This is a long timescale for stable comagnetometer operation. Many of the experimental challenges include stabilizing systematic contributions to the signal without feeding back on the signal itself. This would remove the sensitivity to the fields of interest.

The experiment was placed on a large optical table with horizontally mounted pump and probe beams. Five-layer $\mu$-metal magnetic shields with a 60 cm outer (40 cm inner) diameter provided a magnetic shielding factor of $10^6$. One of the earliest stability problems was identified in the relative motion of the pump and probe beams. A special nonmagnetic stainless steel optical table with a submerged central area allowed for rigid mounting of the optics around the shields [75]. Relative motion was monitored by raising the pumping intensity such that the sensitivity to misalignment increases while the sensitivity to other fields decreases. Also, slow changes in the birefringence of the cell walls induced additional optical rotation in the probe beam unrelated to the atoms. A means of measuring the probe beam background by blocking the pump light was developed. With the atoms depolarized, only the background signal contributed to the measured optical rotation of the probe beam.

The experiment was later moved to Princeton University where a complex system of monitors and feedbacks were incorporated to stabilize the comagnetometer signal against systematic drifts (Fig. 3.4). These improvements occurred over years with full details in Refs. [28, 76]. Briefly, the frequency of the 1 W pump laser is monitored with a grating spectrometer. A variable waveplate raises and lowers the intensity for misalignment measurements as well as stabilizes the intensity. A Fabry-Perot cavity monitors the probe laser frequency, with a barometric pressure dependence that has been reduced using an Invar tube. The current in a tapered amplifier is adjusted to maintain a consistent probe intensity. A Pockel cell nulls the circular polarization
of the probe beam to reduce $L_x$ and pumping effects. Despite these feedbacks, the external cavity lasers occasionally mode hop leading to discontinuities in the signal which are removed in the final analysis. Greater temperature stability was achieved by stabilizing the temperature of the laser mounts to $< 10$ mK.

Shifts in the position of the beams are observed from long term thermal drifts of the optical elements and the breadboard. Four quadrant photodiode position detectors continuously monitor beam positions and provide feedback signals to piezo controlled mirrors. Small deviations in probe beam position are particularly symptomatic due to optical rotation from the linear dichroism of the cell. A “sweet spot” on the cell face can be identified near the center of the cell where the leading order dependence
cancels. A translating lens and deviator allow for independent control of the angle and position of the probe beam to aid in locating the “sweet spot”.

A complex series of modulation techniques were developed to reduce the second order effects to the signal (Eq. \ref{eq:3.43}). Within our literature, these are referred to as “zeroing routines” and reduce the dependance of the signal on the $B_z$, $B_y$, $B_x$, $L_x$, and $L_z$. In addition, a $^3$He polarization feedback has been developed. These have been adjusted over the years, and details on the most modern implementation of these routines are discussed in Section \ref{sec:4.2}. A complex LabView program was also developed to automate data acquisition and coordinate various feedbacks and background measurements. LabView and the associated National Instruments data acquisition cards handle all of the data acquisition. These tools were chosen such that during a program crash, voltages on the cards remain uninterrupted such that transient signals do not disturb the equilibrium of the spins in the comagnetometer.

A Faraday modulator made from Tb-doped glass modulates the polarization of the probe beam at 4.8 kHz and is measured with a Standford Research Systems SR830 DSP lock-in amplifier. The high current required in the faraday modulator requires that it be water cooled. This provides better long term stability than a simple polarizer or balanced polarimeter scheme and reaches a sensitivity of $10^{-8}$ rad/$\sqrt{\text{Hz}}$. At this level, moving air currents contribute to noise. Density variations in moving air modulate the index of refraction leading to optical rotation. Large Lexan boxes around the optics reduce convective air currents. All electronics with cooling fans are placed on an equipment rack above the optical table. Furthermore, evacuated glass tubes along the axis of the shields reduce the optical path length through atmosphere. A large foam box surrounds the entire optical table to reduce airflow and provide additional thermal insulation. Still, opening the door to the room introduces a wave of air from the hallway and a noticeable shift in the signal can be observed.
To reduce diurnal drifts, laboratory air vents were turned off and sealed. An air conditioner continuously runs to maintain the room at a steady temperature and move excess heat to an adjoining room. Acoustic foam fills the gaps between the nested magnetic shields, and a fiberglass bundle surrounds the outermost shield. Furthermore, pedestal mounts and forks are used exclusively for greater rigidity in mounting optics. Tightening all of the table forks has led to an observable reduction in thermal drift. A myriad of temperature sensors around the experiment record various thermal drifts. Systematic studies of the thermal sensitivity of individual elements have been performed by heating different elements and searching for a correlation in the signal. Humidity sensitivity was even discovered as a result of certain epoxies.

These features improve the long-term performance of the comagnetometer and achieve a short term sensitivity of \(5 \text{ fT} / \sqrt{\text{Hz}}\) in the range 0.1-1 Hz. Long term drift quickly dominates the signal at lower frequencies and dramatically reduces sensitivity on the timescale of one sidereal day.

**Previous Lorentz Violation Results**

It was in this configuration that preliminary results on Lorentz and \(CPT\) violation have been collected. These measurements spanned 15 months and contained many days of noise compromised data. Of these, the best 120 days were selected on the criterion that the root-mean-square noise was less than 100 fT (Fig. 3.5). Four systematic errors were removed: (1) any correlation between the signal and another, separate measurement, (2) purely linear or, more rarely, second-order polynomial drifts, (3) sudden jumps in the signal associated with mode hops of the lasers, and (4) temporary excursions from a nominal trend.

Due to large amounts of systematic drift, data in the time domain is analyzed using a Fourier analysis method. One considers a sidereal variation in the calibrated
data with an arbitrary phase,

\[ A_x \cos (\Omega_\oplus t) + A_y \sin (\Omega_\oplus t) + \text{noise}. \]  

(3.44)

Multiplying by sine and cosine references and averaging the data over an integral number of periods returns one half the amplitudes \( A_x \) and \( A_y \). The final averaged amplitudes from this preliminary search are

\[ A_x = (-0.76 \pm 0.74) \text{ fT} \]

\[ A_y = (+0.59 \pm 0.81) \text{ fT}. \]
An adjustment to Local Sidereal Time (LST), the proper geometrical factors, and scaling to energy units connect this result to the SME framework.

While these results are impressive, they are only comparable to the existing limit released by the Harvard-Smithsonian group presented a few years prior to these results \[29\]. Here, co-located hyperpolarized nuclear spin ensembles of \(^3\)He and \(^{129}\)Xe form a \(^3\)He/\(^{129}\)Xe noble-gas spin maser. The \(^{129}\)Xe spin precession frequency is locked to a hydrogen maser and the \(^3\)He is free to precess. The ratio of spin precession frequencies is monitored and a sidereal variation in the precession of the \(^3\)He would indicate a possible Lorentz-violating field. Over the course of a year, 90 useable sidereal day values provide

\[
A_X = (+1.41 \pm 1.45) \, \text{fT}
\]

\[
A_Y = (-0.16 \pm 1.39) \, \text{fT}
\]

where these amplitudes correspond to LST\(^5\). While the Harvard-Smithsonian results have a slightly larger absolute uncertainty in fT, the geometry of their measurement is better optimized for a measurement of \(\vec{b}_{\perp}\). Using the method in section 5.4.5, the Harvard-Smithsonian group presents a limit of \(\tilde{b}_{n\perp} < 1.1 \times 10^{-31} \text{ GeV}\) whereas CPT-I presents \(\tilde{b}_{n\perp} < 1.4 \times 10^{-31} \text{ GeV}\). The comagnetometer results are consistent, but not sensitive enough to surpass the previous measurement.

### CPT-I Upgrades

Several innovations were introduced to the CPT-I installation to further suppress long term drifts \[43, 77\]. The lasers were replaced with distributed feedback (DFB) diodes (Fig. 3.6). Wavelength control was achieved through temperature stabilization of the diode rather than a macroscopic mount. A tapered amplifier on the pump beam

\(^5\)This result is reported in nHz which we have translated to our preferred units of fT for a more direct comparison.
provides the high pumping rates required for the misalignment measurements. The forced hot air heating system was replaced with a high frequency ac heating scheme. A twisted pair of high resistance wire wrapped around the oven carries \( \sim 1 \) A of current at 20 kHz and dramatically reduces nonuniform heating and vibrations from airflow variations. A photo-elastic modulator (PEM) replaces the Faraday modulator in the optical rotation measurement. This provides better performance at frequencies lower than 1 Hz most likely due to the improved thermal stability. The Pockel cell for removing birefringence in the probe beam is replaced with a stress plate. A piezo stack compresses a piece of rigidly held glass to introduce a birefringence that can be used to cancel a small circular polarization of the probe beam.

Finally, the optical path length of the pump beam is dramatically shortened using a single mode fiber. The flap on the table bridges the gap in the depression for the magnetic shields (Fig. 3.7). The top of the flap is well insulated by the Lexan enclosure while air can move freely underneath. Differential heating causes the board to flex with changing temperature. The single mode fiber reduces the net effect of these fluctuations but still retains some sensitivity.

The upgrades improved sensitivity in the 0.1-1 Hz range (Fig. 3.8). The peak at 11 Hz is the resonance at the compensation frequency. Below the compensation point, the noise drops as the comagnetometer compensates for magnetic fields until it is limited by noise from the probe beam or drift in the comagnetometer. The peaks at 2 and 3 Hz are from mechanical resonances of the optical table and can be filtered. Below 0.1 Hz, \( 1/f \) noise continues to dominate. The peak at 0.05 Hz comes from the duty cycle of the cooling system. On the timescale of a sidereal day, the noise remains hundreds to thousands of fT.

Initial return a Lorentz violation search indicated that despite the improvements in short-term sensitivity, systematic noise on the timescale of a sidereal day had not been significantly improved. This is a strong indicator that a new apparatus designed
Figure 3.6: CPT-I comagnetometer upgrade. PD: Photodiode, Pol: Polarizer, FI: Faraday Isolator, PEM: Photo-elastic Modulator, BS: Beam Sampler, I: Integral Feedback Controller, VT: Vacuum Tube. Image Credit: [43].

Figure 3.7: CPT-I comagnetometer upgrade showing the table flap and single mode fiber. Image Credit: [43].
Figure 3.8: The optimized noise spectrum from CPT-I represents an improvement from [28] in suppressing 1/f noise at low frequencies. The compensation frequency is 11 Hz. The dramatic drop at high frequencies is from the low pass filter on the lock-in amplifier. The peaks at 2 and 3 Hz are mechanical resonances of the optical table. The peak at 0.05 Hz is from the cooling system.

from the ground up would prove more fruitful. In the meantime, the optimized noise spectrum (Fig. 3.8) provides excellent sensitivity for a search for an anomalous spin dependent force if a high density spin source is placed outside the outermost shield and reversed on a timescale of several seconds. The 0.75 fT/√Hz sensitivity achieved at 0.18 Hz is a useful point for such a search. The details of the spin source are discussed in Chapter [6] and the results are presented in Ref. [42]. In the meantime, it is sufficient to discuss the development of CPT-II, where we can take advantage of the short-term sensitivity and long-term stability of the comagnetometer for an improved Lorentz and CPT violation search.

3.4 CPT-II Apparatus

CPT-II introduces a more compact design to reduce optical pathlengths and the dependence on mechanical drifts (Fig. 3.9). This results in a simplified setup with fewer feedbacks than CPT-I. All of the optics and lasers are enclosed in a 700 L bell jar pumped down to 2 Torr of atmosphere. Evacuation of the entire optical path
improves thermal stability and removes effects of air convection. The pump beam is mounted vertically such that the sensitive y-direction is in the horizontal plane. The entire apparatus is placed on a rotating platform for reversals of the orientation on a timescale much faster than a sidereal day. This feature serves as a lock-in amplifier to modulate a possible Lorenz-violating field with respect to the apparatus to take advantage of the short term sensitivity. This changes the way in which measurements are performed.

3.4.1 CPT-II Design

A compact design reduces the possible optical pathlength for drift and also allows for simpler temperature control. The optical layout is only a factor of 2-3 smaller than CPT-I but is small enough to demonstrate improvement while still making use of conventional optical components. A brief description of the features of the major components of the new apparatus are provided here.

Magnetic Shields

Details on the magnetic shields used in this experiment are described in Ref. [78]. Briefly, three layers of $\mu$-metal and an inner MnZn ferrite layer provide an overall shielding factor of $1.2 \times 10^8$ in the y-direction (Fig. 3.10). The outermost shield has a diameter of 23 cm providing almost a factor of 3 reduction in size from CPT-I. The 10 mm thick, 104 mm inner diameter ferrite has a higher electrical resistivity than traditional $\mu$-metal to decrease the effects of Johnson noise from thermally conductive...
Figure 3.9: Schematic of the CPT-II comagnetometer on a rotating platform. An equipment rack dominates the lower half of the apparatus. All of the optics and breadboards are located within a 700 L bell jar pumped out to 2 Torr of atmosphere.

electrons in the shield. The shields have < 2 cm optical access holes along three orthogonal directions. The μ-metal shields close with a common can and lid geometry. The ferrite is a brittle ceramic, so an overlapping lid design is impossible to machine. The shield is in three pieces: an annulus with two opposing lids. The mating surfaces are polished to ~ 1 μm to complete the magnetic circuit when closed. The ferrite is very brittle and extra care is always used when opening and closing the shields, particularly to avoid scratching the polishes faces.

The Curie temperature of the ferrite is near 150°C. A fiberglass epoxy G10 jacket with interwoven water cooling lines maintains the ferrite at 40°C even with the bell jar evacuated and the nearby oven hot. Top and bottom water-cooled G10 plates hold the ferrite lids in place since the polished faces can slip. Thermally conductive gap pads between water cooling jacket and the ferrite distribute gentle pressure across
the ferrite lid from brass screws holding it in place. After degaussing the shields, a 
< 2 nT remnant field typically remains.

**Inner Vacuum Space/Field Coils**

The space between the ferrite and the oven contains a cylindrical G10 inner vacuum space for improved thermal insolation. The lower lid is made from a PEEK high temperature plastic, while the upper lid is made from G10 and suspends the oven from above. Six 5/8 inch PEEK tubes extend from the inner vacuum space through the shields and seal with anti-reflection coated windows. All joints seal with Viton o-rings. This space is continuously pumped and maintains a pressure of 2 mTorr. The only significant heat loss mechanism is through radiation. The inner surfaces of the vacuum space are coated with LO/MIT™-I low emissivity paint from Solec to further reduce heat loss.

The inner vacuum space also serves as a structure for magnetic field coils. These apply uniform fields in each direction as well as the five independent gradients. A home built low noise current source with a Hg battery as an internal reference provides up to 7 mA of current for a stable compensation field. Fine adjustments to this field
as well as the $B_x$ and $B_y$ coils are provided by computer controlled voltages and 50 kΩ, 20 kΩ, 20 kΩ resistors respectively. The directions of the field coils have been measured using a fluxgate magnetometer calibrated against Earth’s magnetic field to provide a consistent check on the response of the spins. A positive voltage applied to the $B_y$ coil produces a field in the direction of the lock-in amplifier on the equipment rack that we use as a reference. A simple switch allows us to reverse the direction of the $B_z$ coil. We refer to the two positions as “Forward” and “Reverse”; a positive voltage applied to the $B_z$ coil in the “Reverse” condition results in an upward directed field.

**Oven**

A boron nitride oven houses the comagnetometer cell (Fig. 3.10). This synthetic ceramic is chosen for its high thermal conductivity and easy machinability. The six rectangular panels are held together with PEEK screws and nuts. Two opposite faces have a meandering path for a twisted pair of resistive nichrome wire with double glass silicone insulation. An aluminum nitride potting compound holds the wire in place and provides good thermal contact to the oven housing. Circular ports allow optical access for the pump and probe beams. As is typical for this type of design, the faces of the cell with optical access are coldest where the cell can radiate the most. Radiation shields project outward from the oven to limit the aperture for radiation loss. Additional heating elements wrapped around the projections attempt to compensate for the loss. Still, K has a tendency to condense on these regions and slowly reduce optical transparency over a few months. One would prefer to make the stem the coldest region of the cell; however, a separately tunable stem heater drives the K blob to the neck of the cell to preserve the spherical symmetry for the hyperpolarized $^3$He often making this the warmest region (Section 4.6.1).
The total resistive load of the oven is 180 Ω. Several 10 kΩ platinum RTDs provide temperature monitoring. Many of these are left as open circuits so that no dc current flows during operation to create a magnetic field inside the shields – particularly one correlated with the temperature of the oven. An RTD close to the cell for temperature feedback uses an ac excitation to continuously monitor the temperature. A home-built, compact, audio amplifier provides ∼ 1 A of current near 300 kHz. Depending on the load to the stem heater, this frequency is decreased because the amplifier operates near the edge of its bandwidth. All necessary electrical connections enter this vacuum space through a multi-pin Fischer connector in an access can connected to one of the PEEK tubes outside the shields (Fig. 3.11).

Optical System

The pump beam enters the oven vertically, and the probe beam enters horizontally (Fig. 3.9). This places the leading sensitivity to anomalous fields in the horizontal plane such that a reversal of the rotating platform will reorient this axis. Two custom 304 stainless steel breadboards form the upper and lower deck of the optical layout (Fig. 3.11). A large 12 inch circular hole extends through the lower breadboard for placement of the shields. A 2 inch hole in the upper breadboard allows pump light to enter the shields after reflecting off a mirror mounted at 45°. The lower breadboard is 4 inches thick for greater rigidity in optical rotation measurements while the upper board is only 2 inches thick. These breadboards are mounted on a mechanical vibration isolation platform with a resonance at 1 Hz since a more typical compressed air damping system is difficult to implement within a vacuum.

Two DFB lasers from Eagleyard Photonics provide the tens of mW of light near 770 nm for the pump and probe beams. Several diodes gave their lives in the course of the experiments here. A Zener diode and germanium diode network along with a simple low pass filter in parallel with the diode have reduced electronic transients
that would result in instant laser death. These lasers typically run near 773 nm at room temperature and must be cooled to $\sim -10^\circ$ to approach the D1 transition in K. The internal thermal electric cooler (TEC) by itself does not provide enough cooling power, so additional external TECs cool a copper plate mounted to the back of the diode to $\sim 13^\circ$C. Since these lasers operate in vacuum, the aluminum laser mount carries chilled water from the same loop cooling the ferrite shield. The mounts have been mechanically reinforced for greater rigidity under rotation of the platform. We typically observe drifts of 10 pm/day for diodes near the end of their lifetime and < 0.2 pm/day for well-behaved diodes.

The aspect ratio of the beams is reshaped by an anamorphic prism pair. Spherical lenses shape the pump and probe beams to illuminate the cell. The pump beam is expanded to beyond the size of the cell, and a rigidly mounted 15.8 mm diameter aperture selects the spatially flat center of the beam to provide 8.3 mW of spatially uniform pumping light to the cell. Probe light at 770.759 nm is expanded to a 6 mm beam waist to illuminate a large number of atoms, but still avoid distortion.

\[ \lambda = \Omega^2 \]

\[ W_{\text{Wedge}} \]

\[ F_{\text{Calcite}} \]

\[ 45^\circ \text{ Mirror} \]

\[ \lambda/2 \]

\[ \text{Laser} \]

\[ \text{Anamorphic Prism} \]

\[ \text{Breakout Boxes} \]

---

7 During the summer months in Princeton, even the external TECs operate below the dewpoint causing condensation to appear on the diode faces. Once the bell jar has been pumped out, this is no longer an issue.
at the edges of the spherical cell. The combination of absorption by the vapor and condensation of metallic K on the windows decreases the 13.2 mW of light incident on the cell to 2.4 mW.

The intensity of both beams can be controlled either by adjusting the current to the diode or rotating a half waveplate prior to a calcite polarizer. A stepper motor on the waveplate mount for the pump beam allows adjustments when the bell jar is evacuated. A shutter controlled by a stepper motor can quickly block and unblock the pump beam for measurements of pumping by the probe beam. Immediately before entering the inner vacuum space, the pump beam passes through a quarter waveplate to circularly polarize the beam.

The probe beam layout includes a optical polarimeter. Two crossed calcite polarizers on either side of the cell, a quarter waveplate, a photo-elastic modulator (PEM) and photodiode make all of the optical rotation measurements. Measurements of optical rotation occur between the first polarizer and the quarter waveplate (Section 4.1.3). A stepper motor on the quarter waveplate provides fine adjustments even when the bell jar is closed. As with CPT-I, the PEM demonstrates lower noise than a Faraday modulator on long timescales; it is also more amenable to operating in vacuum. Anti-reflection coated wedges sample a small fraction of each beam and monitor the position on a four quadrant photodiode position detector.

**DAQ System**

A small form factor computer on the rotating platform handles all of the data acquisition. Two PCI-6229 National Instruments cards read 32 analog differential input channels and have 8 analog outputs. The computer also handles GPIB communication with the wavemeter, lock-in amplifier, and laser drivers on the platform. USB communication with the 24-bit counter and waveplate drivers is also available. The computer uses a wireless internet connection to communicate with the lab. Every-
thing on the computer can be monitored from a terminal in the lab in real time using the built-in Windows\textsuperscript{TM} remote desktop connection.

**Motor and Platform**

A 4.4 kW Yaskawa ac servomotor rotates the entire 1500 lb platform. The motor is mounted to one of the support legs 1 m from the gearbox with a ratio of 1:102. A 1 inch steel shaft transfers the rotation through a high-flex coupling. A pillow block also supports the shaft. The original shaft was cleaved and fell to the floor while rotating one day from a slight misalignment of the motor and repeated turning. The coupling and pillow block have led to a reliable operation of the rotating platform.

The motor is controlled by a Yaskawa motion controller. This allows for tuning of the PID in the servomotor and coordinated programmed moves given a velocity profile and distance of revolution (Table 3.4). Our two common settings are “slow” and “moderate”. A built-in filter smoothes the edges of the velocity profile and slightly increases the time calculated to complete a rotation. Commands are sent to the motion controller through the serial port on a computer located in the lab. A simple LabView program translates common commands from a user interface to the text strings understood by the motion controller. This program can repeat motions after fixed interval delays and coordinate these moves with the computer onboard the platform. The communication is handled through the NI-Datasocket Server over the wireless internet connection.

In the process of finding the correct PID values for the servomotor, the platform was occasionally set into large oscillations. Several large stop buttons around the lab cut power to the motor for a hard emergency stop. The motor will suddenly lock and the experiment spins 10-15° across the floor. We considered bolting the experiment to the floor, but have chosen to let the energy dissipate through friction with the floor.
rather than in the gearbox or shaft. Service to the worm gear, shaft, or motor would cause serious delays to the project.

The central axis of the platform carries a vacuum line as well as the 60 Hz ac line power to the platform. A rotating vacuum seal and an electrical slip ring allow continuous rotations of the apparatus in either direction. The earlier Huntington Labs differentially pumped rotating vacuum seal was replaced when the o-rings failed. It has been upgraded to a Ferrotec 1 inch hollow shaft ferrofluid rotating seal. The ferrofluid o-rings are expected to provide a longer working lifetime than the previous implementation. We have enjoyed two years of operation without trouble from this device.

Around the vacuum seal, we have installed a incremental rotary encoder from Gurley Instruments. The 1 inch hollow shaft encoder fits precisely around the vacuum tube. A simple quadrature counter from J-Works reads the quadrature output and a simple LabView program on the platform translates the number to a rotation angle of the platform. The 11250 counts on the disc, four read heads, and an interpolating function can distinguish rotations from translations to provide a precision of 0.001° in the relative angle between the base and platform. This incremental encoder survived most of the Lorentz violation measurements but currently loses a large number of counts when rotating through one quadrant, probably from mechanical misalignment of the disc and heads. The number of counts lost each revolution varies, so the encoder quickly loses position. Reliable operation over several months has been achieved, but future experiments should consider a more robust solution.

<table>
<thead>
<tr>
<th></th>
<th>Slow</th>
<th>Moderate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>0.15 rad/s²</td>
<td>0.38 rad/s²</td>
</tr>
<tr>
<td>Deceleration</td>
<td>0.15 rad/s²</td>
<td>0.38 rad/s²</td>
</tr>
<tr>
<td>Max Speed</td>
<td>0.60 rad/s</td>
<td>1.20 rad/s</td>
</tr>
<tr>
<td>Reversal Time</td>
<td>10.4 s</td>
<td>5.2 s</td>
</tr>
</tbody>
</table>

Table 3.4: Platform reversal specifications at each of the two common speeds.
Bell Jar

The 700 L, 304 stainless steel bell jar encloses all of the optics and lasers. It has a 3/16 inch wall thickness and 34-7/8 inch inner diameter. An I-beam and hand winch in the ceiling rated to 2000 lb can raise and lower the 300 lb bell jar. This winch can also lift the entire experiment from the eyebolts in the 1 inch thick aluminum baseplate. The baseplate is 36 inches in diameter and passes all of the electrical, vacuum, cooling water, and optical fiber feedthroughs. A large diameter Viton o-ring makes a tight seal when the bell jar rests flat against the o-ring on a bed of clean vacuum grease.

A large number of electrical connections are supplied by 3 sets of 50-pin ribbon cables. The ribbon cables pick up excess 60 Hz noise from the room, but simply wrapping them in aluminum foil, insulting them with tape, and grounding one end reduces the pickup. Twisted pair ribbon cable would prevent possible crosstalk, but this has not been an observed issue. Several Fischer connectors and BNC connections provide feedthroughs for more specific channels.

Once the bell jar is lowered, there is no longer optical access to the interior. It was earlier proposed to add viewports to optically monitor the position of the vibration isolation platform \[79\]. Lighting levels around the room differ and there was concern that correlations with reversals may appear on photodiode monitors. No optical access was added and monitoring of the vibration isolation platform with respect to the baseplate is achieved using two non-contact inductive position sensors on either side of the stage (Fig. 3.9).

A dry roots pump at 9 cfm evacuates the bell jar below 1 Torr within 30 minutes. Off-gasing from plastics and epoxies within the bell jar is expected, though this vacuum is well-maintained over weeks. The large volume results in low sensitivity to off-gassing with only an increase to 4 Torr observed after a few weeks. At this point, experiments are simply paused to pump down to 1 Torr once again. When
not evacuating the bell jar, the dry pump backs a Molecular Drag Pump (MDP) that continuously pumps on the inner vacuum space necessary to maintain the 2 mTorr inner vacuum. This area is heated to 185°C and continuously offgasses. If pumping stops, the pressure will reach hundreds of mTorr within 20 minutes failing to insulate the oven as evidenced by rising temperatures around the coil form and ferrite.

The bell jar reduces the effects of air convection on the position of the laser beams (Fig. 3.12). Without the bell jar, the root-mean-square motion is hundreds of nanometers. Simply lowering the bell jar reduces these effects to $< 10$ nm with a knee just below 10 Hz. Pumping out the air further improves the motion but is then limited by thermal drift. At the time of this comparison, water cooling on the lasers had a leak and was disabled. To solve the thermal equilibration issue, the bell jar can be filled with $^4$He. He has an factor of six larger thermal conductivity and a factor of ten smaller index of refraction with respect to vacuum than air. Allowing the He to
settle and equilibrate over a few minutes greatly reduces the drift. In the time since this comparison, the cooling issues have been solved. A 1 Torr vacuum performs even better than the He since its index of refraction is even closer to that of vacuum.

**Equipment Rack**

The equipment rack has a 25 inch × 25 inch aluminum frame and stands 36 inches tall. This size provides a compact design, but provides ample space for key pieces of commercial equipment such as the lock-in amplifier, computer, and laser drivers. In addition, a small frame 300 W chiller supplies cooling water to the laser mounts and ferrite. Custom electronics have been have been designed in some instances for a more compact design including analog position detector circuits, a temperature controller feedback, and an audio amplifier. All of the equipment related to comagnetometer operation except vacuum pumps resides on the platform to allow continuous rotations without a large number of rotating feedthroughs.

**3.4.2 CPT-II Performance**

The improvements of the bell jar and compact design provide suppression of long term drifts by almost an order of magnitude in frequency over CPT-II (Fig. 3.15). The peak at 2.5 Hz is a mechanical vibration while the peak at 5 Hz appears suspiciously like a second harmonic. Only the first peak is observed on an accelerometer. The compensation frequency here is at 10 Hz. Below the compensation point, the sensitivity is limited by the white electronic noise of the photodiode amplifier and lock-in amplifier system to an effective magnetic field of 0.7 fT/√Hz. Below 0.05 Hz, the probe beam is limited by drift. Below 0.03 Hz, the comagnetometer is no longer limited by the probe beam, but begins to drift as well. The noise at 1/sd is still large, but the comagnetometer achieves an effective sensitivity of 2 fT/√Hz at 0.023 Hz.

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8This is most likely from a drifting $B_z$ field as the $^3$He polarization drifts away from the compensation point without feedback.
Figure 3.13: Image of the CPT-II apparatus on a rotating platform. The lower half is dedicated to an equipment rack. A 700 L stainless steel bell jar surrounds the optics and lasers. A view of the interior of the bell jar showing the optics and vibration isolation platform has been superimposed on top of this image. A platform around the experiment simplifies access to the optics when the bell jar is raised. The vacuum pumps are seen at the bottom in the foreground. The shaft for the motor can be seen on the leftmost leg. Large Helmholtz coils surround the experiment to reduce Earth’s magnetic field. The coils are supported by inexpensive, but stable, “mounting” cans.
Figure 3.14: The four faces of CPT-II. The SRS lock-in amplifier on the lower equipment rack provides a handy reference. Using a right handed spin-up convention, we rotate the apparatus to view all four sides.

Figure 3.15: The optimized noise spectrum from CPT-II represents a dramatic improvement in suppression of $1/f$ noise. The compensation frequency is 10 Hz. The peak at 2.5 Hz is a mechanical vibration. The peak at 5 Hz appears to be a higher harmonic that does not appear on an accelerometer.
This timescale is short enough to provide both low noise and long enough to reverse the orientation of the comagnetometer. The motor is capable of executing a reversal in 6 s, however it is important to let the vibration isolation platform and the spins damp for several seconds before recording the signal. Rotations every 22 s allow ample time for reversal, damping, and 3-7 s of signal averaging before the next reversal.

The spectrum in Fig. 3.15 was recorded with the platform at rest over 900 s. Rotations of the platform are expected to disturb the equilibrium and introduce additional drift. This additional drift is much smaller than the drift over a sidereal day, so modulating the direction of the experiment improves the measurement methodology.

**Noise Optimization**

Optimizing low frequency noise takes substantial effort and patience. Often, it is unclear where the drift originates. A correlation analysis with the signal and other parameters can reveal the source of the drift. The comagnetometer signal has contributions of drift from the probe beam and the atoms. Separation of these two signals can help identify the source of the drift as presented in Fig. 3.15. The comagnetometer can best be measured when the $^3$He polarization has reached a stable equilibrium over many hours. This allows the compensation point to be maintained without an active feedback. The probe signal can be distinguished by increasing the compensation field by several mgauss and blocking the pump beam. This depolarizes the alkali atoms and pins the spins along the direction of the field to desensitize the response from the atoms. Typically, the comagnetometer spectrum is taken first and followed by the spectrum of the probe beam alone. It takes many hours to re-equilibrate the $^3$He polarization after depolarizing the alkali atoms by blocking the pump beam.
3.4.3 An Improved Lorentz Violation Search

The noise of the comagnetometer on the timescale of 100 s is on the order of the limit set by the Harvard-Smithsonian group and CPT-I (Section 3.3.2). One would expect that simply reversing the experiment in an arbitrary direction multiple times could quickly average down to below the sensitivity of the previous results. A simple “string analysis” technique (Appendix E) can determine the reversal-correlated amplitude response and reject any remaining longer-term drift. However, the rotating platform introduces systematic effects that must be accounted for; any correlation with a reversal results in a systematic effect.

The most dominant systematic effect comes from the comagnetometer’s sensitivity to rotations as a gyroscope. In CPT-I, a response from Earth’s rotation enters as a stable offset to the signal. Here, the response is correlated with reorientation of the y-direction of the experiment. It achieves maximal overlap with the Earth’s rotation axis along the N-S direction in the lab (Fig. 3.16). Based on the latitude of the lab in Princeton, NJ, the expected amplitude of response from Eq. 3.31 is

$$\frac{\Omega_\oplus}{\gamma_n} \left(1 - \frac{\gamma_n Q}{\gamma_e}\right) \cos (40.345^\circ) = 271 \text{ fT}$$

(3.45)

where $\Omega_\oplus = 2\pi/(86164.09 \text{ s})$ to account for both the rotation of the Earth and its orbital motion about the sun with respect to a celestial inertial frame (Appendix B).

The large gyroscope response breaks the symmetry of reversals in the lab and creates a natural basis for a Lorentz violation test. Since the Earth’s rotation rate is stable to better than a part in $10^{-8}$, repeated measurements along either the N-S or E-W direction should provide a stable background response. A sinusoidal variation at a sidereal frequency in either Earth-fixed response would provide evidence for a possible Lorentz-violating field. The large gyroscope background limits the experiment to operate in an Earth-fixed frame. Measurements over the course of an entire
Figure 3.16: Earth’s rotation as a function of heading of the comagnetometer. Each point represents 8 reversals of the apparatus. Error bars are smaller than the size of each point and are omitted. The curve is a fit with an amplitude of 277 fT and phase of 1.1°W of the expected direction. Rotation conventions are discussed in Section 4.1.1.

day are still required to search for a sidereal variation in the reversal-correlated amplitude. However, the reversals of the platform allow repeated measurements with the 2 fT/√Hz sensitivity in the short term behavior, a significant improvement over CPT-I. However, changes in the measured gyroscope response can limit this technique.

Reversals along the E-W axis have the greatest sensitivity to measurements of \( \tilde{b}_\perp \) but are most sensitive to accurate reorientation of the platform. A 0.2° variation in the positioning of the platform leads to 1 fT noise along the E-W axis. This is a second order effect for measurements along the N-S axis and therefore further suppressed. Since there is a large offset in the N-S response, imbalanced changes in sensitivity or the calibration of the comagnetometer lead to variations in the signal. A 0.3% change in the sensitivity not captured by the calibration produces a 1 fT change in the N-S response. Furthermore, the sensitivity of N-S measurements to \( \tilde{b}_\perp \) is suppressed by \( \sin (40.345°) \).
Figure 3.17: A typical long measurement run of reversals along both the N-S and E-W directions. The scatter is 2-3 $fT$ which is already on the order of the previous limit [29].

In this work, we choose to make measurements along both the N-S and E-W directions throughout the course of the day. This allows for discrimination of systematic effects along each measurement axis from changes in positioning of the platform as well as sensitivity calibrations over the course of a day. Furthermore, a real Lorentz-violating response would appear out-of-phase in the response of both orientations which will aid in further distinguishing a real signal from systematic effects.

Typical long term measurements of both the N-S reversals and E-W reversals have a scatter of 2-3 $fT$ over the course of a sidereal day, and this performance can be maintained over weeks at a time (Fig 3.17). It is clear that a possible sidereal amplitude must be smaller than this scatter which is already a dramatic improvement over the previous results (Fig. 3.5). In one day of measurements, the sidereal variation is already on the order of the previous limit at 1 $fT$. However, several additional systematic effects must be addressed before discussing the full details on the Lorentz
violation search. The earliest measurements of the gyroscope response resulted in a signal facing 20° W of the local N-S axis and amplitudes 15-30% too large. This launched a detailed investigation of the behavior of the comagnetometer gyroscope as well as optical rotation measurements on the rotating platform. These details are discussed in Chapter 4 as they influence the Lorentz violation results in Chapter 5.
Chapter 4

Rotating Comagnetometer Measurements

The CPT-II apparatus on a rotating platform introduces a new measurement scheme for an improved Lorentz violation search. Much of the underlying comagnetometer operation is unchanged. Platform reversals provide a clean measurement of the rotation of the Earth from the gyroscope response of the comagnetometer. The total response of a reversal measurement can also be influenced by background effects correlated with other local anisotropies. This chapter describes operation of the K-$^3$He comagnetometer in the CPT-II apparatus and highlights new effects from platform reversals.

4.1 Reversal Measurements

The noise spectrum in Fig. 3.15 demonstrates that measurements made at 0.023 Hz can take advantage of the short term sensitivity of the comagnetometer. A typical reversal measurement interval consists of continuous recording of the signal for 8-10 reversals every 22 s. Approximately every 200 s, acquisition is paused to adjust the compensation field from drifts in the $^3$He polarization (Section 4.2). The first 3-7 s
contributes to a measurement while the apparatus is stationary. The platform is then reversed over 6 s. Before the next 3-7 s of signal acquisition, several seconds pass for damping of the spins and the vibration isolation stage. Every time the motor begins to move, a trigger timestamp is saved to a file. The clocks on the two computers are synchronized to a navy timeserver with variations less than 20 ms between them. This allows for consistent selection of the signal with the experiment at rest prior to the trigger. The reversal correlated amplitude is then determined using an overlapping 3-point string analysis (Section E.3).

### 4.1.1 Rotation Conventions

A positive voltage applied to the $B_y$ coil points the same direction as the lock-in amplifier on the platform. We use this side of the platform as a reference (Fig. 3.13). Typically, we start measurements with reference to the local south position facing the computer terminal (Fig. B.5). Rotations are most often in the “spin up” direction according to the right hand rule convention, but continuous rotations in either direction are allowed. Each reversal rotates in the same direction to avoid the $0.2^\circ$ of backlash in the worm gear.

We define the heading of the comagnetometer as the direction in which the lock-in amplifier points. We use the common convention of degrees measured clockwise from True North. If the lock-in is pointing east, the heading is $90^\circ$. This is opposite from the typical direction of applied rotations (and increasing counts on the encoder) but simplifies matters relating to the direction of the gyroscope response. For ease of visualization, we extend headings beyond values of $0 - 360^\circ$ to indicate repeated measurements. The orientation of the comagnetometer is simply the heading modulo $360^\circ$.

---

$^1$See Appendix B for a determination of True North with respect to the lab.
The sign of the the comagnetometer response to fields can be reversed by reversing the direction of the spins. As described in Section 3.4.1, the $B_z$ coil points up in the “Reverse” position. In the descriptions that follow, we will refer the reader to the correct coil orientation and heading where relevant. A proper calibration (Section 4.3) will clarify this ambiguity.

### 4.1.2 Gyroscope Measurements

Each of the points in the gyroscope map in Fig. 3.16 represents 8 measurements from 7 reversals of the platform. A typical interval for such a measurement is in Fig. 4.1a where the raw signal has already been multiplied by an overall calibration value (Section 4.3). The applied rotation introduces a large response every time the platform reverses. Structure in this response comes from motion of the vibration isolation platform and imperfections in the platform bearings. The signal quickly damps to the steady state value after the platform comes to rest. The overall offset of the signal is arbitrary and comes from the relative angle of the polarization optics in the polarimeter (Section 4.1.3).

The signal at rest is averaged to a central value (Fig. 4.1b). The uncertainty of each point is determined from the standard deviation of the signal before averaging and the effective noise bandwidth of the measurement (Section E.2). A 3-point overlapping string analysis determines the reversal correlated amplitude and removes linear drift between reversals along the entire measurement interval. The uncertainties in each string point are much larger than the scatter of successive string points due to the correlation between overlapping strings (Fig. 4.1c). These string points are averaged and the standard error determined with a correction for non-independent points. The first point is often distinct from the average of the rest of the points due to a change in the background drift at the start of regular rotations. Prior to each interval, the apparatus remains at rest for 30-60 s of feedback and calibration routines.
Figure 4.1: Typical rotation interval for measurements a) Calibrated signal with large transients from rotations. b) Data is selected while the apparatus is stationary (yellow). c) 3-point overlapping string analysis of selections.

For this particular interval, a value of -276.4 fT with a statistical uncertainty of 0.4 fT is measured. The negative amplitude confirms that the measurement began in the south position. These measurements can be repeated along different headings to produce the points in Fig. 3.16. The statistical uncertainty in each measurement is smaller than the size of the displayed point and is typically omitted from plots for ease of interpretation.

4.1.3 Optical Rotation Measurements

Optical rotation measurements in CPT-II use a PEM to modulate the polarization of the light to separate the signal from $1/f$ noise (Fig. 4.2). The first calcite polarizer determines the initial linear polarization axis. Faraday rotation from the atoms rotates the plane of polarization by an angle $\theta$. Next, a quarter waveplate with one of
its axes nominally aligned with the calcite polarizer contributes a circular component proportional to $\theta$. A Hinds Instruments PEM with its modulation axis oriented at $45^\circ$ modulates a bar of fused silica at $\omega = 2\pi \times 50$ kHz. The mechanical stress in the crystal introduces a time varying birefringence with a retardation angle $\alpha = 0.08$ rad. A final calcite polarizer crossed with the original polarizer selects the time varying intensity of the transmitted light.

The evolution of the polarization can be predicted using the Jones Matrices \[80\]. The PEM is modeled as a time-dependant quarter waveplate and the cell modeled as a half waveplate. A rotation matrix determines arbitrary orientations of each optical element. In the case that the polarizers are well aligned and the cell creates an optical rotation angle $\theta$, the time-varying intensity on the photodiode is given by

$$I(\theta, t) = I_0 \sin \left( \theta + \frac{1}{2} \alpha \sin (\omega t) \right) \simeq I_0 \alpha \theta \sin (\omega t) + \frac{I_0 \alpha^2}{4} \sin^2 (\omega t) \quad (4.1)$$

where $I_0$ is the initial intensity, and we have made the approximation that $\theta, \alpha \ll 1$. In the case that $\theta = 0$, the second harmonic response dominates the intensity (Fig. 4.3). The first harmonic provides a signal proportional to the optical rotation from the cell and is measured by lock-in amplifier referenced to the PEM.

Nominal alignment leads to the behavior described in Eq. 4.1, but it is interesting to consider the deviations from this case. The individual and combined behaviors can be easily confirmed with numerical simulations. These simulations have been
separated into interesting cases to produce simple analytic expressions. If the quarter waveplate is rotated by an angle $\chi$

$$I(\theta, \chi, t) \simeq I_0 (2\chi^2 - 2\chi\theta + \theta^2) + I_0\alpha(\theta - \chi)\sin(\omega t) + \frac{I_0\alpha^2}{4}(1 - 2\theta^2 - 4\chi^2 - 4\theta\chi)\sin^2(\omega t)$$

(4.2)

and the first harmonic responds in the same way as optical rotation from the atoms. For a small adjustment $\delta$ to the final polarizer,

$$I(\theta, \delta, t) \simeq I_0 (\delta^2 + \theta^2) + I_0\alpha(\theta - 2\delta^2)\sin(\omega t) + \frac{I_0\alpha^2}{4}(1 - 2\delta^2 - 2\theta^2)\sin^2(\omega t)$$

(4.3)

leads only to a second order correction in $\delta$ to the first harmonic response. Similarly, a rotation of the PEM by an angle $\zeta$ produces only a second order correction to the optical rotation signal

$$I(\theta, \zeta, t) \simeq I_0\theta^2 + I_0\alpha\theta(1 - \zeta^2)\sin(\omega t) + \frac{I_0\alpha^2}{4}(1 - 2\theta^2 - 2\zeta^2)\sin^2(\omega t).$$

(4.4)

We can model the addition of circular birefringence to the cell or intervening optics by considering a quarter waveplate with its fast axis nominally aligned with the incident
polarization. For a small rotation $\phi$ of this waveplate,

$$I(\theta, \phi, t) \simeq I_0(\theta^2 + \frac{\phi^2}{4}) + I_0\alpha\theta \sin(\omega t) + \frac{I_0\alpha^2}{4}(1 - 2\theta^2 - \frac{\phi^2}{2})\sin^2(\omega t) \quad (4.5)$$

and there are no corrections to the first harmonic. Of all these possible misalignments, the leading change to the first harmonic response comes from a rotation of the first polarizer and the quarter waveplate as these define the axis of optical rotation. Therefore, these optics must be rigidly mounted to avoid undesired changes in the optical rotation signal.

Each of these misalignments to the nominal case leads to a dc offset in the photodiode response (Fig. 4.3). In principle, this effect can be removed by adjusting optical elements until the optimal configuration can be found (Eq. 4.1). For example, a remnant birefringence can be removed by adjusting the final polarizer

$$I(\theta, \phi, \delta, t) \simeq I_0(\delta - \frac{\phi}{2})^2 + I_0\alpha\theta(1 - 2\delta^2)\sin(\omega t) + \frac{I_0\alpha^2}{4}(1 - 2\delta^2 - \frac{\phi^2}{2})\sin^2(\omega t), \quad (4.6)$$

however, experimentally this is not the case. A minimum for the offset can be found that is difficult to reduce below 5% of the total response. Simulations indicate that a number of configurations should lead to a null offset though they neglect any spatial non-uniformity in optical rotation or birefringence. Simply, if there is a different response on two halves of the beam profile, an adjustment of the final polarizer can only compensate for one half at a time. The sum of the two signals leads directly to an offset in the presence of a non-uniformity. This offset is mostly an experimental curiosity and does not contribute significantly to the measurement itself.

In the optical rotation measurements, care is taken to remove extra elements from the optical rotation axis such as dielectric mirrors. Mirrors would yield greater flexibility in beam alignment, but the difference in reflectivity of different polarizations can influence measurements. Measurements reach a noise noise floor of $4 \times 10^{-8} \text{ rad}/\sqrt{\text{Hz}}$. 

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Figure 4.4: Deflection of the probe beam horizontal position measured on a four quadrant photodiode during a typical reversal interval. The 200 nm spikes occur while the experiment is accelerating. The position settles to a fixed value at rest. By raising the legs of the experiment off the floor, the breadboards can be leveled to reduce the shifts in position observed at rest.

Frequencies above 0.1 Hz reach the floor whereas longer term signals are limited by drifts and $1/f$ noise.

### 4.1.4 Tilt

Prior to this work, it was reported that the lasers do not change position under rotation of the platform [79]. This claim came from position sensors whose effective sensitivity had been reduced by placing the detector at the focal point of a lens. It is common to sacrifice sensitivity by reducing the beam size to fit within the 1 cm diameter detector face using a lens. At the focal point of the lens, the decrease in sensitivity to translations of the beam is almost infinite.

As initially constructed, the rotation axis leans 0.5° from vertical. Given this anisotropy, the apparatus experiences changes in gravitational field in different positions. The probe beam breadboard is designed to be monolithic, but shifts in beam
position on the order of 50 nm are observed (Fig. 4.4). Larger deflections occur under acceleration of the platform but return to a consistent value at rest.

The tilt is monitored using two orthogonally placed C-29 tilt sensors from Advanced Orientation Systems Inc. (AOSI) mounted near the center of the upper breadboard. These provide 1 \( \mu \)rad sensitivity over a range of 3 degrees with a bandwidth of 2 Hz. Simply elevating two legs of the apparatus \( \sim 1 \) cm orients the rotation axis within 0.005° of vertical and suppresses the correlations at rest. For a typical polarimeter arrangement, the optical rotation as a function of tilt angle is 1.4 \( \mu \)rad/deg (Fig. 4.5). It is interesting that the optical rotation magnitude is not symmetric about the vertical. This is likely due to a redistribution of weight on the vibration isolation platform.

A series of measurements to determine the source of the beam walk have been undertaken. The response persists for even a minimal polarimeter on the edge of the breadboard with only the necessary optical elements. Several polarizers, mounts, and waveplates have been tightened, loosened, and swapped with no qualitative difference in response. Gentle pressure on laser mounts leads to an optical rotation response.
These mounts have been mechanically reinforced and the effect reduced. Stressing the breadboard near polarization optics by hand can also induce a small optical rotation response. No particularly offensive optic has yet been identified.

A 50 nm shift in position of a 5 cm tall waveplate mount with respect to the base could appear as a 1 $\mu$rad rotation of the waveplate (Eq. 4.2). This is smaller than the observed effects before leveling the apparatus. Imperfections in the birefringence of the optics along different beampaths likely leads to an enhancement of this effect. Capacitive coupling of the PEM modulation frequency onto the photodiode amplifier has been observed as an additional tilt effect. This and other effects have been removed by shielding and strain relieving all cables.

### 4.1.5 Faraday Effect

The Faraday effect describes the rotation angle $\theta$ in the plane of polarization of linearly polarized light as it passes through a medium with a magnetic field $B$ parallel to the direction of propagation. The rotation is given by

$$\theta = VBl$$

where $V$ is the Verdet constant of the material and $l$ is the propagation length through the medium. The Verdet constant can vary with temperature and wavelength.

Earth’s magnetic field has a preferred direction in the lab. The magnetic shields and magnetic field compensation of the comagnetometer are enough to suppress Earth-field size effects on the atoms to below 0.1 fT. A reversal of the apparatus reverses the external magnetic field and the contribution of the Faraday rotation to the optical rotation signal. The leading optical contributors to the Faraday effect are the polarizer, stress plate, vacuum windows, lens, and quarter waveplate (Fig 3.11). The Verdet constants and thickness of each element is listed in Table 4.1.
Large 2.4 m square Helmholtz coils surround the experiment to reduce the changes in magnetic field under reversals to below 5 mgauss. A three-axis fluxgate magnetometer on the lower breadboard monitors all three components of the magnetic field during measurements. Based on the Verdet constants in Table 4.1, an optical rotation amplitude of 4 $\mu$rad is predicted from the measured 0.44 gauss magnetic field. The observed effect is 11.2 $\mu$rad and is well-aligned with the field direction (Fig. 4.6). The high permeability shields concentrate the field in the region of the optics and the fluxgate is located 15 cm from the magnetic shields. This factor of 2-3 enhancement is enough to account for the discrepancy.

### 4.1.6 Optical Interference

A scan of the probe beam wavelength produces oscillations in the optical rotation signal characteristic of an interference pattern. Sensitivity to motion of the optical components has already been identified on the scale of 50 nm (Section 4.1.4) and these shifts in pathlength can likewise influence the phase of the interference pattern under reversals of the platform (Fig. 4.7).

Observed interference fringes vary the optical rotation response from 5-200 $\mu$rad. The source of the pattern is expected to depend on the length of the cavity causing the interference. In Fig. 4.7, the high frequency fringes indicate a cavity on the scale of 0.5 m whereas the lower frequency gives a cavity length of 5 cm. These lengths do not correspond to any particular optic or spacing between optics. Modifications
Figure 4.6: a) Optical rotation from reversals in an applied magnetic field. b) Fluxgate response measured along the probe beam propagation direction. Both responses have the same phase.

Figure 4.7: A slow scan of the probe laser wavelength produces an interference pattern in the optical rotation signal of the probe beam. The phase of the pattern can change with reversal of the apparatus.
to the optical arrangement can dramatically change the frequency of the observed pattern, but only in a seemingly unpredictable way. The fringes persist even when the experiment is well-leveled and have a relatively stable phase over short timescales.

### 4.1.7 Optical Rotation Background Measurements

Reversal correlated changes in the probe beam from the beam walk and Faraday effect can be reduced to 100 nrad by controlling the tilt and the local magnetic field. To remove residual probe background contributions from our signal, a background monitoring scheme has been developed. A relatively large $B_z$ field of 0.65 mgauss desensitizes the atoms and provides a real time measurement of the probe background. In each position, the $B_z$ field can be smoothly raised for a correlated measurement of the background for 2 s before returning it to the compensation point for a measurement of the comagnetometer response (Fig 4.8).

This scheme was implemented for the second half of the Lorentz violation measurements in Chapter 5. The long term behavior of the the probe beam background
can be effectively removed in measurements and provides a dramatic improvement in the long term stability of the the measured gyroscope response (Fig. 4.9). The background removal subtracts residual beam walk and Faraday rotation effects as well as any other change such as photoelastic effects on the windows to the inner vacuum space.

The high $B_z$ measurements remove most of the background, but retain sensitivity to the $B_x$ and $L_x$ fields (Eq. 3.24). These fields have been shown to drift over longer timescales than a typical reversal interval. Therefore, high $B_z$ measurements provide both the probe background and a small constant contribution from the comagnetometer. A 0.01 µgauss shift in the $B_x$ field would provide an estimated 6 fT response from the comagnetometer in the high $B_z$ limit. Since only a monitor of the background offset between each reversal is of interest, this contribution is for the most part neglected, but remains an important technical detail in background measurements.
4.2 Comagnetometer Zeroing

The long list of second and third order effects in Section 3.2 lead to possible systematic effects. These can influence the signal to obscure a measurement of anomalous fields. A scheme to stabilize these parameters to a small fixed value has been developed in Ref. [28]. These procedures are referred to as “zeroing routines” despite only ever reaching a small value in the parameter that is being “zeroed”. The following section describes the implementation of the zeroing routines in the CPT-II comagnetometer with special interest given to any correlated changes with reversals of the apparatus.

4.2.1 Quasi-Steady State Measurements

The lists of contributions to the signal in Section 3.2 all describe the steady state response of the comagnetometer. The fast damping of the nuclear spins (Eq. 2.72) implies that the steady state response can be achieved in a few seconds, so a quasi-steady state measurement scheme has been developed [28]. A modified square wave can be applied

\[ B(t) = B_0 \tanh(s \sin(\omega t)) \]  (4.8)

where \( \omega \) is the frequency of measurement and \( s \) describes the sharpness of the transition. This function has the feature that all time derivatives are continuous and the field efficiently transitions between field values in finite time. With a typical sharpness value of 10 and \( \omega = 2\pi \times 0.2 \) Hz, the field transitions in 600 ms (Fig. 4.10). As long as \( \omega \ll \omega_n \) and the slope \( B_0 \omega s \ll |B^n \omega_n| \), the \(^3\)He spins will efficiently move from one steady state value to another. The comagnetometer response can be averaged and compared at the constant values of the waveform. An elaborate LabView program provides the modulations and selects the appropriate intervals to serve as a quasi-steady state lock-in amplifier.
Figure 4.10: A example square wave modulation at $\omega = 2\pi \times 0.2$ Hz. A sharpness value of 10 transitions the field in 600 ms.

### 4.2.2 Compensation Point

Several hours of optical pumping produces an equilibrium polarization of $^3$He at a modest applied field of tens of mgauss. Once the equilibrium polarization has been reached, the field can be slowly and smoothly lowered to find the compensation point. Any sharp changes in the applied magnetic field will induce transverse oscillations of the spins that decay with the long $T_2$ of the $^3$He.

There are two common methods used to identify when the compensation point has been reached. At large fields ($B_z^a \gg 20$ µgauss), $D_S$ dominates the signal (Eq. 3.20). As the applied field nears the compensation field, $B_z$ in the denominator no longer suppresses terms in numerator. A sharp dispersive response in the signal is observed due to the product of $B_yB_z$ in the numerator. The sharpest part of this feature is close enough to the compensation point that an automated routine can provide fine adjustments to zero the $B_z$ field (Section 4.2.3).

Near the compensation point, the nuclear spins are quickly damped (Eq. 2.72). Far from the compensation point, a $B_y$ field in the form of a modified square wave excites the nuclear spins. By comparing the damping time as the $B_z^a$ field is slowly adjusted as before, a dramatic decrease in the damping time can be observed. At this
point, the automated zeroing routine can adjust the applied field to precisely locate the compensation point.

4.2.3 Zero $B_z$

Assuming that the comagnetometer is already close to the compensation point as described in Section 4.2.2 a simple quasi-steady state modulation procedure can evaluate the value of the applied compensation field. Considering Eq. 3.43, the response of a $B_y$ modulation is proportional to $B_z$. In effect, the modulation takes a derivative of $S_L$ with respect to $B_y$,

$$\frac{\partial S_L}{\partial B_y} \propto \frac{B_z}{B_c}. \quad (4.9)$$

By comparing the response to a $B_y$ modulation at various $B_z$, a zero crossing can be identified. A numerical simulation of $S_L$ for a $B_y$ modulation at various $B_z$ is shown in Fig. 4.11. Near the compensation point, a simple two step measurement can determine the slope and the crossing point. Common parameters for this procedure and other zeroing routines is in Table 4.2.

It is not clear that this procedure produces a $B_z$ field that is precisely zero. There is a finite contribution from other higher order terms (Eq. 3.19), the largest of which is $-\Omega_z/\langle \gamma_n B_c \rangle$. Considering the rotation of the Earth, this would cause a shift in the $B_z$ value of 230 fT which is much smaller than the resolution of the magnetic field coil. Noise from the computer contributes an RMS variation of 2 pT to the $B_z$ field. Likewise, drifts in the $^3$He polarization lead to a typical variation of 20 pT in $B_z^a$ over the course of an hour, with longer term drifts following the average of the $^3$He polarization. By adding up the contribution to the current from the low noise current supply and the computer, a value for $B_c$ can be determined better than 0.5%.
4.2.4 Zero $B_y$

Once a small value for $B_z$ has been reached, a procedure to reduce the effects of a drifting $B_y$ field can be performed. This is similar to the zeroing of the $B_z$ field except the roles of the fields are reversed,

$$\frac{\partial S_L}{\partial B_z} \propto \frac{B_y + \beta_y^m}{B_c} + \frac{L_x \gamma_e}{R_{tot}} + \frac{\gamma_e \Omega_x}{R_{tot} \gamma_n} \quad (4.10)$$

where here the higher order terms amount to measurable offsets. In terms of stabilizing the signal, it is most pertinent to reduce the effects of a drifting $B_z$ field since this value changes with the $^3$He polarization.

The presence of $\beta_y^m$ in Eq. 4.10 does not play an appreciable role in a Lorentz violation search. The value of $\beta_y^m$ is already constrained to less than 1 fT [29] and its contribution to the signal is suppressed by $B_z/B_c$ which is a small number. The noise on the $B_y$ coil from the computer is 4 ngauss and the zeroing variations are typically 50 ngauss.
4.2.5  Zero $B_x$

If the same technique as described for $B_y$ and $B_z$ is used to zero $B_x$, a zero crossing will be avoided. However, a second derivative in $B_z$ will provide the appropriate response,

$$\frac{\partial^2 S_L}{\partial B_z^2} \propto \frac{\gamma_c}{R_{tot}} \left( \frac{B_x}{B_c} - \frac{\gamma_c}{R_{tot}} - \frac{\gamma_e \Omega_y}{R_{tot} \gamma_n} \right).$$  \hspace{1cm} (4.11)

If $B_z$ is already close to zero, an asymmetric modulation of $B_z$ will act as a second derivative. Reference to Section 3.2 indicates that there are a large number of terms proportional to $B_z^2$, so it is likely that $B_x$ is not actually zero. However, $B_x$ provides a value such that a $B_z^2$ modulation is suppressed. Since $B_z$ is the most quickly drifting parameter, this is a useful procedure.

The $B_x^a$ field has similar noise characteristics to $B_y^a$, and Eq. 4.11 has many more anomalous field contributions (Section 3.2.9). These are already constrained below the resolution of the magnetic field coil. Measured variations in the $B_x^a$ field are 25 nagauss.

4.2.6  Orientation Dependent Zeroing Shifts

These zeroing routines have been developed previously, but we have suggestively displayed the largest higher order terms. It is clear from Eqs. 4.10 and 4.11 that an orientation dependence in the zeroing routines will appear with reorientation of CPT-II. Figure 4.12 shows a clear out-of-phase behavior for each of these fields. More noise persists on the $B_x^a$ value from a $B_z^2$ modulation (Eq. 4.11) whereas the $B_y^a$ value comes from a $B_z$ modulation (Eq. 4.10). The overall offset comes from contributions of beam misalignment, remnant magnetic fields, and lightshifts. Remnant magnetic fields are expected on the order of 20 µgauss and are difficult to evaluate without an

\[ ^{2} \text{This likely results from pumping along the x- and y-directions (Section 3.2).} \]
additional sensitive magnetometer. In practice, offsets larger than 100 \( \mu \)gauss imply that the lasers are poorly aligned.

In terms of the applied field,

\[
B^a_y = B_c \left( -L_x \frac{\gamma_x}{R_{tot}} - \frac{\gamma_x \Omega_x}{R_{tot} \gamma_n} \right) - B_{y \text{ offset}}
\]  

(4.12)

where the offset \( B_{y \text{ offset}} \) is explicitly considered. As predicted, the \( B^a_y \) field is more negative when the comagnetometer is in the east position. Similar behavior holds for \( B^a_x \),

\[
B^a_x = B_c \left( +L_y \frac{\gamma_y}{R_{tot}} + \frac{\gamma_y \Omega_y}{R_{tot} \gamma_n} \right) - B_{x \text{ offset}}
\]  

(4.13)

which agrees with Fig. 4.12 in predicting a more positive \( B^a_x \) field in the north position.

The orientation dependence of the zeroing routines can lead to a systematic bias in a reversal measurement if the fields are not re-zeroed in every position. In specifically considering the rotation terms with the product of estimated small offsets in the \( B_z \) value, this orientation dependance contributes a \( < 1 \) fT correlation with reversals.
along the E-W axis from an offset in the $B_y$ field. The effect along the N-S axis is further suppressed by an extra factor of $B_z \gamma_e/R_{tot}$ to $< 0.01$ fT. These effects are smaller than those required in this thesis and have been ignored in measurements.

The phase of the lock-in amplifier is arbitrary, but consistent throughout all of the measurements in this work. In the “Reverse” orientation, the slope of the $B_x$, $B_y$, and $B_z$ zeroing is positive. The $B_y$ and $B_z$ slopes become negative in the “Forward” orientation, but $B_x$ remains positive since it is determined by $B_x^2$. This implies that a positive voltage applied to the $B_x$ coil produces a field oriented the opposite direction from the probe beam propagation as shown in Fig. 3.9. This is in agreement with the shift in Fig. 4.12.

### 4.2.7 Zero $L_x$ and $\vec{s}_m$

The probe laser is nominally linearly polarized. A small circular polarization leads to a finite $L_x$. Likewise, this circular contribution will lead to a finite pumping rate. If the circular component to the probe polarization can be removed, then both of these effects have been reduced. The circular component of the probe beam can be adjusted using a stress plate to squeeze a glass microscope slide and introduce a birefringence to compensate for birefringence in the beam path. Coarse adjustment of the stress plate is controlled by screws on the mechanical mount with finer control provided by a voltage controlled piezo stack wedged between the glass and the mount.

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<thead>
<tr>
<th>Modulation Type</th>
<th>Zero $B_z$</th>
<th>Zero $B_y$</th>
<th>Zero $B_x$</th>
<th>Zero $L_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude ($\mu$gauss)</td>
<td>3.9</td>
<td>3.3</td>
<td>9.9</td>
<td>79</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Step Size ($\mu$gauss)</td>
<td>3.3</td>
<td>5.2</td>
<td>5.2</td>
<td>(0.05 V)</td>
</tr>
<tr>
<td>Lock-in $\tau$ (ms)</td>
<td>30</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.2: Typical zeroing parameters for CPT-II.
To evaluate the birefringence in the probe beam, the pump beam is blocked to depolarize the atoms. Depolarized atoms reorient with the local magnetic field and the pumping of the probe beam. Near zero field, a clear optical rotation response follows the pumping direction from the probe beam. This response can be observed by adjusting the $B^o_z$ field by -79 $\mu$gauss (Table 4.2) which is approximately the value of $B^e$. The sign reversal with pumping orientation provides clear signal for a feedback mechanism.

When the pump beam is blocked by the shutter, the $^3$He polarization starts to drift, and a quickly changing $B_z$ field dominates the signal response. Shuttering the beam for less than 0.5 s is sufficient to locate the zero crossing without domination by the drift. This procedure is too fast to be performed by hand without a significant loss of $^3$He polarization and must be automated for accurate adjustment of the stress plate.

This procedure does not truly zero the $L_x$ contribution during normal comagnetometer operation, but instead removes the average contribution over the entire probe beam. Any inhomogeneities in the pump and probe overlap will lead to an offset. An alternative is to use the $L_xL_z$ term in Eq. 3.43 to remove the contribution that participates in measurements, but this often presents difficult to interpret results. The method described here is sufficient for these experiments. Even if there remains a finite $L_x$ contribution, it is valuable to maintain $L_x$ at an unknown but stable offset.

4.2.8 Zero $L_y$ and $L_z$

The $L_y$ term provides a first order contribution to the signal; however, there is no source of light along the y-direction to produce a lightshift. One might think that a misalignment of the pump laser can project along the y-axis to introduce a non-negligible $L_y$. Such a shift was directly introduced in an attempt to develop a $L_z$ feedback (Section 4.4.2) by injecting a small fraction of the pump beam along the
y-axis. A scan of the pump wavelength across the resonance should produce a clear dispersive shape from the lightshift. This behavior is not observed and is more consistent with the pump and probe beams determining the axes of the comagnetometer. Therefore, sensitivity to $L_y$ is ignored.

The $L_z$ term is controlled by adjusting the pump beam wavelength. In the presence of a single optical transition in K, it would be simple to adjust the pump beam wavelength to the resonance of the D1 transition in K. The $^3$He buffer gas creates a poorly measured pressure shift, so several absorption profiles have been measured to determine the central wavelength of the D1 transition in the comagnetometer cell. Precise determination of the central wavelength of these profiles are limited by multiple systematic effects including interference fringes and spontaneous emission from the laser.

The lightshift from the nearby D2 transition shifts the zero lightshift point from the center of the D1 absorption line (Eq. 2.10). We predict a shift to a higher wavelength of +0.0044 nm from the D2 line and estimate the zero lightshift point to be at 770.084 nm as measured on our Burleigh WA1500 wavemeter. This wavelength is in good agreement with effects that depend on the value of $L_z$ (Section 4.4). The wavelength is continuously monitored on the Burleigh wavemeter through an optical fiber vacuum feedthrough and manually corrected for deviations larger than 2 pm.

### 4.2.9 Zero Pump-Probe Orthogonality

CPT-I requires an elaborate feedback system to maintain the pump and probe beam orthogonality over several hours (Eq. 3.17). The thermal isolation of the bell jar in CPT-II reduces such mechanical drifts. The zeroing procedure for the orthogonality described in Ref. [28] cannot be implemented using a single DFB diode laser; it

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3 An HP Agilent 86120B wavemeter gives a reading of 770.088 nm for the same light source, so it is clear that there is no absolute calibration.
does not provide enough pumping power to evaluate the contribution at high pump intensity under normal operating conditions to evaluate $\alpha$.

There are three degrees of freedom that contribute to “good” pump and probe alignment: the pump beam, the probe beam, and the $B^a_z$ field. Proper alignment is achieved when the pump and probe beams are orthogonal with the pump beam and $B^a_z$ field also parallel. This is a challenging parameter space to explore. We have found success in limiting the search space by placing a flat mirror on the quarter waveplate to retroreflect the pump beam. It is simple to evaluate the retroreflection on the back of the aperture (Fig. 3.11). If the pump beam is well centered on the window and the retroreflection centered on the aperture, its alignment is complete.

Further adjustments are made to the probe alignment. It is roughly centered on the two vacuum windows that are 30 cm apart. The vertical alignment is evaluated by blocking and unblocking the pump beam. Overlap in the pump and probe produces a clear optical rotation signal that can be reduced through adjustments to the vertical alignment of the probe beam. These can typically be reduced to below 0.5 mrad of optical rotation.

A final check is evaluated by raising the $B^a_z$ field after executing the $B_x$ zeroing routine. Fine adjustments to the pump beam alignment can reduce the change in optical rotation to $< 5$ mrad. If the pump beam is adjusted, adjustments to the probe beam vertical alignment are repeated. The entire alignment procedure is performed prior to pumping out the bell jar and left in place for the duration of the experiment. If there is a change in the mechanical equilibrium from evacuation of the bell jar, these changes are only small effects and are deemed acceptable. A scheme that automatically feeds back on the atom response would be preferable, however the procedure described here has led to consistent results and low drifts in the comagnetometer signal.
4.2.10 Zeroing Convergence

Each of the described zeroing procedures depends on the values of the fields already being small. Therefore, it is insufficient to consider each routine independently. Close to the compensation point, these procedures converge on typical values after several iterations. It is important to zero the $B_z$ field frequently as this is the most quickly drifting parameter due to its dependance on the $^3\text{He}$ polarization. We commonly observe coupling between the $B_y$ and $B_x$ routines due in part to the linear dependance of the $B_z$ field on $B_y$ and the quadratic dependance on $B_x$. Similarly, the $L_x$ and $B_y$ contributions are also coupled. Zeroing over several iterations cycles through these couplings and converges on stable values. Values that only require small adjustments from repeating the zeroing routines after a 20-30 minute interval are considered stable if the $B_z^a$ field has also tracked the $^3\text{He}$ polarization.

The $L_x$ zeroing routine causes a significant drop in the equilibrium $^3\text{He}$ polarization from briefly blocking the pump beam. This procedure requires additional repetitions of the $B_z$ zeroing to ensure that the compensation point has been properly readjusted before other routines are executed. Adjustments to the $L_x$ routine typically lead to small refinements to the $B_y^a$ field in the $B_y$ zeroing procedure. The particular schemes executed are discussed in Section 5.1.2.

4.3 Calibration

The gyroscope contribution provides a large systematic offset in the reversal correlated amplitude along the N-S direction. A discrepancy between the sensitivity of the comagnetometer and the calibration leads to noise in repeated measurements. For stable amplitude measurements ($< 1$ fT), the calibration technique should have a precision of at least $1/300$ and be accurate to within a few percent. New calibration
methods have been developed in this work to identify systematic effects and improve
calibration precision and accuracy. These techniques are discussed here.

4.3.1 Traditional Calibration: $B_yB_z$ Slope

The leading prefactor $P_z\gamma_e/R_{tot}$ in Eq. 3.43 determines the sensitivity of the comagnetometer to anomalous fields. From an experimental perspective, it is simple to define the calibration as the appropriate value by which to multiply the voltage to determine the response in fT,

$$\beta = \kappa S_L$$

where $\beta$ is the field to be measured. This includes amplification by both the lock-in amplifier and the photodiode amplifier which have remained fixed throughout the experiment.

In CPT-I, an anomalous field is difficult to apply directly. A calibration method based on the zeroing of the $B_z$ field has become the “traditional calibration” technique [28]. For $B_z \lesssim R_{tot}/\gamma_e$, the $B_yB_z$ dependance is well described by

$$S(B_yB_z) = \frac{P_z\gamma_e R_{tot}}{R_{tot}^2 + \gamma_e^2(B_z + L_z)^2} \left(\frac{B_yB_z}{B_c}\right)^2$$

where only the largest term in the denominator has been considered. This form presents a clear dispersion curve in $B_z$ and can be observed under a symmetric $B_y$ modulation (Fig. 4.13). The slope at the zero crossing can be used to determine $\kappa$ in terms of known parameters

$$\kappa = \left(\frac{B_c}{\Delta B_y} \frac{d}{dB_z}(\Delta S)\right)^{-1}$$

where $\Delta S$ is the change in signal from the modulation amplitude $\Delta B_y$. $\kappa$ can easily be extracted from the two point zeroing that occurs every $\sim 200$ s due to drifts in the
polarization. The step size in Table 4.2 determines the separation of the two point zeroing in $B_z$.

The “traditional calibration” has been validated in CPT-I against a known rotation. A piezo stack wedged between the optical table and an immobile block gently rocks the table at frequencies below 1 Hz. Monitoring the six degrees of freedom of the optical table on inductive non-contact position sensors and comparing the motion to the signal response gives agreement to within 10%.

### 4.3.2 $\Omega_y$ Rotations

The ability to measure the Earth’s rotation rate with a signal to noise of $10^3$ in CPT-II reveals a clear 20% discrepancy in a rotation calibration compared to the traditional method. The value of $\kappa$ is consistently smaller for the rotation measurement. The leading sensitivity to $\Omega_y$ is (Eq. 3.31)

$$
S(\Omega_y) = \frac{P_e \gamma_e \Omega_y}{R_{tot} \gamma_n} \left(1 - \frac{\gamma_n Q(P_e)}{\gamma_e}\right)
$$

(4.17)
where the second term gives a valuable 0.5% correction to the response. In Princeton, NJ, the predicted amplitude response for a reversal measurement is \( S(\Omega_{y}^{\oplus}) = 271.2 \) fT. There are a number of background effects (Section 4.1) and additional rotation terms (Section 4.4) that have already been accounted for.

Figure 4.14 determines a calibration of 142.0 pT/V which disagrees with the 188.0 pT/V found using the traditional method (Section 4.3.3). The phase of the response is within 3° of the expected N-S axis in the lab.

Rotations enter the Bloch equations in the same way as anomalous fields, so one would more likely trust the rotation method to calibrate sensitivity to an anomalous field. For the Lorentz violation experiment, this method of calibration is insufficient since a sidereal variation is searched for within the Earth’s rotation rate measurement. An independent method of calibration is required to separate these effects. In the following sections, we describe several measurements to investigate this discrepancy and develop an easily repeatable and robust calibration method.

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Footnote: 4 Here, we have made the reasonable assumption that \( Q(P^e) \approx 5 \).
4.3.3 $\Omega_x$ Sensitivity Through Zeroing

An orientation dependance in the zeroing of the $B_x$ and $B_y$ fields has already been discussed in Section 4.2.6. The rotation terms that cause these shifts can be used as a check in the calibration value. For example, an $\Omega_x$ term contributes to the zeroing of the $B_y$ field (Fig. 4.15). One would expect rotational dependencies to be small, but the $\gamma_e/R_{tot}$ prefactor enhances the effect to produce a shift amplitude of 0.420 $\mu$gauss.

Equation 4.12 predicts the amplitude of the expected shift in $\Delta B_y^a$. Earth’s rotation

$$\frac{\Omega_x}{\gamma_e} = -\Delta B_y^a \frac{R_{tot}}{\gamma_e B_c}$$

(4.18)

can be predicted in magnetic units with known values of $R_{tot}$ and $B_c$. In general, $R_{tot}$ is not directly measured and is instead inferred from the width in the $B_z$ resonance in Eq. 4.15 (Fig. 4.13). A least squares fit to Fig. 4.13 provides $R_{tot} = 362$ 1/s and $L_z=0.201$ $\mu$gauss. These values predict a Earth’s rotation rate to be 355 fT which is in poor agreement with the expected value near 270 fT.

The $R_{tot}$ value is in agreement with the traditional calibration technique. It is likely that a magnetic field gradient broadens the resonance in Fig. 4.13 to increase the measured $R_{tot}$ value (Section 4.3.6). All of the atoms experience the same rotation, so a rotation produces no gradient. If a gradient has broadened the resonance, the correct value of $R_{tot}$ from Eq. 4.18 should be 276 1/s.

Measurements of the $B_x$ and $B_y$ zeroing shifts have traditionally not used the reversal procedure and string analysis described in Section 4.1.2. For the measurement in Fig. 4.15, the $B_z$ and $B_y$ field are zeroed in each location as the the platform incrementally moves $7.5^\circ$ to the next position. These measurements are susceptible to linear drifts in the $B_z$ and $B_y$ fields that are likely from thermal drifts. It is unclear what causes the kink at 300$^\circ$, but reversals and the string analysis would produce a cleaner measurement.
Figure 4.15: $B_y^a$: Forward: $B_y^a$ for the $B_y$ zeroing as the heading of the comagnetometer is reoriented. The phase of the total response deviates by $5^\circ$ from the expected alignment. Note the sign reversal of the shift from Fig. 4.12 for flipping the spin orientation.

4.3.4 $\Omega_z$ Rotations

Under typical conditions, rotations about the z-axis are normally suppressed by $10^{-5}$ compared to rotations about the y-axis. The CPT-II implementation introduces the ability to apply a continuous $\sim 0.1$ Hz rotation about the z-axis, several orders of magnitude larger than Earth’s rotation rate. Because the comagnetometer is much more sensitive to rotations about the y-axis as opposed to the other directions, any applied rotation about the x- or z-axis should consider a projection along the y-axis to determine the net effect. Inspection of Eq. [3.33] reveals several terms that can be varied to determine the $\Omega_z$ dependence of the signal. Nominally, $L_x < 0.1$ $\mu$gauss and is difficult to control with the stress plate. The product $B_y\Omega_z$ provides a simple method of determining the coefficient of an $\Omega_z$ term by rotating the platform at a constant speed and varying the $B_y$ field.

Figure 4.16 displays the signal response under rotations of the apparatus for nine values of the of the $B_y$ field detuned from the nominal $B_y^a$ field determined in the south position. The apparatus rotates $1080^\circ$ rotation at $0.1915$ Hz determined by the
Figure 4.16: $B_y^a$: Forward. Response of comagnetometer to several $1080^\circ$ revolutions of the platform at 0.1915 Hz at different $B_y$ offsets. The net shift is modified by detuning the $B_y^a$ field. While rotating, there is a variation in the response periodic with a full rotation due to small variations in tilt.

programmed velocity profile of the servomotor\(^5\). There is a periodic signal associated with each rotation interval. Tilt sensors indicate a correlated response from wobble while the platform is in motion. The tilt has been optimized to produce small changes for positions $180^\circ$ apart while the experiment is at rest. A simple digital filter removes this variation before averaging the shifted signal. The shift is negative for positive detunings of the applied $B_y$ field and positive for negative detunings. The nominal zeroing value is repeated in the first, fifth, and ninth rotation to check against possible mechanical drifts from continuously rotating the platform.

The response in Fig. 4.16 can be repeated for several rotation frequencies\(^6\) (Fig. 4.17). The linear behavior in the shift with adjustments in $B_y$ is as predicted,

---

\(^5\)There is a smoothing function applied to the velocity profile, but this rate is accurate to 1%.

\(^6\)A 0.3 Hz rotation is within the capability of the motor, but moving the 1500 lb apparatus this fast is an impressive sight and is not commonly performed for fear of damaging the rotation mechanism.
Figure 4.17: $B_y$: Forward. Shift in comagnetometer signal under several rotation frequencies of the platform. A misalignment in the comagnetometer axes and the rotation axis mixes the sensitivity to $\Omega_y$ and $\Omega_z$. The change with $B_y$ detuning can distinguish the two effects.

as is the sign change in reversing the direction of $\Omega_z$. The decreasing shift for increasing $B_y$ detunings for positive rotations is also in agreement$^7$ with Eq. 3.33.

The slopes of the fitted lines can provide a sensitivity measurement (Table 4.3). In general, these values disagree with the traditional method by 30%, but are within a few percent of the $\Omega_y^\oplus$ measurement. These measurements indicate that rotation measurements are consistent but do not agree with an equivalence to magnetic fields.

It is interesting that the $\Omega_y^\oplus$ and $\Omega_z$ measurements differ by a few percent. There exist higher order $B_y\Omega_z$ terms (Eq. C.27), but the leading $-B_y\Omega_z^2/(\gamma_n B_c)^2$ term only contributes a 0.1% change to the measured slopes. It is more likely that there is additional flexing of of the breadboard while the experiment undergoes a uniform centripetal acceleration. Such an effect could explain to a linear shift in the measured slope as observed in Table 4.3.

This effect could be compensated for using the tilt sensors; however, the AOSI tilt sensors have a bandwidth of 1 Hz and the acceleration from a rotation pushes

$^7$Note that the $B_yB_z$ slope is negative in the “Forward” coil orientation (Fig. 4.13).
<table>
<thead>
<tr>
<th>Method</th>
<th>Sensitivity</th>
<th>Uncertainty</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_z^{(1)}$</td>
<td>-147.8 pT/V</td>
<td>± 0.6 pT/V</td>
<td>1.27</td>
</tr>
<tr>
<td>$\Omega_z^{(2)}$</td>
<td>-149.4 pT/V</td>
<td>± 0.6 pT/V</td>
<td>1.26</td>
</tr>
<tr>
<td>$\Omega_z^{(3)}$</td>
<td>-150.1 pT/V</td>
<td>± 0.8 pT/V</td>
<td>1.25</td>
</tr>
<tr>
<td>$\Omega_z^{(4)}$</td>
<td>-144.5 pT/V</td>
<td>± 1.2 pT/V</td>
<td>1.30</td>
</tr>
<tr>
<td>$B_y B_z$</td>
<td>-188.5 pT/V</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$\Omega_y^\infty$</td>
<td>-143.05 pT/V</td>
<td>± 0.26 pT/V</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 4.3: Comparison of sensitivity slopes for the $\Omega_z$ measurements in Fig. 4.17. The uncertainty represents the statistical uncertainty from the fit. In general, the rotation measurements disagree with the traditional method by 30%.

them into the nonlinear region of their calibration curve. We also expect that a large rotation can drive the comagnetometer away from the compensation point (Eq. 3.19), $\Omega_z/\gamma_n \approx 5$ nT which is a reasonable fraction of the compensation field of 244 nT in these measurements. The $B_z^a$ field remains at the same value determined by the $B_z$ zeroing while the apparatus is at rest for all measurements (Section 4.2). None of these effects are large enough to account for the discrepancy with the traditional calibration method.

**Horizontal Intercept**

The shift is not symmetric about the nominal $B_y^a$ zeroing value. The horizontal intercept in Fig. 4.17 can be used to estimate the alignment of the platform’s rotation axis with the axes of the comagnetometer. The comagnetometer is most sensitive to rotations about the y-axis, so we consider the case where the y-axis of the comagnetometer is tipped into the rotation axis of the platform by a small angle $\theta$,

$$S(\Omega) = \frac{P_e z \gamma_n}{R_{tot}} \left( - \frac{\Omega B_y}{\gamma_n B_c} \cos \theta + \frac{\Omega}{\gamma_n} \sin \theta + X \Omega \cos \theta \right)$$

(4.19)
where only the leading rotation terms are considered and $X$ contains the terms relating to $L_x$. Simplification of this expression for $\theta \ll 1$ yields

$$\Delta S = \frac{P^e_{\gamma e}}{R_{\text{tot}} \gamma_n} \left( - \frac{\Delta B_y}{B_c} + \theta + X' \right)$$

(4.20)

where we explicitly express the signal shifts $\Delta S$ and magnetic field detuning $\Delta B_y$ as displayed in Fig. 4.17. It is clear from this expression that both $\theta$ and $X'$ contribute to the horizontal intercept. Estimates of the probe lightshift give $L_x \approx 100 \text{ fT}$, so $X'$ is on the order of $10^{-7}$. The intercept at 11 $\mu$gauss implies $\theta \approx -0.26^\circ$. The probe beam has been aligned by centering the beam to within 1 mm on each of the vacuum tubes windows along the 30 cm optical path. The precision of the alignment is on the order of the measured value.

The measurements rely on the sensitivity to rotation to dominate over other offsets as indicated by the convergence of the slope at different rotation rates to a similar intercept. The rotation dependence of the zeroing (Section 4.3.3) sets the nominal zeroing value to within a 1 $\mu$gauss offset which can contribute to an estimate of $\theta$. Here, $B_y$ was zeroed in the south position, so the offset does not contribute. It would be possible to adjust the stress plate and look for a shift in the horizontal intercept to confirm the estimate of $L_x$. Products of rotations like $\Omega_x \Omega_z$ (Section C.1.5) may play a role in features in the signal during a continuous rotation, but these have been filtered along with the tilt effects.

### 4.3.5 Low Frequency $B_x$ Modulation

The steady state solution does not provide any more obvious options for developing a meaningful calibration technique. However, the low frequency oscillatory response of the comagnetometer reveals new insight (Eq. 2.81). A low frequency $B_x = B_0 \cos (\omega t)$
modulation provides the following response

$$S(B_0 \cos(\omega t) \hat{x}) = -\frac{P_z \gamma_e}{R_{tot}} \left( \frac{\omega B_0 \sin(\omega t)}{\gamma_n B_c} \right)$$

(4.21)

with the replacement $B_c \approx -B^n$ to be consistent with the experimentally known values. All of these values are readily available and can provide a simple determination of $P_z \gamma_e / R_{tot}$.

The low frequency response demonstrates clear linear suppression of the modulation as a function of frequency as long as $\omega \ll \omega_n$ (Fig. 4.18). The measured amplitudes are positive which is in agreement with the coil in the "Reverse" orientation, and the sign of the response is also in agreement with the orientation of the $B_x$ coil. This technique was developed late in the Lorentz violation search and detailed measurements of all of the calibration techniques under the same conditions were not available for direct comparison.

With $B_c = 179.9$ nT and $B_0 = 1.30$ μgauss, the measured sensitivity is 129 pT/V. At the same time, the traditional calibration provides a value of 155 pT/V. This
20% difference is similar to the discrepancy between values from the traditional calibration and rotations. This strongly suggests that a systematic effect influences the traditional calibration method.

An automated calibration method for using the a slow modulation of the $B_x$ modulation has been developed. A 0.645 $\mu$gauss cosine modulation over two full periods is applied to the $B_x$ coil to return a sine wave response. There is a jump in the signal from quickly shifting the $B_x$ coil to the start of the cosine waveform. The second period is recorded to reject any discontinuous features at the start of the waveform. The response is measured at 0.5 Hz then at 0.3 Hz. The slope between these two points and the known value of $B_c$ from the $B_z$ zeroing determines the sensitivity.

The drawback of the low frequency $B_x$ calibration is that it is inherently slow. The procedure typically takes 12 s to execute. In principle, a measurement at one frequency can determine the slope if the line is assumed to pass through the origin. The raw values from each measurement are saved individually along with the slope in case a comparison with a one point zeroing is of interest. It has been sufficient to use the two point zeroing.

The product of the low frequency $B_x$ calibration and the gyroscope response provides an effective measurement of Earth’s rotation of 277 fT. This is within 2% of the predicted value of 271 fT. The dominant systematic effect is likely in the measured value of $B_c$. This value is determined from the sum of the applied magnetic field to reach the compensation point. There is already an expected 2 nT remnant magnetic field inside the shields. It would be possible to confirm this experimentally by reversing the directions of the spins and measuring the compensation field in each orientation. This is a moderately time consuming process and was not checked experimentally over the course of measurements. Of the calibration methods described

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8 A sharper discontinuity is present from applying a sine waveform from the discontinues derivative.
in this section, the low frequency $B_x$ modulation is the most efficient and accurate method.

### 4.3.6 Traditional Calibration Revisited

The slope at $B_z = 0$ in Fig. 4.15 determines $\kappa$ and can be decreased by an inhomogeneity in the $B_z$ field. A $dB_z/dz$ gradient broadens this resonance and increases the measured value of $\kappa$. In the presence of a linear gradient, the $B_z$ zeroing will correctly identify the center of the gradient as the zero. The net effect can be simply represented as the sum of two resonances at different $B_z$ values spatially separated by the width of the probe beam,

$$
\frac{\partial S}{\partial B_y} = \frac{1}{2B_c} \left[ \frac{P_e^\gamma_c R_{tot}(B_z + \delta B_z)}{R_{tot}^2 + \gamma_e^2(B_z + \delta B_z + L_z)^2} + \frac{P_e^\gamma_c R_{tot}(B_z - \delta B_z)}{R_{tot}^2 + \gamma_e^2(B_z - \delta B_z + L_z)^2} \right]
$$

(4.22)

where $\delta B_z$ represents the difference in magnetic fields of the two resonances. The gradient

$$
\frac{dB_z}{dz} = \frac{2\delta B_z}{d}
$$

(4.23)

where $d$ is the width of a uniform probe beam. Figure 4.19 shows a simulation of the resonance broadening due to a 10 $\mu$gauss/cm gradient with $R_{tot} = 276$ 1/s. Equation 4.15 fits reasonably well to the broadened resonance with $R_{tot} = 307$ 1/s. It is entirely reasonable that this effect was not observed previously. The astute reader may have noticed that $R_{tot} \neq R_p + R_m + R_{en} + R_{sd}$ in Table 3.2 which is likely due to the broadening of the gradient.

The gradient not only influences the slope of the $B_z$ resonance, but it also affects the sensitivity of the comagnetometer. The product of the calibration and the gyroscope response clearly informs the effectiveness of the calibration techniques. By applying a known $dB_z/dz$ gradient and zeroing the $B_z$ field to remove any $B_z$ offset, the effective amplitude varies by 10% for a 5 $\mu$gauss/cm gradient using the traditional
Figure 4.19: Simulation of the $B_z$ resonance with and without a $dB_z/dz = 10 \text{ \mu gauss/cm}$ gradient. The net effect is to broaden the resonance and increase the value of $R_{tot}$ that normally would have been extracted from a fit to Eq. 4.15.

calibration while the low frequency $B_x$ calibration varies by less than 1% (Fig. 4.20). Applying the gradient causes the $^3$He polarization to drift, so the case with no applied gradient is repeated at the beginning and end of the measurement as a consistency check.

The offset in Fig. 4.20a without an applied gradient indicates that a magnetic field gradient already exists. The model in Section 3.2 assumes that the alkali-metal polarization is a constant, but the pump propagation model in Eq. 2.15 indicates otherwise. The model predicts that the polarization is roughly 50% at the center of the cell and has a polarization gradient of 0.33/cm. This predicts a magnetization gradient from the alkali of $21 \text{ \mu gauss/cm}$ which can account for the measurement of the gyroscope at 345 fT instead of 277 fT using the traditional calibration (Fig. 4.20).

Evaluating the slope of Eq. 4.22 to determine the effect of the gradient on the traditional calibration yields

$$\kappa \propto \frac{R_{tot}}{P_z \gamma e} \left(1 + \frac{\delta B_z^2 \gamma^2 e}{R_{tot}^2} \right)^{-1} \quad (4.24)$$
Figure 4.20: Calibrated gyroscope amplitude under an applied $dB_z/dz$ gradient. a) Traditional zeroing technique. b) Low frequency $B_x$ calibration. The traditional calibration is severely affected by the gradient, while the low frequency $B_x$ calibration is more robust.

with $L_z = 0$. As expected, the gradient will always broaden the resonance to increase the reported value of $\kappa$. This effect was not reported in CPT-I, likely because the alkali density was a factor of 4 lower at 160°C.

4.4 Gyroscope Phase

The probe beam background effects discussed in Section 4.1 can lead to an apparent shift in the phase of the comagnetometer gyroscope response of several degrees. Even with these effects reduced below a significant level, phase shifts in the gyroscope response persist from drifts in the vertical fields. These contributions lead to first order sensitivity to reversals along the E-W direction.
4.4.1 Phase Behavior

The leading rotational sensitivity is in the y-direction

\[
S(\Omega_y) = \frac{P_z \gamma_e \Omega_y}{R_{tot} \gamma_n} \left( 1 - \frac{\gamma_n Q}{\gamma_e} + \frac{\gamma_e^2}{R_{tot}^2} (B_z + L_z)^2 \right). \tag{4.25}
\]

Rotations about the z-direction are normally suppressed by $10^{-5}$, but there is a finite contribution to the rotations about the x-direction

\[
S(\Omega_x) = \frac{P_z \gamma_e \Omega_x}{R_{tot} D_S \gamma_n} \left( \frac{\gamma_e (B_z + L_z)}{R_{tot}} \right). \tag{4.26}
\]

where again one of the leading correction terms is

\[
\phi = \frac{\gamma_e}{R_{tot}} (B_z + L_z). \tag{4.27}
\]

Introduction of $\phi$ into the sensitivity to $\Omega_y$ and $\Omega_x$ terms provides

\[
S(\Omega_x) = \frac{P_z \gamma_e \Omega_x}{R_{tot} D_S \gamma_n} \phi \tag{4.28}
\]

\[
S(\Omega_y) = \frac{P_z \gamma_e \Omega_y}{R_{tot} \gamma_n} \left( 1 - \frac{\gamma_n Q}{\gamma_e} - \phi^2 \right) \tag{4.29}
\]

where this form is suggestive a phase shift with $\sin(\phi) \approx \phi$ and $\cos(\phi) \approx 1 - \phi^2/2$ for $\phi \ll 1$. It is only suggestive because Eq. 4.29 lacks the factor of 2 in the cosine dependence of a small angle and expansion of $D_S$ in Eq. 4.28 will give a $\phi^2$ term. Likewise, there is no correction to the $\Omega_x$ sensitivity from the $\gamma_n Q/\gamma_e$ term in Eq. 4.28 as there is in Eq. 4.29. This prediction holds for both the steady state solution (Section 3.2) and the toy model (Appendix C).

To demonstrate this behavior, several maps of the gyroscope response were acquired with the $B_z^a$ field detuned from the compensation point (Fig. 4.21). Each point represents 5 reversals of the apparatus. It is clear that a nonzero $B_z$ introduces a
Figure 4.21: $B_x^o$: Reverse. Measured phase shifts from detuning the $B_x^o$ field from the compensation point. Each point represents 5 reversals of the apparatus and the lines are a cosine fit with an arbitrary phase. This study was taken prior to the development of the low frequency $B_x$ calibration, so it suffices to forgo the calibration.

phase shift from sensitivity to Earth’s rotation along the x-direction of the comagnetometer. Prior to the detuning, the traditional calibration method varies less than 0.5% over the course of these measurements\footnote{The measured calibration is 247 pT/V from the $B_yB_z$ slope.} but does not determine the sensitivity\footnote{At the time of these measurements, only the traditional calibration was well-developed. Away from the compensation point, this method does not return an accurate calibration.} with $B_z \neq 0$. Equations 4.28 and 4.29 imply that only a phase shift should be observed to a large degree for $\gamma B_z/R_{tot} \ll 1$. Some of the applied $B_z$ values are not always small, so expansion of $D_S$ in Eq. 3.29 is not valid. In either case, a non-zero $B_z$ should decrease the contribution to $\Omega_y$.

The response can be quantified in terms of the amplitude and phase for several $B_z$ values. For negative detunings, the total measured amplitude increases (Fig. 4.22a). There is also an asymmetry in the total amplitude with the sign of the $B_z$ detuning. This effect is likely due to the real change in sensitivity from the $dB_z/dz$ gradient. The phase behaves with a linear change as expected with a slope of $1.4^\circ/\mu$gauss (Fig. 4.22b). Combined with the 200 nagauss fluctuations in finding the $B_z$ zero value
(Section 4.2.3), E-W reversals are expected to be limited to $\sim 1 \text{ fT}$. This is slightly smaller than the observed variation (Fig. 3.17).

Fluctuations and drifts in the lightshift from the pump laser contribute in an identical manner from changes in the intensity or wavelength of the laser. It is simplest to consider small changes in the pump wavelength to confirm the gyroscope response. Changes in intensity dramatically alter the pumping rate and several hours must pass for the $^3\text{He}$ to reach a new equilibrium polarization. The traditional calibration method can give a reasonable value of the sensitivity (Fig. 4.23a), but as with the $B_z$ detuning, the total amplitude behaves in some complicated way. As discussed in Section 4.3, the gradient causes the shift in amplitude from the expected 277 fT.

The phase response behaves as predicted (Fig. 4.23b). Assuming all other systematic effects have been properly accounted for, the crossing has a heading of 90° when $\lambda = 770.087 \text{ nm}$. This is close to the zero lightshift point predicted in Section 4.2.8. The slope provides a correction for pump wavelength drifts of $0.2^\circ/\text{pm}$ or equivalently $1.07 \text{ fT/pm}$. The pump laser has demonstrated stability < 0.1 pm over the short term, but can drift several pm/day. A lightshift feedback has been considered to stabilize these long term drifts (Section 4.4.2).
It is interesting that finite values of $B_z$ and $L_z$ serve to twist the sensitive direction of the comagnetometer into the x-direction. In CPT-II, the vertically oriented spins produce a signal when an anomalous field in the y-direction causes them to precess into the x-direction. For spins already projecting onto the x-axis, a change in the vertical fields causes the spins to precess in the horizontal plane to mix the x- and y-axes.

An accurate measured phase of the gyroscope response is an important feature of the comagnetometer for anomalous field searches. The gyroscope response determines the axis of sensitivity to all fields including the interesting anomalous fields $\beta_y^e$ and $\beta_y^n$. Agreement between the axes as defined by the light fields and the rotation of the Earth suggests that other fields are at most a small contribution to the total response. At the compensation point and with the pump beam at the zero lightshift point, the gyroscope crossing is measured at a heading of 88.5° in agreement with the absolute certainty in the orientation (Appendix B).
4.4.2 Lightshift Feedback

The sensitivity of E-W reversals to drifts in the pump wavelength of 1.07 fT/pm inspired several attempts in producing a robust $L_z$ feedback. Inevitably, these attempts failed due to correlations between the pumping rate and lightshift. Results from these attempts are briefly reviewed here.

**Attempt 1: Traditional**

A means of evaluating $L_z$ has been developed in Ref. [28] using Eq. 4.15. A change in the wavelength of the pump beam introduces an asymmetry in the $B_z$ resonance (Fig. 4.24a). Fitting these asymmetric dispersion curves to Eq. 4.15 provides a value of $L_z$ for each pump wavelength. These values determine a clean dispersion curve as a function of wavelength$^{[11]}$ (Fig. 4.24b). This curve predicts the zero lightshift point to be at $770.0905 \, \text{nm} \pm 2.5 \, \text{pm}$ which is close to the value predicted by other means (Section 4.2.8). This value is likely to be dominated by systematic effects from the $dB_z/dz$ gradient (Section 4.3.6). The full-width of the dispersion is $0.320 \, \text{pm}$ (Eq. 2.8) and larger than that expected for the pressure broadened linewidth of K in $^3\text{He}$. This is again consistent with the effect for a magnetic field gradient. The traditional method is not a good choice for a $L_z$ feedback since it lacks the desired 1 pm precision in determining the zero lightshift point and requires many time consuming $B_z$ resonance maps.

**Attempt 2: Injection of $L_z$ into $L_y$**

The comagnetometer has first order sensitivity to $L_y$ (Eq. 3.43). It may be possible to evaluate the $L_z$ lightshift by injecting a fraction of the pump beam along the comagnetometer y-axis. A shutter can block and unblock this lightshift beam to directly measure the contribution from this source. As the pump wavelength is adjusted, a

---

$^{[11]}$The errorbars here represent the statistical uncertainty only.
Figure 4.24: a) Asymmetry to the $B_z$ resonance from an $L_z$ contribution at various pump wavelengths. b) Map of the fitted $L_z$ values from asymmetry in the $B_z$ resonance. This method identifies $L_z = 0$ at $\lambda = 770.0905$ nm $\pm 2.5$ pm where the uncertainty accounts for the statistical variability only.

Figure 4.25: a) A third circularly polarized laser near 770.1 nm provides dispersive-like lightshift behavior when injected along the y-direction. This curve does not fit well to a dispersion curve (red), likely due to unaccounted shifts from the associated pumping rate. b) The zero crossing of the lightshift beam (a) has a different value depending on the wavelength of the pump beam.

A clear dispersive profile is expected. The additional pumping along the y-axis introduces a shift in the $B_y$ zeroing of $\sim 20$ $\mu$gauss (Eq. 3.21) and can be compensated for with the appropriate adjustment to the $B_y$ field. These measurements have been performed and were characterized by small signals and inconsistent results.
A change in wavelength of the pump beam changes both $L_z$ and $L_y$ simultaneously as well as the pumping rate from both beams. These changes are directly coupled and cannot be separated. Likewise, as a check, a change in the handedness of circular polarization simultaneously reverses both the lightshift and the pumping rate effects. An AOM was introduced in an attempt to break any coherence between the pump and lightshift beams but provided little change. An independent circularly polarized laser was later introduced as the lightshift beam and produced dispersive-like behavior as a function its wavelength (Fig. 4.25a). This might at least provide an indication of the zero lightshift point. A dispersion curve does not fit well indicating that the behavior is more complicated than predicted. Also, the zero crossing shifts with a change in pump wavelength (Fig. 4.25b). This scheme has been abandoned as a viable $L_z$ feedback.

**Attempt 3: Large $L_x$ from a Far Off Resonant Laser**

The $L_xL_z$ term in Eq. 3.43 provides an opportunity to determine $L_z$ if $L_x$ can be varied. The linearly polarized probe beam already produces a small $L_x$. Adjustments to the stress plate can increase the $L_x$ contribution to the signal but also introduce an additional background optical rotation signal. Wavelength adjustments of the probe beam again lead to correlated pumping rate changes that enter to leading order in the x-direction (Eq. 3.16).

Far from resonance, $R_p \sim 1/(\nu - \nu_0)^2$ but $|\vec{L}| \sim 1/(\nu - \nu_0)$. An available 200 mW at diode at 808 nm would introduce a lightshift of 1.7 $\mu$gauss with a pumping rate of only 4 1/s. To combine the two beams, a FF801-Di01 dichroic beamsplitter from Semrock replaces the mirror before probe light enters the cell (Fig. 3.11) transmitting 808 nm light but reflecting the 770 nm light from the probe. A shutter once again controls the 808 nm lightshift beam to compare correlated changes in the signal with
the introduction of the lightshift beam. An FF01-769/41 filter at normal incidence prior to the photodiode prevents detection of the lightshift beam on the photodiode.

The lightshift beam along the x-direction produces shifts in both the $B_x$ (Eq. 3.24) and $B_y$ (Eq. 3.21) zeroing. These are typically compensated with adjustments to $B_x^a$ and $B_y^a$ on the order of 25 $\mu$gauss which can be dramatically changed by adjusting the alignment of the lightshift beam. Repeated measurements of the zero lightshift point have a scatter of 5 pm. Adjustments to the alignment of the lightshift beam likewise shift the value by up to 10 pm. This scheme has been abandoned as a viable $L_z$ feedback due to large scatter and the sensitivity to the lightshift beam alignment.

Reference Cell

Ultimately, the ideas presented here have failed to produce a feedback mechanism capable of stabilizing the pump laser to 1 pm. There has been discussion of including an additional vapor cell on the platform. This would indeed provide a stable reference with which to lock to the center of the absorption peak or the zero lightshift point given any number of standard techniques. The additional hardware of heating a second cell adds complexity to the apparatus that was not attempted in this work. It has been sufficient to continuously monitor the pump wavelength using a Burleigh WA-1500 wavemeter on board the platform.

4.5 AC Response

Traditionally, the steady state response is the main focus of the comagnetometer signal [28, 43]. In developing the low frequency $B_x$ calibration, a series of measurements of the ac response of the comagnetometer have been performed. Some interesting observations from these investigations are summarized here.
4.5.1 Low Frequency Response

The low frequency response is evaluated in Section 2.5.2 and used in developing an accurate calibration (Section 4.3.5). The low frequency $B_x$ modulation produces both an in-phase and out-of-phase component not listed in Eq. 2.81. Expanding Eq. 2.78 to the next highest order in $\omega$ produces

$$P_e(t) \approx \frac{P_{e\gamma_e}}{R_{tot}} \left( \frac{\omega B_x \sin(\omega t)}{\gamma_n B^n} - \frac{(\gamma_e B_e/Q + \gamma_n B^n)\omega B_x \cos \omega t}{(\gamma_n B^n)^2 R_{tot}/Q} + O(\omega^3) \right)$$

(4.30)

correctly predicting the sign of both phase responses and the scaling with $\omega$ (Fig. 4.26). However, the in-phase response fails to continue to follow the polynomial behavior in $\omega$ at the lowest frequencies.

This behavior also persists for a low frequency $B_y$ modulation. Further expansion of Eq. 2.78 for a $B_y$ modulation provides

$$P_e(t) \approx -\frac{P_{e\gamma_e}}{R_{tot}} \left( \frac{\omega^2 B_y \cos(\omega t)}{(\gamma_n B^n)^2} + \frac{(2\gamma_e B_e/Q + \gamma_n B^n)\omega^3 B_y \sin(\omega t)}{(\gamma_n B^n)^3 R_{tot}/Q} + O(\omega^4) \right)$$

(4.31)

also correctly predicting the sign and frequency dependance of the phases (Fig. 4.26). This analysis ignores the effects of $R_n$ which can become more significant at the lowest frequencies. Keeping this contribution,

$$P_{\perp}(t) = -\frac{1}{2} \left[ \frac{(B_x + iB_y)P_{e\gamma_e}(R_n + i\omega)/Q + R_{tot}(\gamma_n \lambda M^n P_{n_z} + \omega - iR_n^{n})/Q + \omega(R_{tot} + i\gamma_n \lambda M^n P_{n_z} + i\omega)}{(B_x + iB_y)P_{e\gamma_e}(R_n - i\omega)/Q e^{-i\omega}} + \frac{(B_x + iB_y)P_{e\gamma_e}(R_n - i\omega)/Q e^{-i\omega}}{(B_x + iB_y)P_{e\gamma_e}(R_n + i\omega)/Q + R_{tot}(\gamma_n \lambda M^n P_{n_z} - \omega - iR_n^{n})/Q - \omega(R_{tot} + i\gamma_n \lambda M^n P_{n_z} - i\omega)} \right]$$

(4.32)
Figure 4.26: Reverse: Low frequency response of the comagnetometer to modulations of the transverse magnetic field ($B_x = 1.3 \mu$gauss, $B_y = 3.9 \mu$gauss). Deviations from the simple polynomial behavior predicted by Eq. 4.30 and 4.31 are likely due to the $dB_z/dz$ gradient. A numerical simulation with a 20 $\mu$gauss/cm gradient in the $B_z$ field correctly predicts the behavior. The low frequency $B_x$ calibration is used to determine the overall scale of the model.

which reduces to Eq. 2.78 in the limit that $R_{tot}^n \to 0$. Expanding this expression to low orders of $\omega$ produces a frequency independent amplitude,

$$P_\perp(t) = -\frac{(B_x + iB_y)P_{z,e}\gamma_e R_{tot}^n \cos(\omega t)}{-iR_{tot}R_{tot}^n + \gamma_e \lambda M_e P_{z,e} R_{tot}^n + \gamma_n \lambda M_n P_n} + \mathcal{O}(\omega)$$

(4.33)

which contributes only an in-phase contribution to the $B_x$ modulation and an out-of-phase contribution to the $B_y$ modulation. A numerical simulation indicates that an extremely large $R_{tot}^n \sim 4$/hour would be necessary to account for the behavior in Fig. 4.26.
Section 4.3 indicates the importance of considering a magnetic field gradient. Introduction of a 20 μgauss/cm gradient by the method in Section 4.3.6 produces the curves in Fig. 4.26. Like the introduction of $R_{\text{tot}}^n$, the $dB_z/dz$ gradient introduces a frequency independent amplitude\(^{12}\). The simulation incorporates $B^n = 170$ nT and $B^e = 80$ μgauss which are close to the experimentally measured values. The slope of the out-of-phase $B_x$ modulation determines the calibration for these measurements. All of the behavior matches well except for the lowest frequencies of the out-of-phase $B_y$ response. These effects are suppressed by $\omega^3$ and it is suspected that the signal to noise in this region is poor. The additional suppression of the $B_y$ modulation forced the applied field to be increased by a factor of 3 to achieve a reasonable signal to noise.

Previously, the effects of a magnetic field gradient from the nonuniform K polarization has been considered in terms of contributing noise \(^{28}\). In this work, the effects of the gradient have been measured in two ways, through the low frequency response and the traditional calibration technique. Ultimately, the model is in agreement with the experiment and demonstrates that the low frequency calibration method is insensitive to the magnetic field gradient (Fig. 4.20b) as a feature of the K-³He comagnetometer.

### 4.5.2 AC Heater Coupling

In investigating the long term behavior of the comagnetometer, a correlation between the ac heater amplitude and the signal was discovered. An analog feedback loop adjusts the heater amplitude to maintain a consistent temperature of the oven. Fluctuations in either the heater amplitude or its coupling strength to the signal cause systematic drifts. Quickly turning on and off the ac heater generates a dramatic dc shift in the signal.

\(^{12}\)Simplification is complicated and not illuminating, so this result is not reproduced here.
This coupling is not through electronics or a ground loop. Blocking the pump beam depolarizes the alkali atoms and no change in the signal is observed upon turning on and off the heater; the atoms experience a field. The heater contains resistive wire in a twisted pair to reduce the dipole moment of the wire. Likewise, there is a blocking capacitor to remove any dc component to the ac signal. The comagnetometer responds to a high frequency magnetic field as $1/f$ (Eq. 2.63). It is interesting to observe that the coupling to the heater signal scales as $1/f^3$ (Fig. 4.27). The comagnetometer calibration at the time of this measurement was 400-500 pT/V, so these shifts create an effective magnetic field of 20 pT. The coupling also scales with the power to the oven and is slightly smaller for a triangle wave than a sine wave which as unexpected.

If the spins precess quickly in an ac field, the time average of the length of the spin projection vector decreases. Such a decrease would result in an offset in the zeroing of magnetic fields (Section 1.2) that depends on the heater frequency. A comparison of the heater at 49 kHz and 173 kHz does not provide a consistently observable shift in the values found by the zeroing routines for the $B_x$, $B_y$, and $B_z$ fields. This confirms that there is no significant dc magnetic field introduced by the heater to shorten the
Table 4.4: Measured 50 kHz magnetic field pickup at the location of the cell from the six heater elements on the oven. The stem is the smallest because it is farthest from the cell. It is interesting that the effects of the assembled oven reduce the coupling an order of magnitude.

effective spin vectors. Such an effect would scale as $1/f^2$ and is inconsistent with Fig. 4.27.

Despite the twisted wire pair, it is interesting to measure the magnetic field pickup on a small coil for each of the six heating panels. Shielding the return wire on the pickup loop is important to reduce capacitive pickup. At 50 kHz, a response of $\sim 1 \text{nT/mA}$ for each panel and 70 pT/mA at the center of the assembled oven (Table 4.4). The stem heater produces the smallest effect because it is farthest from the position of the cell. The effect at the center of the assembled oven is reduced by more than an order of magnitude likely due to destructive interference from contributions of all panels.

The effects of heater variations are reduced by operating the heater at the limits of the amplifier bandwidth near 170 kHz to take advantage of the $1/f^3$ behavior of the coupling. Commercial high frequency amplifiers are bulky and do not fit conveniently on the rotating platform. A home-built audio amplifier supplies 1 A of current at 170 kHz. The amplifier has been operated up to 300 kHz, but typical heater loads of 200 $\Omega$ decrease its bandwidth. Operating at these higher frequencies reduces the
offset to 500 fT. During normal operation, the power to the oven varies 0.5-1% over 30 minutes and only contributes to long term drifts in the comagnetometer signal.

4.5.3 Sensitivity Optimization

Previously, a sensitivity optimization scheme has been developed to improve the pumping of the alkali polarization which we refer to colloquially as “Routine 13”. At 50% polarization, the value $P_{z/\gamma_e}/R_{\text{tot}}$ reaches a maximum. A high frequency modulation of the $B_y$ field at 110 Hz determines the prefactor in Eq. 2.78 when the alkali atoms experience a low magnetic field. Adjustments to the alignment of the pump beam as well as the wavelength provide an extremum in the response that corresponds to the optimum polarization.

In the work on CPT-II, it has been noted that the extremum will often correspond to a noticeably skewed alignment and low $^3$He polarization from poor spin-exchange optical pumping efficiency. It is not clear why this is the case, but it is suspected that the polarization gradient plays a significant role in this procedure. In this work, it has been preferable to use the procedure described in Section 4.2.9 to align the pump beam. Once aligned, adjustments to the intensity can optimize the sensitivity by comparing the magnitude of the gyroscope response at various pump intensities.

4.6 Symptomatic/Enhanced $^3$He Relaxation

Comagnetometer operation benefits from an increased compensation frequency for faster dynamics. A larger $B_c$ also suppresses second order terms in (Eq. 3.43). Each of these parameters are proportional to the $^3$He polarization. The polarization is not limited by pump beam intensity, but rather by symptomatic relaxation effects of $^3$He. Practical operation of the comagnetometer requires a compensation field of several hundred nT to provide a compensation frequency of several Hz.
4.6.1 Self-Relaxation

The assumption that the magnetization of K and \(^3\)He is uniform within the comagnetometer cell correctly captures most of the dynamics of the spins. However, the asymmetry of the cell leads to a significant gradient in the dipolar field from \(M^n\). Strong self-relaxation of \(^3\)He along its own gradient is observed (Eq.2.24). This is the dominant \(^3\)He relaxation mechanism in the comagnetometer and limits the equilibrium \(^3\)He polarization.

The largest contribution to self-relaxation is the asymmetry of the cell at the stem where it has been pulled-off the vacuum manifold. To mitigate this effect, we prepare the cell such that a blob of K metal plugs the neck of the cell. This is often achieved by heating the cell with a heat gun to just above the melting point of K at 63\(^{\circ}\)C. Surface tension holds most of the blob together. Gentle shaking and reorientation of the cell while hot can maneuver the blob towards the stem. The blob typically catches near the neck of the stem. After some practice, a sharp flick of one’s wrist can snap the blob into position. As the cell cools, different \(^3\)He pressures push bubbles through the blob until it hardens or surface tension holds it in place.

Typically, we evaluate the blob placement before reinstallation of the cell in the shields. We place the cell in a large convection oven and uniformly heat the cell until the K melts. If the blob hardens before the density of gas has equalized on either side of the blob, we observe a dramatic drop of the blob down length of the stem. This requires repositioning of the blob once again. If the cell passes this test, we mount it inside the magnetic shields. Historically, the blob does not always maintain its position due to the nonuniform distribution of heat around the oven. To avoid frequent removals of the cell, CPT-II incorporates an adjustable stem heater to redistribute heater power to the small volume behind the blob to pushing it back to the neck of the cell.
Optical access near the stem is restricted when the shields and inner vacuum space are closed, so we evaluate the final placement of the blob by measuring $^3$He relaxation in the absence of pumping (Section 2.2.2). Figure 4.28 shows a measurement of the decrease $B_z$ field to find the compensation point as the $^3$He relaxes in a fixed field of 2.8 mgauss. The solid curve represents a fit to a simple exponential decay. It is clear that the decay decreases faster than the exponential as the polarization decreases. A fit to a simple heuristic model

$$P(t) = P_0 e^{-at - bt^2} \quad (4.34)$$

more accurately describes the relaxation where $P_0$ is the initial polarization and the $a$ and $b$ coefficients contribute to the linear and nonlinear decay respectively. Successive increases in power applied to the stem show diminishing values of $b$ and lengthening of $T_1$. Once an equilibrium compensation field of 200-300 nT has been achieved, we find this sufficient for practical comagnetometer operation. Occasional adjustments of the stem heater can maintain the polarization for several weeks. After several
weeks of operation, the position of the blob shifts due to diffusion of $^3$He through the liquid metal. One could wonder if the gradient in $M^e$ significantly contributes, but in this case, placement of the blob would make little difference. It is clear that the $^3$He polarization is limited by the gradient in its own dipolar field.

### 4.6.2 Magnetized Comagnetometer Cell

After one particular re-installation of the comagnetometer vapor cell, it became difficult to locate the compensation point and virtually impossible to stabilize the polarization once found. Typical magnetizations of several mgauss were only achieved at relatively high magnetic fields ($B^o_a = 10$ mgauss). Slowly lowering the $B^o_a$ field to find the compensation point as described in Section 4.2.2 caused the polarization to quickly drop. Following the compensation point as it plummeted only served to increase the relaxation effect. A new or severely increased field dependant polarization relaxation mechanism had been introduced. A measurement of the $^3$He relaxation provided a clean exponential decay demonstrating that self-relaxation (Section 4.6.1) was not responsible. Our usual methods of measuring the compensation point failed here, but a simple NMR tipping pulse and measurement of the initial free induction decay (FID) amplitude provided a signal proportional to the $^3$He polarization.

While placement of the K blob was not the issue, a foreign object within the shields can produce a fixed magnetic field gradient. Relaxation along gradients is suppressed by $(B^o_a)^2$ (Eq. 2.24) and could explain the sudden increase in relaxation upon nearing the compensation point. An attempt to compensate for possible gradients by engaging combinations of gradient coils failed. These fields work well for suppressing uniform gradients but would could not possibly account gradients from a localized object. Rather than open the shields, systematic measurements of the $T_1$ of the cell provided a surprising result: $T_1 \propto B^o_a$ rather than the expected $T_1 \propto (B^o_a)^2$ (Fig. 4.29).
This behavior has been previously measured in vapor cells exposed to large magnetic fields ($\sim 10$ kgauss). Figure 4.30a (reproduced from Ref. [84]) shows the measured $T_1^{-1}$ in the low-field limit for two vapor cells, 14A and 20B, in which 20B has been exposed to a large field while 14A has not$^{13}$. The behavior of $T_1^{-1}$ of the comagnetometer cell (Fig. 4.30a) is strikingly similar despite the orders of magnitude difference in applied fields. The large increase in relaxation at low magnetic fields is a signature of this effect. Note also the asymmetry in relaxation rates on either side of applying zero field. Systematic studies the comagnetometer relaxation were not undertaken as we are more motivated by tests of fundamental symmetries and this is mostly a technical issue for restoring comagnetometer operation. The effect appears to be caused by $^3$He relaxing at the walls of the cell after the walls have become magnetized in a large magnetic field. The authors suspect that ferromagnetic impurities on the surface of the glass become magnetized at high field. The local magnetic field gradient near these sites accelerate $^3$He relaxation. These effects are referred to as “$T_1$ hysteresis”, a change in $T_1$ based on the history of magnetic fields applied to the cell.

$^{13}$The group at the University of Utah prefers to plot $T_1^{-1}$. 

Figure 4.29: Measurements of $T_1$ at various fields. Relaxation along a fixed gradient would give $T_1 \propto (B_z^2)^2$ whereas the measured response is linear.
Fortunately, magnetizing a comagnetometer cell does not render it useless. $T_1$ hysteresis effects can be reversed by degaussing the cell \[85\]. Here, the cell is placed between two poles of an electromagnet and rotated perpendicular to the field using a dc motor as the field magnitude slowly decreases. This device is colloquially referred to as the Electrified Cell Spinner (ECS) and is useful for systematic hysteresis studies. Rotation of the cell in the decreasing field randomizes the ferromagnetic domains within a site to reduce the net magnetization. We were able to approximate operation of this device by placing the cell on a G10 rod within an 1.1 T electromagnet. The cell was slowly spun by hand while a colleague ramped down the magnet over 60 s. It seems important to continue to rotate the cell until it is fully removed from the magnet in case a remnant magnetization exists even at zero current. Re-installation of the cell after degaussing restored the cell to normal operation.

We did not intentionally magnetize our cell and how it occurred is still an open question. We often carry the cell to a neighboring room to reposition the K blob. It is possible that the cell was placed close to a set of magnetic coils providing a field of at most $\sim 10$ gauss while in transit. Considering that most cells do not suffer from magnetization effects until exposed to 10 kgauss, this is a surprising occurrence. It
has been shown that even exposure to low fields (< 50 gauss) will cause the onset of $T_1$ hysterisis [85]. A factor of two change in $T_1$ in a cell exposed to 30 gauss field has been observed [84]. Also, work at the University of Utah reports that these effects are largest for Pyrex cells. Our comagnetometer is constructed from higher density aluminosilicate glass to prevent diffusion of $^3$He through the glass. Nevertheless, it is clear that it is possible to magnetize the comagnetometer cell and care should be taken to prevent these events.
Chapter 5

A Test of Lorentz and \textit{CPT} Violation

We performed a search for a Lorentz- and \textit{CPT-}violating field coupling to the neutron spin using the CPT-II apparatus described in Chapter 3. The reversal technique described in Section 4.1 allows for removal of the long term drift in the signal response. The careful understanding of the comagnetometer behavior in Chapter 4 provides a clear understanding of the signal response. We search for sidereal oscillations in the signal to constrain an anomalous spin coupling of an extra-solar origin and improve the existing limit on the neutron spin by a factor of 30. These results have been published in Ref. [30] and represent the highest energy resolution of any spin anisotropy experiment. The following chapter provides additional details not included in the publication.

5.1 Lorentz Violation Search

The Lorentz violation search includes data for 143 days from July 2009 to April 2010 (Fig. 5.1). Collection is separated into two distinct parts represented by the summer and winter. The long time span provides an important separation between sidereal
Figure 5.1: Complete nine month collection interval for the Lorentz and \(CPT\) violation search. The large gap in the middle provided an opportunity for continued comagnetometer behavior studies and improvements to the apparatus. The two large collection intervals occur during the summer and the winter. The sign of the calibration maintains a consistent calibrated response despite a reversal of the spins.

and diurnal signals with the Earth on the opposite side of the sun while a point on the surface faces the same star. Both the N-S and E-W signals are collected as they are susceptible to different systematic effects (Section 3.4.3). Also, the direction of the \(B_z\) field has been reversed as a further check of systematic effects. Early measurements suffered from repeated interruptions evidenced by the shifts in the E-W alignment. These issues were solved during the Fall of 2009 with several upgrades to the long term operation of the apparatus. The winter data demonstrates noticeable improvement in the robustness of measurements achieving uninterrupted operation over several weeks.

5.1.1 Measurement Protocol

Long term measurements for the Lorentz violation search proceed as follows: The apparatus typically starts in the south position with the magnetic fields and lightshifts
well-zeroed. Data acquisition begins, and a trigger signal is sent to the motor via the Labview Datasocket server. The motor waits 7-10 s and reverses the experiment. An odd number of reversals are performed every 22 s. An extra rotation of 90° occurs after the same delay. The signal at rest is averaged and the reversal correlated amplitude determined by the string analysis.

Data acquisition pauses at the end of fixed number of intervals for the $B_z$ zeroing routine. This defines one record of data. The next reversal sequence repeats starting in the west position. This pattern is repeated such that every other record makes measurements along the E-W or N-S axis. Also, each time a different axis is measured, the apparatus starts in the opposite position compared to the previous record along that axis. This serves to average any effect related to the starting location.

Reversal measurements are separated into several large files ranging from 1.5-10 days in length. They are labeled by the starting sidereal day and span sd 3492.86 to sd 3756.33 (Table D.3). Short duration files typically indicate that an error interrupted operation. Longer files are typically stopped when convenient to investigate the most recent results of the experiment. A search for a sidereal variation relies on several days of uninterrupted measurements. Over intervals less than one day, linear drifts in the data fit well to a sine wave clearly distinguishable from a true sidereal variation in longer measurements. Longer measurements beyond a week in length begin to separate sidereal and diurnal effects.

Both computers save all time values in sidereal days from January 1, 2000, in GMST which is convenient for later analysis of the sidereal signal. The calculation from the local computer time is described in Appendix A. The two computers are independently synchronized with a navy time server using freely available Dimension 4 software to query the server every 3 s and synchronize the computer clocks. Typical corrections are on the order of 10 ms.
At the beginning of each record, the local sidereal time is determined. The times during the record follow the hardware clock on the data acquisition card and are calculated from the sample rate typically at 200 Hz. In parallel, the computer controlling the motor saves the sidereal time at each instance that the motor begins to rotate the platform. A comparison of times recorded for a simultaneous event shows that the computers are well synchronized within 20 ms. This is sufficient to reliably coordinate selection of 3-7 s samples of data separated by 22 s.

The computer onboard the rotating platform saves several classes of data referred to as “fast”, “medium”, “relaxed”, and “slow”. These designations classify signals into different types for ease in processing. The comagnetometer signal and other high bandwidth parameters such as the tilt sensors and position detectors are sampled at 200 Hz and averaged to lower sample rates. The signal follows the sample rate in Table D.2 while the other parameters are saved to the medium rate at 2.5-10 Hz. Various temperature readings have long time constants and are saved at 1 Hz in the relaxed data. The slow data acquisition handles coordination of slow bandwidth measurements, namely the encoder and the wavemeter. These communicate through USB devices and cannot be easily synchronized with the other data at the same high bandwidth. These values are sampled every 3 s as determined by a software timing rather than the hardware timing of the rest of the analog channels. Each of these values are saved as single precision binary numbers except the sidereal time which is saved as a double precision binary number. This sampling and averaging scheme maintains manageable file sizes without significant loss in compression.

5.1.2 Zeroing Schedule

Frequent execution of the zeroing routines maintains sensitivity of the comagnetometer to the anomalous spin-dependent fields and limits the effects of higher order terms on the signal. The $B_z^a$ field must be adjusted between 50-200 ngauss every 200-300 s
due to drifts in the $^3$He polarization. This timescale limits the length of consecutive reversal measurements. Other fields only demonstrate significant drifts over the timescale of hours. These timescales divide zeroing into the minor and major zeroing routines. The minor zeroing routines occur before each record along the N-S or E-W axis to maintain a small $B_z$ field and evaluate the sensitivity of the comagnetometer. The major zeroing routines occur every 7 hours$^1$ and execute a sequence of all the zeroing routines (Section 4.2).

The particular zeroing sequences executed in the experiment evolved over the course of measurements. In the summer, the following sequences were executed

Minor Zeroing : $B_z$

Major Zeroing : $B_z B_y B_z L_x B_z B_z B_y B_z B_x B_z$.

This sequence constrained drifts in the $B_z$ field to 50 ngauss between minor zeroing events, though it was typical to observe a 500 ngauss/hour decrease in $B_z^a$ field corresponding to loss of $^3$He polarization (Fig. 5.2). Closing the shutter for a few seconds for the $L_x$ zeroing drops $B_z^a$ by 8 $\mu$gauss from loss of spin-exchange optical pumping. Repeated zeroing of the $B_z$ field compensates for the loss and tracks the return to equilibrium. A polarization feedback routine similar to that described in Ref. [28] was not implemented due to the steady decline in overall polarization. The set point would likely need to be adjusted on daily intervals and could lead to daily correlations.

A similar procedure continued into the winter data collection. At this point, optical rotation background measurements of the probe beam were implemented (Section 4.1.7). Periodically increasing the $B_z$ field increases the $^3$He equilibrium magnetization by 25 $\mu$gauss by suppressing $^3$He relaxation along magnetic field gradients for

$^1$This value is chosen such that it is not an integer multiple of a diurnal or sidereal day to avoid a correlation at these frequencies.
Figure 5.2: Summer: Adjustments to the $B_z^a$ field using the $B_z$ zeroing to track the $^3$He magnetization. The regular drops in magnetization occur from closing the shutter during the $L_x$ zeroing.

a significant fraction of the record (Eq. 2.24). This leads to additional drifts in the $B_z$ field, so the $B_z$ zeroing executes twice in each minor zeroing routine to compensate.

In file 3692.34, a procedure to maintain the $^3$He polarization during the zeroing was added. A routine to maintain a high $B_z$ field ($HB_z$) raises the $B_z^a$ field by the same 0.65 $\mu$gauss as the background measurements for 8 s. An additional routine for a longer high $B_z$ field ($HB_z L$) raises the $B_z^a$ field for 60 s to compensate for the polarization loss from the $L_x$ zeroing. The zeroing sequence becomes

Minor Zeroing : $B_z$ $HB_z$ $B_z$ $HB_z$ $B_z$ $HB_z$ $B_z$

Major Zeroing : $B_z$ $HB_z$ $B_z$ $HB_z$ $B_z$ $B_y$ $HB_z$ $L_z$ $HB_z L$ $B_z$ $HB_z$ $B_y$ $B_z$ $HB_z$ $B_x$ $B_z$.

In file 3734.60, the low frequency $B_x$ calibration was introduced to improve upon the traditional method in the $B_z$ zeroing routine (Section 4.3.5). At this time, the range on the stress plate piezo was insufficient to fully compensate for the birefringence of the probe beam. Rather than open the bell jar to adjust the mechanical screws
for increased range and delay the experiment, collection continued with the following refinements to the routines

Minor Zeroing: \( B_z \) \( HB_z \) \( B_z \) \( HB_z \) \( Cal \) \( B_z \)

Major Zeroing: \( B_z \) \( HB_z \) \( B_z \) \( HB_z \) \( B_z \) \( B_y \) \( HB_z \) \( B_x \) \( B_z \) \( HB_z \) \( B_z \)

where the \( Cal \) method performs the low frequency \( B_x \) calibration and the \( L_x \) routine has been removed. This provided reasonable performance for the remainder of the Lorentz violation search.

The major zeroing routines execute on a timer every 7 hours along with the minor zeroing routines at the end of a reversal sequence. The orientation dependance of the \( B_x \) and \( B_y \) zeroing routines (Section 4.2.6) introduces possible shifts in the \( B_z \) and \( B_y \) fields depending on the orientation of the apparatus on this timescale. Beginning in file 3498.72, execution of the major zeroing routines was delayed until the apparatus was aligned along the N-S axis. This is accomplished by comparing the encoder reading to be within 2\(^\circ\) of the starting heading modulo 180\(^\circ\) reducing the offset in the \( B_y \) zeroing but maximizing the effect in the \( B_x \) zeroing. In terms of drift, the offset from \( B_x \) is suppressed by \( B_z^2 \), so it can be neglected as long as the \( B_z \) zeroing routine executes regularly.

During the winter data collection, reading of the encoder values would fail after a few days of operation from a firmware error in the JSB524 counter. The major zeroing routines were executed manually at approximately 7 hour intervals for a short time. Shortly after failure of the counter, accessing the DLL files would freeze the operating system and require a hard restart of the computer. The counter at the base of the experiment is difficult to access without some disassembly. Since the variations in platform placement are too small to substantially influence the correlated reversal signal (Section 5.2.5), a simple scheme of passing a text string containing one of the
four resting platform positions from the motor to the rotating computer through the Datasocket server was introduced at file 3669.09. A simple comparison of the text string allowed execution of the major zeroing routines with the experiment in the correct position.

Just prior to file 3734.60, a clear orientation dependence in the $B_x$ zeroing routine was observed which had been previously obscured by noise in previous investigations. At this point, major zeroing routines were executed only in the south position to avoid long term shifts in both the $B_y$ and $B_x$ zeroing. Future experiments will likely use a more sophisticated scheme of adjusting the $B_{y}^{a}$ and $B_{x}^{a}$ fields by a known amount during each record to compensate for the $\Omega_{x}$ and $\Omega_{y}$ contributions to the $B_{z}$ and $B_{z}^{2}$ terms respectively. These ideas have been discussed but not implemented at this time.

### 5.1.3 Reliability

Many times during the acquisition, a failure would occur that would stop operation. While much of the operation of the experiment is fully automated, several errors required human intervention. During the summer months, a NI-DAQ error would halt operation from a miscommunication between the two data acquisition cards. This was later solved with a computer upgrade with two separate PCI slots rather than one slot and a PCI bridge. During the first few weeks of the winter dataset, the LabView program on the onboard computer would lock up and fail to quit. This required a full restart of the computer. It was later deemed to be a result of a faulty 24-bit counter and a memory overwrite error. Also, in the early spring, Princeton’s Office of Information Technology (OIT) would occasionally disable local internet claiming that illegal devices were present on the network. In the short term, this was solved by connecting increasingly longer ethernet cables to any available active ethernet port and having many stern conversations with OIT representatives. An active internet connection
allows for reliable synchronization of computer clocks with the navy timeserver, and active network access is required to reliably pass NI datasocket triggers.

Both computers contain minimal software for undistracted operation. Windows updates have been disabled to prevent unwanted interruptions for the data acquisition system. Several interlocks disable the ac heater if certain components exceed preset temperature limits. A simple error handling system determines when applied values on the data acquisition system near the hardware limits. An email client sends a daily email during normal operation as well as notifications of simple errors. The email client can also send a text message to the operator’s cell phone. Uninterrupted operation over three weeks has been achieved, though regular supervision is recommended to gather long uninterrupted datasets.

5.1.4 Least Squares Fits

Due to the large systematic drifts in the CPT-I data, the amplitude of sidereal oscillation was determined by multiplying the comagnetometer signal by a sinusoidal response of arbitrary phase and averaging the product over integer periods (Section 3.3.2). The elimination of this long term drift allows for a more direct analysis by directly fitting the gyroscope signal to a sidereal oscillation with arbitrary phase using a least squares fit

\[ S_R = A_x \cos(\Omega_\oplus t) + A_y \sin(\Omega_\oplus t) + c \]  

(5.1)

where \( S_R \) is the reversal correlated amplitude, \( A_x \) and \( A_y \) are the amplitudes of the two sidereal phases, and \( c \) is a constant offset. This fitting occurs independently for both the N-S and E-W measurements. Nominally, the E-W measurements would have zero offset, but imperfect alignment with the gyroscope signal creates a small but finite offset.
Figure 5.3: Example long term collection of the N-S and E-W amplitude with a sinusoidal fit to a sidereal frequency.

The sidereal time is saved in GMST, but the lab is located at -74.652° W latitude. Prior to fitting, the times are adjusted by $t \rightarrow t - 74.652/360$ to LST for straightforward interpretation of the $A_X$ and $A_Y$ amplitudes in celestial galactic coordinates (Appendix A). Fits occur on datasets that are continuous and longer than one day. Fits to several sidereal cycles are preferred for greater separation of sidereal and diurnal effects.

### 5.1.5 Combined Measurements

The N-S results are most sensitive to variations in comagnetometer sensitivity and calibration while the E-W results are sensitive to variations in orientation of the comagnetometer and drifts in the $B_z$ and $L_z$ fields. Throughout a given day, measurements of both of these directions distinguishes variations of these two systematic effects. Furthermore, a true sidereal variation provides an out-of-phase response in
both signals, distinct from any Earth-frame field. The amplitudes of independent fits in the N-S and E-W measurements (Eq. 5.1) can be combined

\[ A_X \rightarrow - \left( \frac{A_{NS}^X}{\sin \chi}, A_{EW}^Y \right) \]
\[ A_Y \rightarrow - \left( \frac{A_{NS}^Y}{\sin \chi}, -A_{EW}^X \right) \]  \hspace{1cm} (5.2)

where \( \chi = 40.3453 \degree \) is the latitude in Princeton, NJ. At \( t = 0 \), the celestial \( \hat{x} \) points overhead which corresponds to a projection south for an experiment in the Northern Hemisphere. A sidereal component is most sensitive when oriented in the equatorial plane of the Earth, therefore, the sensitivity of N-S measurements to anomalous fields is suppressed by the the sine of the latitude.

5.2 Correlation Analysis

The least squares method can be limited by long term drifts in the signal. These drifts are smaller than those observed in the CPT-I collection but still persist. A correlation analysis of the signal with each of the other recorded parameters has been performed. The only finite correlations observed occur from drifts to the \( L_z \) and \( B_z \) fields. This section will describe several parameters of importance and their contribution to the analysis.

5.2.1 Calibration

The low frequency \( B_x \) modulation calibration scheme (Section 4.3.5) was not developed until file 3734.60 near the end of the Lorentz violation search. For consistency in the measurement, the traditional calibration method has been used to calibrate all files, though the total amplitude of the gyroscope response has been rescaled to 277 fT as determined by the low frequency \( B_x \) calibration. In comparing the two calibration
techniques, the traditional calibration is more sensitive to $^3$He polarization drifts at the beginning of the file. Near sd 3746, both calibrations capture a sensitivity change, but near sd 3749, the traditional calibration presents a drift that is not present in the low frequency calibration. This is clearly a failure of the traditional calibration as the rotation signal should be constant (Fig. 5.5). Overall, the low frequency $B_x$ calibration more closely matches changes in sensitivity of the comagnetometer.

### 5.2.2 Faraday Effect

Large square Helmholtz coils surround the experiment to reduce Earth’s magnetic field and correlated optical rotation signals from reversal of the apparatus in this field. No feedback scheme has been implemented on these coils, though a fluxgate on the platform monitors changes in the local field along all three coordinate axes. Magnetic field changes along the x-axis provide the largest contribution to the optical rotation signal. A clear long term change in the field under reversals of the apparatus (Fig. 5.6b) can be observed without a dramatic change in the corresponding signal.
Figure 5.5: N-S response calibrated with each of the practical techniques. a) Traditional calibration. b) Low Frequency $B_x$ calibration. It is clear that the deviation at sd 3749 in Fig. 5.4 is due to the traditional calibration and not a change in the sensitivity.

(Fig. 5.6a). Perhaps a correlation exists at sd 35005.5, but this is not consistent with a correlation between sd 3499.5-3500.

These changes in local magnetic field are due to slow ramping (0.25-0.5 T/min) of a 14 T superconducting magnetic in a nearby lab. Clear structure matching the particular ramping routine rates can be observed under closer inspection of the fluxgate response. The fluxgate is mounted 30 cm from from the rotation axis of the platform and is also sensitive to the magnetic field gradient. Section 4.1.5 predicts that the 2 mgauss change in field corresponds to 50 nrad of optical rotation. Given the sensitivity of the comagnetometer, these changes are on the order of 0.5 fT and below the typical scatter (Fig. 5.6a).

### 5.2.3 Pump Lightshift

Two diode lasers have served as the source of pump light with the replacement made in the gap between the summer and winter data collection. The diode used in the
Figure 5.6: a) E-W reversal amplitude. b) Long term correlation behavior of the magnetic field measured by the fluxgate outside the shields. The change in local magnetic field is from ramping of a 14 T superconducting magnet in a lab down the hall. These changes do not significantly influence the measurement.

summer was near the end of its useful lifetime and was subject to large drifts of 1 pm/day. Corrections for drifts in the pump wavelength can significantly improve the stability of the measured reversal amplitude. The laser used in the winter collection drifts 0.1 pm/day, so no correction is needed.

Significant drift in the pump beam wavelength introduces a change in the $L_x \Omega_x$ term (Section 4.4) and leads to a drift in the long term response of the E-W reversals (Fig. 5.2.3a). It is clear that the drift in the pump wavelength is correlated with the change in the E-W reversal response (Fig. 5.7b). Here, 30 consecutive wavelength measurements are averaged and a cubic spline used to interpolate the values for the E-W reversal domain. Prior to averaging, outliers have been automatically rejected by considering groups of 10 points and rejecting any values beyond 2 standard deviations from the mean.
Figure 5.7: Pump lightshift drift corrections. The gap at sd 3552.5 is from a data acquisition error in the middle of the night. a) E-W reversals before correcting for changes in $L_z$. b) Long term pump wavelength measurements. c) E-W amplitude adjusted for drifts in the pump beam wavelength.

Measurements of the gyroscope phase shift described in Section 4.4 provide a correction to the pump beam wavelength drift (Fig. 5.7c). This correction has been applied prior to rescaling the reversal measurements to 277 fT and uses a value of +1.33 fT/pm to account for the drifting pump beam. This effect is second order in the N-S reversals and need not be applied. The gap at sd 3552.5 is due to a data acquisition error in the middle of the night. The experiment was promptly restarted in the morning and continued without error until the end of the file.

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2If the direction of the spins are reversed, the sign of this correction should also reverse.
Figure 5.8: a) E-W reversal amplitude under the influence of $B_z$ drift showing a clear asymmetry in the central amplitude from starting in the east or west position. b) Reduction of the influence of the $B_z$ drift using the $m_4$ procedure and extrapolating to the time of the $B_z$ zeroing.

### 5.2.4 $B_z$ Drifts

Like the pump wavelength drift, a drifting $B_z$ field introduces sensitivity to $\Omega_x$. The $B_z$ field drifts on a faster timescale than the pump wavelength due to the changing $^3$He polarization (Fig. 5.2) and must be accounted for differently. E-W reversal measurements show a clear bifurcation in the reversal measurements depending on whether the experiment started in the east or west position (Fig. 5.8a). The $B_z$ field continues to drift the same direction, but asymmetry in the drift on individual reversal measurements leads to a systematic offset with the starting position.

Early in the search, E-W reversals were averaged in pairs and adjusted using the time derivative of the $B_z$ field to account for this drift. More sophisticated procedures described in Section 5.3.2 perform much better. The $m_2$, $m_3$, and $m_4$ methods each reduce the correlation of the reversal amplitude to $B_z$ drift (Fig. 5.8b).
Figure 5.9: Typical encoder reading for positions of the platforms 180° apart. The positions are taken modulo 90° for comparison on a fine scale. The encoder starts arbitrarily at the 18° mark in the south position. The discontinuity shortly after sd 3542.50 comes from pausing the experiment and adjusting equipment on the rack.

5.2.5 Orientation

Inconsistent placement of the platform in the same position can introduce a systematic effect. The position is recorded continuously every 3 s using an incremental encoder with 0.001° precision. Figure 5.9 shows the encoder value at each rest location before the next trigger. We expect that the scatter is due to imperfections in the teeth of the gears. This variation is remarkably different in comparing the north and west positions (Table 5.1). It is interesting to note that the points cluster along a lower line and scatter upward asymmetrically. This is likely a result of the 0.2° backlash in platform and overshoot. The servo motor comes to a particular value determined by its own encoder, and the large momentum of the platform carries the platform beyond this point. The servomotor encoder cannot recognize this overshoot or compensate for it within the backlash.

Shortly after sd 3542.50, the wavemeter failed to record values over the GPIB. The experiment was paused to remove the wavemeter from the lower shelf to reboot it.
Table 5.1: Typical variations in platform location as measured by the encoder. These are too small to observe in the reversal signal even in the E-W direction.

<table>
<thead>
<tr>
<th>Position</th>
<th>Typical Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>0.001°</td>
</tr>
<tr>
<td>South</td>
<td>0.004°</td>
</tr>
<tr>
<td>West</td>
<td>0.012°</td>
</tr>
<tr>
<td>East</td>
<td>0.006°</td>
</tr>
</tbody>
</table>

In the process, the equilibrium position of the experiment was inadvertently shifted $0.005^\circ$ as seen by the discontinuity at the gap. For long runs, we observe a relative drift of the platform and legs $< 0.002^\circ$ over 10 days. For scale, a $0.02^\circ$ shift in the position along the E-W direction would only produce a 0.1 fT change in the signal. Fortunately, these effects are too small to contribute significantly to our measurements and do not enter the analysis.

While the platform position is well-known from the encoder, the location of the vibration isolation provides an additional free parameter. The mechanical springs and rubber come to a new position and orientation with each reversal of the platform. The relative position of the vibration isolation platform with respect to the aluminum baseplate is recorded using two non-contact position sensors on either side of the platform. This allows separation of a rotation from a translation in the recorded signals. Each of the sensors is calibrated separately by shifting the platform and give the same slope within $0.1\%$.

The reversal correlated orientation of the vibration isolation platform rises to an equilibrium value over 10 hours. The scatter is less than $0.002^\circ$ and the magnitude of the drift is less than $0.010^\circ$. As with the variations in the platform positioning, these changes do not significantly influence the comagnetometer response at the 0.1 fT level. The observed backlash from the vibration isolation platform is $0.09^\circ$, which can almost contribute at the 1 fT level, though all measurements are made with reversals in the same direction.
Figure 5.10: Typical long term variations in the orientation of the vibration isolation stage relative to a $180^\circ$ reversal. Over 10 hours, the platform reaches an equilibrium orientation.

### 5.2.6 Long Term Signal Variations

The fits in Fig. 5.3 are limited by wiggles observed in the data. The source of these variations is unknown. A correlation analysis with the signal and other parameters such as temperatures, laser position detectors, water flow, etc. has been performed. No direct correlation has been observed with any single parameter other than the pump lightshift and $B_z$ drift previously discussed. A fourier transform of the reversal amplitudes may provide insight into time-dependant patterns in the response.

The longest sequence of uninterrupted data combines files 3669.09, 3677.13, and 3685.43. A fourier transform of this combined file produces the spectrum in Fig. 5.11. An FFT requires equally spaced data, and the values in Fig. 5.3 have a typical spacing of $480 \pm 6$ s. A linear interpolating function creates a new domain with equally spaced points of 100 s to meet this requirement\(^3\). The spectrum in Fig. 5.3 shows no peaks at any of the low frequencies indicating that there is no regular pattern to the variations.

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\(^3\)The interpolating function also accounts for minor computer slowdowns that are present in the dataset.
Figure 5.11: Fourier transform of the longest uninterrupted measurements in Fig. 5.3. The bin size is $1.8 \times 10^{-6}$ Hz. The E-W reversals are subject to $B_z$ field drift and exhibit higher noise on short timescales. There is no peak at $4 \times 10^{-5}$ Hz which corresponds with the major zeroing routines.

in Fig. 5.3 Higher frequencies are limited by the averaging of the string points and zeroing schedule. A peak at $1/480$ Hz correctly indicates the spacing between the original points as an artifact from the interpolating function (not shown). It is likely that the larger noise in the E-W signal on short timescales is due to drifts in the $B_z$ field.

The N-S reversals in file 3519.53 present the largest variations all of the files with excursions up to 1.5 fT from the central value (Fig. 5.12). The cause of these variations is unknown, but this file provides a representative case to search for correlations with other saved values. No strong correlation with any single channel has been observed. Some trends arise and are only suggestive of a correlation as they do not persist.

### 5.3 Analysis Methods

The long collection time over nine months makes it difficult to vary experimental parameters, repeat the experiment, and use the scatter in the result to determine a systematic uncertainty. Instead, the same data is used repeatedly, but analyzed
with differing processing techniques. The scatter in processing techniques provides an estimate of the systematic uncertainty in the measurement. The following section describes details relevant to data processing and determination of the systematic uncertainty.

5.3.1 Final Processing Scheme

By the end of the collection, a global routine was developed to analyze all of the data and incorporate all of the pertinent elements to the analysis. Data with the experiment at rest is selected and averaged during each record. Prior to averaging, a digital Butterworth low pass filter is applied to remove oscillations in the signal from $^3$He precession between 5-10 Hz (Fig. 4.1). To prevent artifacts from the digital filter, an extra 2 s prior to each sample is filtered and dropped prior to averaging to the central value. The uncertainty is determined using the scatter and the ENBW (Section E.2).

The string analysis determines the reversal correlated voltages and removes background drift (Section E.3). Five parallel processing techniques are added at this step to account for higher order drifts and to determine the systematic uncertainty (Section 5.3.2). The reversal correlated voltages are combined with the calibration and
a correction applied for drifts in the pump wavelength (Section 5.3.3). Outliers are removed by manually selecting an appropriate domain in every file (Section 5.3.4). These files are saved for straightforward analysis by the least squares fit with several variations (Section 5.3.6). The results are combined to a final pair of amplitudes and the systematic uncertainty determined (Section 5.4.1).

5.3.2 Processing Methods

Five different processing techniques have been developed to analyze each record. The first is a straightforward application of the 3-point overlapping string analysis; the next four modify this process. These techniques are referred to as “simple” and “modification 1-4” using the shorthand s, m1, m2, m3, m4 for convenience. Likewise, the intervals are analyzed with and without the removal of the probe background for a direct comparison of the improvement.

Simple: s

For each element in a record, a three point overlapping string analysis is performed. The overall sign convention is assigned according to the first measurement. The N-S and E-W measurements are separated and each converted to the sign convention north minus south (ns) and east minus west (ew).

The introduction of the probe background measurements in the winter introduces secondary measurements. A 3-point overlapping string analysis on the background measurements provides an overall correlation from the background which we refer to as ns2 and ew2. Subtraction of the signal and background amplitudes demonstrates improvement in the the short term scatter of reversal measurements. Further improvement is achieved by considering the difference in the signal and background with each reversal. The string analysis on these differences generates amplitudes re-
Figure 5.13: File 3669.09, record 1576. E-W reversals a) Example case of the disturbance of the signal from rotating the apparatus and drifts in the $B_z$ field. This first point has no drift due to adequate time to settle during the zeroing routines. b) The resulting string points from the record in (a). The change in drift causes the first point to be disjoint from the rest. The $m_3$ and $m_4$ processes compensate for the drift by fitting a line to the string points. The $m_4$ process disregards the first string point as shown.

ferred to as ns3 and ew3. In the case without a probe background measurement, ns2 and ew2 contain zeros, and ns3 and ew3 are identical to ns and ew.

Modification 1: $m_1$

This modification considers an improvement to the determination of the uncertainty of the signal at rest. In the case that the signal drifts over the averaging interval (Fig. 5.13a), the standard deviation over-estimates the uncertainty. A least squares fit to a line can remove the drift. These files follow the same structure as $s$ and the results are qualitatively the same.

Modification 2: $m_2$

This modification reduces the effect of drift in the $B_z$ field over the entire record (Fig. 5.13a). The overall signal drifts to higher voltages from drifts in the polarization. The drifting $B_z$ adjusts the phase of the gyroscope signal (Eq. 4.28) and contributes differently from the orientation of the apparatus; measurements 1, 3, 5, and 7 drift
less than measurements 2, 4, 6, and 8. Fitting the odd and even measurements to a line with the same slope but opposite sign isolates this effect from the string analysis. Due to the subtle changes in the probe background with each measurement, these fits are performed only after removal of the background.

**Modification 3: \( m_3 \)**

Modification 2 provides a complicated method to remove the \( B_z \) drift from the signal itself. This drift propagates into the string points, but an alternate approach in modification 3 with a linear fit to the string points can also remove this effect (Fig. 5.13b). Two results are reported for the amplitude, the central value (\( \text{ns3c/ew3c} \)) and the y-intercept (\( \text{ns3z/ew3z} \)). The center value is similar to the string analysis without the fit. The y-intercept extrapolates back to the time of the zeroing and estimates the value without \( B_z \) drift. Again, these results are only performed after direct removal of the probe beam background.

**Modification 4: \( m_4 \)**

The most striking feature of Fig. 5.13b is the discrepancy of the trend of the first string point with the rest. Comparison with the signal in Fig. 5.13b indicates that a new trend in the drift occurs during the first reversal. The string analysis accounts for a linear drift, not a change in the drift. It is likely that mechanical stresses on the platform and settling of the vibration isolation stage cause the trend. So far, observations of the vibration isolation stage on the inductive position sensors do not account for this change, but subtle changes in the tilt below the resolution of the tilt sensors could contribute to a rotation of \( \Omega_y \).

The discontinuity in the string points is not present in every record, but consistently dropping the leading point improves the scatter. We conclude that this method best represents the reversal correlated amplitude. A consistent sign convention for
the interval is maintained with the starting condition despite dropping the leading point. Again, intervals are only considered after point by point removal of the probe background.

5.3.3 Calibrated Data

A Matlab structure array organizes the various processed voltages for each file into a single easily addressed structure. Each process is calibrated and corrected for drift in the pump lightshift (Section 5.2.3). The low frequency $B_x$ calibration was not developed until file 3734.60, so for a consistent treatment of the data, the traditional calibration is used throughout the files.

The traditional calibration provides an overall amplitude that is too large and varies across the files (Fig. 5.14). The low frequency $B_x$ calibration consistently provides an overall amplitude of 277 fT and is insensitive to the magnetic field gradient from the polarized alkali-metal vapor. The amplitude of the calibrated signal is
rescaled to 277 fT considering the contributions from the N-S and E-W data added in quadrature.\footnote{One could scale to the 271 fT as predicted by the theory, but we choose instead to rely on the measurement and focus on setting a more conservative limit.}

For the calibration, groups of 10 values are averaged. As with the pump wavelength correction (Section 5.2.3), outliers are automatically rejected prior to averaging. A cubic spline interpolates the calibration value for each amplitude to match the original points. Two parallel blind analyses, each using different software (Matlab and Genplot) evaluate the result in units of fT and check the consistency of the final results.

### 5.3.4 Domains

Automated routines have been considered for rejecting outliers in the reversals similar to those used for determining the calibration and pump wavelength. These and more advanced routines are not perfect and have been rejected for removal of outliers from the calibrated data. Every file is investigated by hand, and all obvious outliers are removed. Typically, the first point in the file is shifted with respect to the rest of the file much like the first string point in a sequence (Fig. 5.13b). These first points are automatically removed and disregarded.

### 5.3.5 Averaging

Groups of consecutive points can be combined into a single value and uncertainty prior to the least squares fit. A weighted average of groups of 1, 2, 6, 12, and 16/18 points are combined with a weighted uncertainty for each of the processes.\footnote{An average of 16 or 18 is chosen to reduce the number of points ignored in the file.} Not every file has a number of points that easily divides by the size of the averaging group. In this case, the remaining data is thrown away. Rather than remove the end or beginning of a file, these regions are maintained. The dropped points are instead
evenly distributed throughout the file. In general, larger groups of averaged points decrease the reduced $\chi^2$ of the least-squares fits to values slightly larger than 1. Care has been taken not to average large groups such that less than 4 averaged points remain per day. In this case, a sidereal frequency would be filtered from the data.

### 5.3.6 Least Squares Fit Refinements

Equation 5.1 describes the sidereal signal of interest in this work. A direct fit using this method can be susceptible to other effects, so four additional methods have been developed to fit the data. In the absence of systematic effects, these methods should give similar results. Only a comparison of the results across all the methods can determine the validity of the result.

**Direct Fit**

A direct fit uses a least squares fit using Eq. 5.1 to the N-S and E-W reversal measurements separately. This is the simplest and most direct of the fitting techniques. This technique returns amplitudes $A_x$ and $A_y$ and their associated uncertainties for each file. Longer term files of several days demonstrate improved separation of possible sidereal oscillations from variations in the background signal.

**Background Fit**

Drifts on a longer timescale than a sidereal day persist in the signal. A low order polynomial can suppress the influence of these drifts on the fitted amplitudes without removing a signal at a sidereal frequency. The order of the polynomial is selected based on the length of the file. The order of the polynomial added to Eq. 5.1 is limited by one less than the number of full sidereal days in the file. For example, a
file of length 3.15 sidereal days will be fitted to the following function

\[ S_R = A_x \cos(\Omega_\oplus t) + A_y \sin(\Omega_\oplus t) + c_0 + c_1 t + c_2 t^2 \]  

(5.3)

where \( c_0, c_1, \) and \( c_2 \) are the coefficients of the polynomial of order zero, one, and two respectively. For these fits, a large integer around 3600 sd is subtracted from \( t \), such that the center of the polynomial is somewhere in the middle of the file. This leads to a more efficient convergence of the overall fit\(^6\).

**Single Fit**

One can also consider the complete collection of each of the N-S and E-W measurements as a single massive set (Fig. 5.1). The dc offset for each file differs particularly for E-W reversals, so the average value of each file is removed before combining measurements. This technique returns a single amplitude and uncertainty for each of the total N-S and E-W collections. The single fit is subject to changing scatter across the entire dataset but can fully distinguish sidereal and diurnal components in the time domain.

**Strong Fit**

A true sidereal signal from a Lorentz-violating field will appear out-of-phase in each of the N-S and E-W reversals. Both the N-S and E-W measurements from the same file can be combined to constrain precisely the correct sidereal response between orientations. The N-S and E-W reversals contribute to the measurement differently, so they are both projected onto the equatorial plane for a direct comparison. The\(^6\)Figure 5.3 provides an example of the background fit technique where the polynomial background has been subtracted from both the data and the fit except for the \( c_0 \) term for display purposes.
Figure 5.15: Strong fit procedure on file 3685.43 where groups of 18 points have been averaged to a single value. The E-W reversals have been shifted forward in time by an integer number of sidereal days so as not to overlap with the N-S reversals, and an extra 0.75 sd matches the phases. The error bars on the N-S measurements are larger than on the E-W measurements due to the projections onto the equatorial plane. The time has been adjusted to LST.

Following transformation prepares a file for the strong fit:

$$R_{EW}(t) \rightarrow R_{EW}(t - N - 0.75) \quad (5.4)$$

$$R_{NS}(t) \rightarrow R_{NS}(t)/\sin(\chi) \quad (5.5)$$

where $R$ is the reversal correlated amplitude, $N$ is an integer number of sidereal days, $\chi$ is the latitude, and $t$ is the local sidereal time. A direct fit is then applied to the transformed dataset as in Fig. 5.15 where the mean value from each orientation is again removed. In general, the N-S measurements will provide lower statistical weight due to to the smaller spatial overlap with the equatorial plane. The results of this fit on file 3685.43 are $A_X = -0.051 \pm 0.093$ fT and $A_Y = 0.029 \pm 0.093$ fT where the amplitudes are reported in terms of galactic celestial coordinates.
Single Day Fit

Each file can also be divided into single days and fitted by day. The advantage of this technique is that it can provide over 100 amplitudes providing a reasonable distribution of scatter in the measured amplitudes across the entire dataset as well as avoiding bias from long term drift removal. Each file is divided into intervals of at least one day where the first $N-1$ days are equal, and the last one carries the remainder. For example, a file of length 4.23 sd is broken into 3 days of 1.05 sd with the last day of length 1.08 sd. In the case of a data acquisition gap greater than 0.1 sd, the gap is treated like the edge of a file. Each of these one-day intervals are fitted to Eq. 5.1 and return an amplitude and uncertainty for both the N-S and E-W measurements for each day selected.

5.3.7 Systematic Checks

Throughout the course of the experiment, three main parameters have been compared to check against a systematic effect in the fitted amplitudes. (1) The N-S and E-W measurements provide a separation of systematic effects in the gyroscope background from Earth’s rotation. Furthermore, a Lorentz-violating field will appear out-of-phase in each orientation to distinguish between Earth-based effects and celestial effects. (2) The direction of the spins in the comagnetometer have been reversed twice throughout the entire measurement. Equal duration of data with the spins up and down under similar conditions have been collected during the winter months. This provides a reversal of spin-based effects from the gyroscope and celestial fields uncorrelated with other signals associated with operation of the apparatus. (3) The entire data collection spans nine months with significant data collected in both the summer and the winter. A comparison of the summer and winter data provides a complete separation of the diurnal and sidereal phases with the Earth on the other side of the sun for the same orientation in space-time.
5.4 Results

The large variety of processing techniques lead to a distribution in the measured amplitudes which provides an estimate for the systematic uncertainty. Within this distribution, several representative files are identified to determine a final measured amplitude. The measured amplitudes are connected to parameters of the SME improving the limit on $\tilde{b}_n$ by a factor of 30. The details of this analysis are described here.

5.4.1 Representative File and Systematic Uncertainty

The fitting techniques, averaging, and processing methods lead to 136 amplitude measurements for both phases of the N-S and E-W measurements where each amplitude is a weighted average across the entire dataset. A plot of the amplitudes shows remarkable consistency (Fig. 5.16). The horizontal lines represent the weighted average of the measured amplitudes, and only a few results are more than a standard deviation from the mean. The $m_4$ ns3c, ew3z file averaged in groups of 18 with the background fit is well centered in three of the four distributions. The $A^{EW}_4$ measurements present a systematic shift correlated with alternating between the ew3c and ew3z methods. The ew3z method accounts for known drifts in the $B_z$ field, so a bias is no unexpected.

5.4.2 Sidereal Amplitudes

Given the representative file in Fig. 5.16, the individual amplitudes from each fit can be displayed (Fig. 5.17). Here, the N-S and E-W measurements have been combined and averaged as described in Section 5.1.5. These amplitudes represent a dramatic improvement over the equivalent results in the previous generation experiment (Fig. 3.5).

---

7The strong fits are omitted in this comparison because they combine the N-S and E-W results into a single amplitude pair.
Figure 5.16: Measured amplitudes from the various processing, averaging, and fitting (except strong) techniques. The horizontal lines indicate the average of the amplitudes. The $m_4$ ns3c ew3z file averaged in groups of 18 with the background fit is highlighted as a representative file. The scatter in the amplitudes is used to identify the systematic uncertainty in this collection.

The summer data has a larger scatter of data points than indicated by their error bars. We believe this is due to fluctuations of the background optical rotation. In the winter, background measurements were implemented to compensate for these effects and reduce the overall scatter. To obtain the final limit, the summer and winter data are averaged separately, scaling the errors by their respective reduced $\chi^2$ before the final average. A comparison of the results taken in the summer and the winter, with the $^3$He spins up or down, and from the N-S or E-W amplitudes provides a check of possible systematic effects (Section 5.3.7). Only the winter data is considered for a comparison of the spin up and spin down states. The N-S error bar is larger than the E-W error bar due to the reduced sensitivity to $A_X$ and $A_Y$ in N-S measurements.
Figure 5.17: Top Panel: Summary of the amplitudes $A_X$ and $A_Y$ for each file as a function of time. Upward and downward triangles indicate the direction of the $^3$He spins. The mean of the amplitudes is well centered on zero and the scatter improves as the experiment progresses. Bottom Panel: Systematic comparisons: SU/SD - spin up vs. spin down (from winter data only), SM/WN - summer vs. winter data, NS/EW - amplitudes from N-S vs. E-W reversals only. The presence of a systematic effect will cause a shift from a reversal of the SU/SD, SM/WN, NS/EW parameters. Each pair agrees within its uncertainty.

(Section 5.1.5). A separation from a reversal of any of these parameters will indicate a systematic effect, yet each provides a consistent result.

Artificial Lorentz Violation Simulation

It is possible that fitting an additional polynomial to the background can remove sensitivity to a sidereal oscillation. As a check of both the processing techniques and combining N-S with E-W measurements, a sidereal oscillation several standard deviations larger than the limit is artificially added into the complete dataset. Execution of the analysis yields the results in Fig. 5.18. These amplitudes agree within 1 aT.
**Simulation:** As a check of the fitting routines, a $+8\sigma$ and $-10\sigma$ sidereal oscillation is introduced for $A_X$ and $A_Y$. The dashed green line represents the expected central value of $A'_X = +0.168$ fT and $A'_Y = -0.210$ fT. These amplitudes are precisely the same as those in Fig. 5.17 with the appropriate shift for the inserted sidereal signal.

Figure 5.18: Simulation: As a check of the fitting routines, a $+8\sigma$ and $-10\sigma$ sidereal oscillation is introduced for $A_X$ and $A_Y$. The dashed green line represents the expected central value of $A'_X = +0.168$ fT and $A'_Y = -0.210$ fT. These amplitudes are precisely the same as those in Fig. 5.17 with the appropriate shift for the inserted sidereal signal.

to those presented in Fig. 5.17 accounting for the known shift of $A_X$ by $+0.168$ fT and $A_Y$ by $-0.210$ fT. This confirms that a sidereal oscillation can be detected and also confirms correct treatment of the sign when combining N-S and E-W reversals. Analysis of the subdivision parameters generates a similar effect for the systematic reversals.

### 5.4.3 Final Average

In determining the final limit to report, six representative files have been identified across the Genplot and Matlab investigations (Table 5.2). The final result is an average of each of the six files. In each case, the summer and winter data have been considered separately and the error bars scaled by the reduced $\chi^2$. This particular consideration reduces the uncertainty by 10%. Files A and B represent the “best” and “good” methods from the Genplot analysis. Likewise, files F and E are the “best” and “good” methods from the Matlab analysis. C includes the first representative
file identified in Section 5.4.1 and D incorporates a “naive” analysis including all the methods. Unless otherwise noted, the reported amplitude and statistical error are the average of each processing method. The systematic error is the standard deviation of the amplitude values.

The six files are as follows: (A) The “best” Genplot method considers the $m_4$ ns3z and ew3z data with averaging of 1, 4, 10, 16, and 20 points. Both the single day fits and background fits are considered. (B) A “good” Genplot method considers all of the $m_3$ and $m_4$ processes and includes the ns3/ew3, ns3c/ew3c, and ns3z/ew3z combinations. (C) The first representative file identified considers the $m_4$ ns3c, ew3z method with groups of 18 points averaged to a single value. The background fit determines the amplitude and statistical uncertainty. The systematic uncertainty is determined from the scatter in the amplitudes reported from the direct and strong fitting techniques. (D) The “naive” method includes all processing, averaging, and fitting methods described in the previous sections. (E) The “good” Matlab method considers both the $m_3$ and $m_4$ processes and includes the ns3c/ew3c and ns3z/ew3z combinations. Unlike the “good” Genplot method, the ns3/ew3 combination is omitted. The direct, strong, and background fits determine the reported amplitude and
uncertainties. (F) The “best” Matlab method includes an average of the $m_4$ process with the ns3c/ew3z and ns3z/ew3z combinations. Again, the direct, strong, and background fits determine the amplitudes and uncertainties.

The $A_X$ amplitudes are spread evenly within 10 aT of a zero value and the $A_Y$ amplitudes have a tighter spread clustered around 30 aT. Each of the reported statistical and systematic uncertainties are tightly grouped. Given the tight cluster of points, an average of the results from methods A-F to a single amplitude pair is reasonable and provides the following result

$$A_X = (0.001 \pm 0.019_{\text{stat}} \pm 0.010_{\text{syst}}) \text{ fT} \quad (5.6)$$

$$A_Y = (0.032 \pm 0.019_{\text{stat}} \pm 0.010_{\text{syst}}) \text{ fT}. \quad (5.7)$$

The amplitude for $A_Y$ is a 1.5$\sigma$ result and is interpreted to be consistent with zero.

### 5.4.4 SME Parameters and Energy Scaling

The comagnetometer is sensitive to the difference in electron and nuclear spin couplings (Eq. [3.9]). The limit on an electron Lorentz-violating field from Ref. [86] is equivalent to a magnetic field of 0.002 fT and is more tightly constrained than Eqs. [5.6] and [5.7]. Therefore, we can ignore a possible signal from electron interactions and set clean limits on Lorentz-violating nuclear spin interactions.

These results can be interpreted in terms of the parameters in the SME [9]. The following 3- and 4-dimensional operators in the relativistic Lagrangian can be constrained from coupling to a spin-1/2 particle

$$\mathcal{L} = -\bar{\psi}(m + b_{\mu} \gamma^5 \gamma^\mu + \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu}) \psi + \frac{1}{2} i \bar{\psi}(\gamma_\nu + d_{\mu\nu} \gamma^5 \gamma^\mu + \frac{1}{2} g_{\lambda\mu\nu} \sigma^{\lambda\mu}) \partial_\nu \psi \quad (5.8)$$
where \( b_\mu \) and \( g_{\lambda\mu\nu} \) are CPT-odd, while \( d_{\mu\nu} \) and \( H_{\mu\nu} \) are CPT-even fields. Five- and six-dimensional operators not included in the SME also lead to higher order spin coupling terms [87, 88]. The energy shift from the comagnetometer is

\[
\delta E = -\mu_{\text{He}}(-\beta^i_\mu)\sigma_i = -\mu_{\text{He}}A_i\sigma_i
\]

(5.9)

where the negative sign on \( \beta^i_\mu \) accounts for the response of the comagnetometer to an anomalous nuclear spin coupling and \( \sigma \) is the Pauli matrix\(^8\).

The \(^3\)He nucleus contains one neutron and two protons. To leading order, the protons are paired in a singlet state leaving the nuclear magnetic moment to be dominated by the neutron. The polarization of each nucleon has been calculated to be \( P_n = +0.87 \) and \( P_p = -0.027 \) for the neutron and proton respectively [89]. The leading energy energy shift of the \(^3\)He nucleus in the SME becomes

\[
\delta E = -P_n\tilde{b}^n_i\sigma_i - 2P_p\tilde{b}^p_i\sigma_i
\]

(5.10)

where \( \tilde{b} \) is a linear combination of SME coefficients\(^9\). In cartesian coordinates with the quantization axis along the z-axis,

\[
\tilde{b}^n_3 = b^3_n - m_ng_{120} - H^n_{12}
\]

(5.11)

where \( m_n \) is the mass of the neutron [17].

Given these relationships, the following limits can be placed for the neutron

\[
\tilde{b}^n_X = (0.1 \pm 1.6) \times 10^{-33} \text{ GeV}
\]

(5.12)

\[
\tilde{b}^n_Y = (2.5 \pm 1.6) \times 10^{-33} \text{ GeV}
\]

(5.13)

---

\(^8\) \( A_i \) is defined as in Eq. 5.2

\(^9\) The factor of 2 comes from the multiplicity of 2 protons.
using the fact that $\mu_{^{3}\text{He}} = -6.707 \times 10^{-17}$ GeV/T. This result represents the highest energy spin anisotropy measurement to date.

### 5.4.5 Interpretation to a Limit

If the final amplitudes reported in Section 5.4.3 were zero, the limit on $|\tilde{b}_\perp| = |\tilde{b}_X^2 + \tilde{b}_Y^2|$ would be simple to determine from propagating the uncertainty. The nonzero amplitudes influence the smallness of the limit especially the 1.5$\sigma$ amplitude for $\tilde{b}_Y$. A simple gaussian interpretation of the parameters can set a conservative limit on $\tilde{b}_\perp$. For example, given the measurements $\tilde{b}_X = x_0 \pm \sigma_x$ and $\tilde{b}_Y = y_0 \pm \sigma_y$, the central values are offset from the origin with 1$\sigma$ uncertainties $\sigma_x$ and $\sigma_y$ (Fig. 5.19). A limit is determined by integrating the probability within a fixed radius centered at the origin that contains 68.3% of the two-dimensional gaussian,

$$0.683 = \frac{1}{2\pi\sigma_x\sigma_y} \int_0^{2\pi} \int_0^R e^{-r\cos(\theta) - x_0^2/(2\sigma_x^2)} e^{-r\sin(\theta) - y_0^2/(2\sigma_y^2)} r dr d\theta \quad (5.14)$$

where $R$ determines the radius of the circle that constrains the total integral. This method produces a limit of $|\tilde{b}_\perp| < 3.7 \times 10^{-33}$ GeV at the 68% confidence level$^{10}$

$^{10}$In other units, this result is equivalent to 49 aT or 1.5 nHz.
Figure 5.20: Summary of published measurements of $\tilde{b}_1^n$ in the last decade.

Others in the field, for instance Ref. [40], follow the same method as Eq. 5.14; however, they assume that both $x_0 = 0$ and $y_0 = 0$ in determining the radius of the limit. This method represents only the precision of the measurement, not the measurement itself. In comparing the work of this thesis with other results, all measurements have been standardized to the two-dimensional gaussian approach.

5.4.6 Competitors

Several experiments have performed searches for $\tilde{b}_1^n$ over the last 10 years (Fig. 5.20). The Harvard-Smithsonian noble-gas maser limit stood as the most precise measurement of $\tilde{b}_1^n$ for a decade although the results from CPT-I achieved comparable sensitivity. In 2009, work in Grenoble using ultracold neutrons presented a less-sensitive limit. Despite the lower sensitivity, the ultracold neutron measurement is particularly interesting because it is the first limit on the unbound neutron. The result is completely independent of nuclear structure and unaffected by model-dependent nuclear corrections. It is also free from possible bound state suppression effects. In addition, the measurement surpasses the sensitivity to the free proton by a factor of 200 \cite{35} to present the best free nucleon limit.
The Harvard-Smithsonan dual noble-gas spin-maser uses the precession of two polarized noble-gas species to search for a sidereal variation in the ratio of $^{129}$Xe/$^3$He precession frequencies. In a static field of 1.5 gauss, the spins precess at 4.9 kHz and 1.7 kHz for $^3$He and $^{129}$Xe respectively. The quantization axis of the spins is oriented along the E-W direction for optimized spatial overlap with the equatorial plane. An inductive pickup coil measures the spin precession of both species and the precession of the $^{129}$Xe is compared against a H-maser. A feedback loop stabilizes the magnetic field leaving the $^3$He free to precess. A sidereal oscillation in the precession frequency of $^3$He is a signature of a Lorentz-violating field.

The $^{129}$Xe/$^3$He maser has similar short term sensitivity to the K-$^3$He comagnetometer but is also able to extend this performance to lower frequencies than in the best noise spectrum in this work (Fig. 3.15). This demonstrates incredible long term stability and suppression of low frequency noise. Using the values listed in Table 1 of Ref. [29], the sidereal amplitudes of each phase are determined$^{11}$ to be

\[ \tilde{b}_X^n = (5.5 \pm 5.7) \times 10^{-32} \text{ GeV} \]  
\[ \tilde{b}_Y^n = (0.6 \pm 5.4) \times 10^{-32} \text{ GeV}. \]  

This result is limited by laser and temperature stability over the course of a day. Placing the $^{129}$Xe/$^3$He maser on a rotating platform and performing frequent reversals would not be as beneficial as it has been for the K-$^3$He comagnetometer. Both the $^{129}$Xe and $^3$He have long coherence times of hundreds of seconds which limits the frequency of reversals to this timescale.

A group in Mainz, Germany also use a comparison of the spin precession of $^3$He and $^{129}$Xe in a co-located vapor cell as a $^3$He/$^{129}$Xe comagnetometer [40]. Both species are optically pumped in spherical vapor cells which are then placed as close as

$^{11}$The similar computation in Ref. [28] fails to take into account the reversals from east and west orientations of the magnetic field.
possible to the SQUID in a magnetically shielded room. A tipping pulse excites the spins into the transverse plane in a guiding magnetic field of 400 nT. The signal is a superposition of both the $\omega_{^3\text{He}} = 2\pi \times 13.4$ Hz and $\omega_{^{129}\text{Xe}} = 2\pi \times 4.9$ Hz precession measured with a low-$T_c$ DC-SQUID magnetometer. From a fit to the free induction decay, a phase difference

$$\Delta \Phi(t) = \Phi_{^3\text{He}}(t) - \left(\gamma_{^3\text{He}} / \gamma_{^{129}\text{Xe}}\right) \Phi_{^{129}\text{Xe}}(t)$$  \hspace{1cm} (5.17)

removes magnetic field effects where the value of $\gamma_{^3\text{He}} / \gamma_{^{129}\text{Xe}}$ is known to a part in $10^9$. A Lorentz-violating field leads to a sidereal oscillation in $\Delta \Phi(t)$ while Earth’s rotation rate leads to a linear phase drift. The Mainz $^3\text{He}/^{129}\text{Xe}$ comagnetometer places the following bounds

$$\tilde{b}_X^X = (+3.36 \pm 1.72) \times 10^{-32} \text{ GeV}$$ \hspace{1cm} (5.18)

$$\tilde{b}_Y^X = (+1.43 \pm 1.33) \times 10^{-32} \text{ GeV}.$$ \hspace{1cm} (5.19)

in 7 runs totaling less than 140 hours. Each run is limited by the $^{129}\text{Xe}$ transverse relaxation time to 8-16 hours by wall relaxation. In principle, longer times are possible. Due to the coherence of the spins, the statistical uncertainty of a run decreases as $1/T^{3/2}$ rather than $1/T^{1/2}$ for independent measurements. This experiment will not benefit from a rotating platform as the coherence times are tens of hours.

Using the method in Section 5.4.5 bounds on $|\tilde{b}_\perp^X|$ can be placed for each experiment consistent with the interpretation in this work. The Harvard-Smithsonian work sets $|\tilde{b}_\perp^X| < 1.1 \times 10^{-31}$ GeV, and the Mainz results sets $|\tilde{b}_\perp^X| < 4.7 \times 10^{-32}$ GeV. If the wall relaxation times of $^{129}\text{Xe}$ can be extended in the $^3\text{He}/^{129}\text{Xe}$ comagnetometer, these results will be the most direct competitor to this work.
5.4.7 Beyond $\tilde{b}^n_{\perp}$

The analysis presented here constrains $\tilde{b}_X$ and $\tilde{b}_Y$ using the signature of a sidereal oscillation. These results are combined to constrain $|\tilde{b}^n_{\perp}|$. Other measurements are possible with the measurements of $A_X$ and $A_Y$, but have not been strongly pursued. Future improvements to this experiment will continue to increase the sensitivity to these and other parameters. Final limits on a wider variety of possible parameters will be determined from an optimized experiment. Some of these parameters are discussed below.

The parameter $\tilde{b}_Z$ is unconstrained for both the proton and the neutron [27]. A method has been proposed to use axial precession of the Earth to take advantage of a small sidereal oscillation of $\tilde{b}_Z$ [90]. The Earth’s axis has been observed to precess about the normal to the ecliptic plane over a period of 25,771.5 years. The presence of a nonzero $\tilde{b}_Z$ would appear as a slow linear drift in the $A_X$ parameter. It is clear from the data that such an analysis will place a limit on $\tilde{b}_Z$ that is not as stringent as the limits placed on $\tilde{b}_X$ and $\tilde{b}_Y$. Combining the Harvard-Smithsonian and the Princeton data should improve this result from a 10-year baseline between each set of measurements. This collaboration is in progress under the direction of the author of [90].

The measurements of $A_X$ and $A_Y$ can also constrain theories of spacetime torsion [91]. Gravity curves spacetime with energy-momentum density as its source, but torsion twists spacetime with some models predicting a spin density as its source. The effects of torsion introduce a preferred orientation for a freely falling observer as is the case for local Lorentz violation, so this limit can also be interpreted as a limit on spacetime torsion [91]. Similarly, continued measurements extending beyond a year would more tightly constrain the boost invariance of the neutron [37]. The signature of this effect is an annual variation in the daily sidereal modulation of $A_X$ and $A_Y$. 

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Table 5.3: Lorentz Violation limits for the neutron, proton, and electron as determined by CPT-I and CPT-II compared with the best limits on each particle.

<table>
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<th>CPT-II (This Work)</th>
<th>CPT-I (2005)</th>
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<td>$</td>
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<td>$</td>
<td>\tilde{\beta}_e</td>
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<td>$2.8 \times 10^{-30}$ GeV</td>
<td>$1.0 \times 10^{-28}$ GeV</td>
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The nonzero contribution of the proton to the $^3$He nucleus (Eq. 5.10) allows a limit of $|\tilde{\beta}_p| < 6.0 \times 10^{-32}$ GeV. This is the most sensitive measurement for proton Lorentz violation $^{27}$. A limit on the electron Lorentz violation from this work is not nearly as stringent as the best limit (Table 5.3). However, this result constrains the electron contribution to the comagnetometer response to set clean limits on nuclear spin couplings. The proton limit can also be used in combination with the $^3$He-$^{129}$Xe maser result $^{29}$ and recent analysis of the nuclear spin content of $^{129}$Xe $^{92}$ to set an independent stringent limit on proton Lorentz violation.

There are many models involving higher order operators not considered within the SME $^{87, 93, 94}$, and limits on these parameters are also improved. Even though only spatial components of spin anisotropy in the Earth’s equatorial plane have been presented here, time-like components of Lorentz violation could also be observed since the solar system is moving relative to the rest frame of the cosmic microwave background (CMB) with a velocity $v \sim 10^{-3}c$ at a declination of $-7^\circ$. Possible anisotropy associated with the alignment of low-order CMB multipoles also points in approximately the same direction $^{95}$.

### 5.4.8 Proposed Improvements

The limit presented here could be further improved by continued measurements following the procedure developed by the end of this collection. An appropriate calibration using the low frequency $B_x$ calibration throughout the entire dataset as well as
implementation of the probe background measurements would provide lower scatter in measured sidereal amplitudes (Fig. 5.17). The 143 days of data collected over nine months have improved the existing limit by a factor of 30. Continued measurements with this scheme would improve the uncertainty by the square root of the number of days, but would likely be limited by any finite amplitudes measured.

During the completion of this work, an improved alkali-metal noble-gas comagnetometer has been developed by replacing $^3$He with $^{21}$Ne. The gyromagnetic ratio of $^{21}$Ne is an order of magnitude smaller than $^3$He resulting in an order of magnitude greater energy sensitivity for the same magnetic field background. The spin-3/2 nucleus of $^{21}$Ne also contributes a significant quadrupole moment. This leads to increased relaxation of the nuclear spins on the order of 90 min over the case of $^3$He, so a hybrid alkali optical pumping scheme has been implemented to achieve a compensation frequency of 10 Hz. The pump laser pumps an optically thin K vapor without significant pumping efficiency loss by absorption through the cell. The polarized K more uniformly polarizes an optically thick Rb vapor for more efficient pumping of the $^{21}$Ne. A probe laser near 795 nm samples the response of the Rb atoms. This K-Rb-$^{21}$Ne comagnetometer can provide at least an order of magnitude improvement in sensitivity to nuclear spin couplings. The details of this comagnetometer are beyond the scope of this work.

This K-Rb-$^{21}$Ne comagnetometer has an order of magnitude greater sensitivity to rotations as well as anomalous nuclear spin couplings. Rotational effects that can be safely neglected in this thesis will play a role in the next generation of experiments. To solve these issues, this work further proposes that the experiment be placed at the South Pole for removal of the Earth-fixed gyroscope background. At either the North or South Pole, the gravity vector and the rotation vector align for suppression of both tilt and gyroscope effects. A landmass and research station are available at the South Pole making this location more attractive.

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Placement of the experiment 100 m from the South Pole will yield 54 aT as the maximal Earth’s rotation amplitude even with the enhanced sensitivity of the K-Rb-\textsuperscript{21}Ne comagnetometer. The total integrated limit from this work is 49 aT, so Earth’s rotation effects can almost be neglected at this location. Furthermore, all of the systematic effects relating to the N-S and E-W reversals described earlier are removed. The $\Omega_z$ calibration method in Section 4.3.4 will still provide an important check of the calibration. If systematic effects in a reversal can be sufficiently controlled, it would also be possible to reverse the experiment along the celestial X- and Y-axes instead of choosing a fixed Earth-frame direction and searching for a sidereal oscillation in the response. This no longer limits new measurements to the period of one day and ultimately leads to faster averaging of the signal.

Plans for an experiment at the South Pole are currently underway. Development of the K-Rb-\textsuperscript{21}Ne comagnetometer on the rotating apparatus has recently placed a new limit on tensor Lorentz violation based on the methods of this thesis \cite{97}. Future work will include upgrades to the CPT-II apparatus for more rugged and automated operation including more sensitive tilt measurements and replacement of the platform with a more rugged air bearing. Experiments and development will continue in Princeton, NJ before shipment of the apparatus to the South Pole. The final measurement is expected to achieve energy sensitivity on the order of $10^{-36}$ GeV (1 pHz) where effects suppressed by two powers of the Planck mass such as dimension-6 Lorentz-violating operators \cite{88} can be observed.
Chapter 6

Nuclear Spin Source

Hyper-polarized $^3$He targets are a well-developed technology for both atomic and nuclear experiments. A nuclear spin source containing $9 \times 10^{21}$ hyper-polarized $^3$He spins has been constructed for tests of long-range spin-dependant forces using a K-$^3$He comagnetometer. The design of this spin source incorporates accurate polarization measurements and frequent reversals of the $^3$He spins. Optimization of the reversals by adiabatic fast passage has achieved losses of less then $2.5 \times 10^{-6}$ per flip. This is key to maintaining a high polarization under frequent reversals. Details in this chapter will be specific to the design and construction of the spin source to complement details regarding the execution of a new limit of long-range spin-dependant forces using the CPT-I comagnetometer in Refs. \[42, 43\].

6.1 Spin Source Design

For a test of a long-range spin-dependant force, a spin source is placed outside the magnetic shields of the CPT-I comagnetometer (Fig. 6.1). The spins in the spin source are reversed every 2.8 s. A correlated response on the comagnetometer will indicate the presence of a spin-spin interaction between the spins in the source and

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1Some of these details have been described in an earlier report [101].
the spins in the comagnetometer. The sensitivity of the measurement scales with the number of the spins in the source and increases with proximity of the spins in the source to those in the comagnetometer. Several spin-dependant potentials are proposed with either a $\mathbf{S}_1 \cdot \mathbf{S}_2$, $\mathbf{S}_1 \times \mathbf{S}_2$, or $3(\mathbf{S}_1 \cdot \hat{r})(\mathbf{S}_2 \cdot \hat{r}) - \mathbf{S}_1 \cdot \mathbf{S}_2$ interactions with several different inverse scalings with the distance $[42]$. An optimal spin source for this experiment has a high density of polarized spins and a compact design. This allows for a search for both $\mathbf{S}_1 \cdot \mathbf{S}_2$ and $\mathbf{S}_1 \times \mathbf{S}_2$ type interactions by reorienting the axis of the spin source.

### 6.1.1 Early Spin Source Considerations

A hyper-polarized liquid $^{129}$Xe spin source would achieve a higher density of polarized spins, but the total volume of liquid $^{129}$Xe is restricted. In addition, placement of a cryogenic spin source as close as possible to the comagnetometer shields presents obvious engineering challenges. A greater number of polarized spins can be developed...
in a high volume, high density $^3$He vapor cell. High pressure vapor cells are subject to unexpected and catastrophic failure, and a practical limit of a 20 atm gas at operating temperature has previously been observed [102]. Several key features were modeled prior to construction to determine the specifications given this constraint.

### 6.1.2 Description

Figure 6.2 provides a schematic of the hyper-polarized $^3$He spin source. A high power laser diode array bar containing 20 individual diodes provides several watts of pumping light. An external cavity with internal lenses in a Littrow configuration adjusts the wavelength to 770.1 nm and narrows the spectrum to a width of 0.1 nm where the pressure broadened linewidth in K is 0.3 nm. The spectrum is continuously monitored using an integrating sphere and spectrometer for occasional cavity adjustments. A liquid crystal waveplate allows for automated control of the handedness of the pumping light. Several cylindrical lenses and mirrors reshape the beam and deliver 2 W of pump light to the cell.

The cylindrical vapor cell is filled with 12.0 amagats of $^3$He, several mg of K, 20.4 Torr of N$_2$ and measures 12.8 cm long with an inner diameter of 4.3 cm. The cell is heated to 190° in a cylindrical G-7 oven by forced hot air. Pyrex windows sealed with Viton o-rings provide optical access along the axis, and the entire assembly is surrounded by an insulating polyamide foam. The oven also supports three axis magnetic field coils (Section 6.2.3). Several windings on top of the insulation form an anti-Helmholtz coil for control of the gradient along the axis. A mirror retroreflects pump light back onto the cell and a photodiode monitors the transmitted intensity. A fluxgate next to the oven monitors the magnetic field of the spin source. The entire assembly is mounted on an 18 × 24 inch optical breadboard for mounting near the
Figure 6.2: Schematic layout of the spin source. LDA: Laser Diode Array, Pol: Polarizer, LCW: Liquid Crystal Waveplate, FG: Fluxgate, CC: Compensation coil, EPR: EPR Coil, BS: Beam Sampler, PD: Photodiode, IS: Integrating Sphere. Image Credit: [43].

comagnetometer. A sheet metal and cardboard enclosure painted flat black prevents photons from the spin source from reaching the comagnetometer.

6.1.3 Design Considerations

The dimensions of the cell are determined by a model predicting the number of polarized spins under a variety of conditions. The model considers spin-exchange optical pumping by either K or Rb and predicts either 3 W or 60 W of laser power respectively to polarize $10^{22}$ $^3$He nuclear spins in a 200 cm$^3$ cell at a maximal pressure of 20 atm at operating temperature. A supply of several laser diode array bars each capable of supplying several watts of power at 770 nm were available in the lab, so K was selected. In hindsight, it would have been useful to consider hybrid optical pumping, as these cells achieve higher polarizations [103].

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2Early in the experiment, photons from the spin source reached an intensity feedback for the pump beam on the comagnetometer to produce a correlated response on the comagnetometer with reversals of the spin source.
The model determines the total number of polarized spins as a function of temperature, $^3$He density, laser power, and aspect ratio of a cylindrical cell. The model considers optical pumping on resonance of a pressure broadened absorption line in Eq. 2.5 and follows from the propagation model in Section 2.1.4. Spin-exchange cross sections are extracted from measured spin-exchange rate constants provided in Refs. [104, 105] to determine the rates at different temperatures (Section 2.2). The $^3$He dipole-dipole relaxation rate in Eq. 2.23 is modified to include a temperature dependence as

$$\frac{1}{T_{1dd}} = \frac{p^n}{744 \text{ amagat} \cdot \text{hour}} \sqrt{\frac{296 \text{ K}}{T}}$$

which holds to a good approximation below 550 K [55, 106].

The model also considers an observed limit to spin-exchange optical pumping cells, the $X$-factor, as a modification to Eq. 2.25

$$P_z = \langle P^a_z \rangle \frac{P^{ne}_{se}}{R_{ne}(1 + X)} + P^{sd}_{sd}$$

where $X$ depends on the surface area to volume ratio of the cell [54]. The plots in Fig. 2 of Ref. [54] provide an estimated value $X = 0.3$ for $S/V = 0.45$ of the cell used in the experiment (Section 6.1.4). Given all of these factors, the model predicts a 40% polarization ($2 \times 10^{22}$ spins) for 3 W of pumping light in a 200 cm$^3$ vapor cell at 190°C. In practice, only 20% polarization is achieved.

Alignment of the pump beam can limit the polarization in optically thick cells through a process known as skew light optical pumping. By pumping atoms along a direction misaligned with the quantization axis, the vapor is no longer transparent and absorption diminishes the propagation of light through the cell. The absorption by the alkali-metal vapor is shown to roughly double at an angle

$$\theta \sim \frac{1}{\sqrt{n_a \sigma}}$$
over a distance $l$. For our cell, $n_a = 9.5 \times 10^{13}/\text{cm}^3$, $l = 12 \text{ cm}$, and $\sigma = 3.5 \times 10^{-12} \text{ cm}^2$ predicts an angle of $9^\circ$. Given the geometry of the optical layout, the beam alignment is restricted far below this level and it is unlikely that this effect is to blame. In addition, it is strongly suspected that pumping of the $D_2$ line also limits the polarization [107]. Experienced individuals in this field recommend using much more laser power than the model indicates to achieve high polarizations [108]. Despite the factor of 2 difference between predicted and measured polarizations, $9 \times 10^{21}$ spins in a compact source that can efficiently reversed has been achieved for a test of long-range spin-dependant forces between nuclear spins.

### 6.1.4 Cell Manufacture

Three cells were hand blown from a 52 mm stock tube of Corning 1720. This glass has not been manufactured for 20-30 years and has been gathering defects while in storage. These defects can lead to catastrophic failure in a high pressure cell, so this tube is resized to freshly melt the glass and remove defects. Remelting the glass also increases the wall thickness to 4 mm for added strength. Failure of a cylinder is most likely along the axis from the hoop stress given by

$$\sigma_h = \frac{p\phi}{2t} \quad (6.4)$$

in the limit where $\phi \ll t$ where $p$ is the pressure inside the cylinder, $\phi$ is the diameter, and $t$ is the wall thickness. Sharp corners at the faces of the cylinder weaken the glass so the ends are curved outward[3]. There is little distortion of an image as viewed along the axis of the cylinder.

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[3] This is why a boiled hot dog splits along the axis, never along the radius.

[4] Curving the windows in like the bottom of a wine bottle will lead to a more concentrated force at the joint and reduced structural integrity. It also further distorts the optical quality of the windows.
The 4 mm wall thickness is selected using the practical and design limits of glass at a given pressure. Glass suppliers recommend a design limit of 0.1 inch wall thickness for a cylindrical cell with inner diameter of 1 inch containing 200 psi \[109\]. Using Eq. 6.4, this recommends a wall thickness of 7 mm for a 12 amagat (300 psi at 190°C) cell. The design limit is set an order of magnitude below the known breaking point of the glass, but the practical limit is only a factor of 2 smaller than the breaking point. The cell is maintained under ideal conditions since it is evacuated and filled with a dry inert gas where all the water has been driven off when heated. Operation of cell between the practical and design limits provides an acceptable trade off between volume and structural integrity.

Any remaining defect can cause the cell to explode, and an explosion from a cell containing 20 atm of helium at 200°C is equivalent to 250 mg of TNT. Each of the three cells are hydrostatically pressure tested prior to filling with $^3$He gas to avoid this issue. The cells are filled with and immersed in distilled water. Compressed nitrogen pressurizes the cell. The criteria that a cell is acceptable for filling is that it maintains a pressure of at least 25 atm for at least 45 minutes. This is 5 atm beyond its intended operating condition. Two cells passed, so a third was tested beyond the specification. It exploded after 45 minutes at 27 atm followed by 5 minutes at 28 atm. These cells are filled with 12.0 amagat of $^3$He using the procedure in Ref. [43], but one failed at the pulloff during initial heating.

The surviving cell has an inner diameter of 43 mm, outer diameter 52 mm, interior volume 186 cm$^3$, and length 13.6/12.5 cm with/without the curved windows \[5\] (Fig. 6.3a). This cell provides the results in all measurements in this chapter. Due to the high pressure even at room temperature, it is best considered a bomb. Great care is always used when handling the cell. Fragments from the broken cell are large

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\[5\] An average length of 12.8 cm is assigned assuming the cell is a regular cylinder.
and extremely sharp (Fig. 6.3b), so protective clothing is always worn when handling the cell (Fig. 6.3c).

6.2 Efficient Spin Reversals

Adiabatic fast passage (AFP) provides a means of efficiency reversing spin orientation by sweeping a transverse, oscillating magnetic field through the resonance determined by $\omega_0 = \gamma_n B_z$. This technique is more effective than a NMR $\pi/2$ pulse if there are inhomogeneities in the applied field [61]. A liquid crystal waveplate reverses the handedness of the circularly polarized pump light for continuous pumping in either orientation.

6.2.1 Adiabatic Fast Passage

For spins along the z-direction, an oscillating magnetic field along the transverse direction $B_y = B_{rf} \cos(\omega t)\hat{y}$ produces two counter-rotating fields at $\omega$ and $-\omega$. In a
frame co-rotating with one of the components,

$$\vec{B}_{\text{eff}} = (B_z - \frac{\omega}{\gamma_n})\hat{z} + \frac{B_{rf}}{2}\hat{y}$$  \hspace{1cm} (6.5)$$

where the effect of the counter-rotating field has been ignored. When the frequency of the oscillating field equals $\omega_0 = \gamma_n B_z$, the effective field along $\hat{z}$ vanishes and the spins precess about $\hat{y}$. An oscillating field at a frequency that sweeps through the resonance at a rate slower than the spin precession will reverse the spin orientation. For efficient AFP, Ref. [110] indicates that the following inequality should be well satisfied,

$$D_{^3\text{He}} \left| \nabla B_z \right|^2 \ll \frac{\dot{\omega}}{\gamma_n B_{rf}} \ll \gamma_n B_{rf}$$  \hspace{1cm} (6.6)$$

where the term to the left is the $^3$He gradient relaxation in Eq. 2.24 expressed in terms of the sweep at resonance. Typically $B_{rf} < B_z$, so the relaxation rate is largest at the resonance. This relaxation rate sets the criteria for the fast condition while the term on the right describes the adiabatic condition. To decrease the lower bound, substantial effort was placed in improving the homogeneity of the magnetic fields over the entire region of the cell (Section 6.2.3).

### 6.2.2 Simulation

To determine efficient AFP parameters, a numerical simulation compares different frequency sweeps. The $^3$He polarization is modeled in the absence of gradient relaxation using

$$\frac{d\vec{P}_n}{dt} = \gamma_n \vec{B} \times \vec{P}_n$$  \hspace{1cm} (6.7)$$

where $B_z = B_0$ and $B_y = A(t)B_{rf} \cos(\phi(t)). A(t)$ scales the amplitude of $B_{rf}$ and $\phi(t)$ describes the frequency sweep where several variations are considered. Efficient flips are produces from a symmetric trapezoidal amplitude function and a linear frequency
sweep. For a sweep over time $\tau$, these parameters are described by

$$A(t) = \begin{cases} 
\frac{t}{\tau_1} : & 0 \leq t \leq \tau_1 \\
1 : & \tau_1 < t < \tau - \tau_1 \\
\frac{\tau - t}{\tau_1} : & \tau - \tau_1 \leq t \leq \tau 
\end{cases}$$

$$\phi(t) = \omega_s t + \frac{1}{2} at^2 \quad (6.8)$$

where $\tau_1$ is the time over which the amplitude is ramped, $\omega_s$ is the starting frequency, and $a$ is the sweep rate. The factor of $1/2$ in $\phi(t)$ is important to accurately describe the linear frequency sweep.

In simulations, the most efficient reversals occur for $B_{rf} = B_0/2$, $\tau_1 = \tau/4$, and a linear frequency sweep between $\omega_0/2$ and $3\omega_0/2$ over 1 s. This achieves losses of $4 \times 10^{-6}$ per flip. Sweeping over a broader range quickly increases the losses to the 1% level. Adjustments to $\tau_1$ are more robust as long as $\tau_1 \ll \tau$. Similar results hold for $B_{rf} \approx B_0$. For reversals every 10 s, we expect an additional relaxation mechanism

$$\frac{1}{T_{1\text{AFP}}} \approx \frac{1}{700 \text{ hours}} \quad (6.9)$$

that can be added to Eq. 2.22. The cell is limited by the dipolar $^3$He collisions with $T_1 \approx 70$ hours, so these reversals will result only in a modest reduction in the polarization of the cell. It is expected that the implementation of such a sweep will result in lower efficiency because the model does not account for the effects of magnetic field gradient relaxation.

### 6.2.3 Magnetic Field Optimization

It is not clear based on the limit in Eq. 6.6 how uniform the magnetic field needs to be in order to achieve efficiency similar to the model. Ultimately, we attempt to create

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6For numerical simulation purposes, $A(t)$ is expressed in terms of hyperbolic tangent functions to provide continuous derivatives.
Coil Calibration Uniformity

<table>
<thead>
<tr>
<th></th>
<th>Calibration</th>
<th>Uniformity</th>
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<tbody>
<tr>
<td>$B_x$</td>
<td>0.688 gauss/A</td>
<td>-</td>
</tr>
<tr>
<td>$B_y$</td>
<td>3.61 gauss/A</td>
<td>$&lt; 10^{-2}$</td>
</tr>
<tr>
<td>$B_z$</td>
<td>4.10 gauss/A</td>
<td>$&lt; 5 \times 10^{-3}$</td>
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Table 6.1: Measured coil specifications closely match the model.

a static $B_z$ field and transverse $B_{rf}$ that are as uniform as is reasonable to achieve. The field of a finite solenoid does not meet the requirements of this application as it would be too long for a compact spin source.

There are several magnetic field optimization schemes in the literature. An attractive approach appears to be an evaluation of the magnetic field over an arbitrary region from many current carrying conductors that can be cast into a linear programming problem [111]. This approach is useful in applications where thousands of turns are used but is impractical for our compact spin source. A more appropriate approach is proposed in Refs. [112, 113] to use increasing numbers of current loops carrying the same current that are appropriately placed to cancel higher order derivatives of the magnetic field at the origin. This approach can be extended to reduce the total gradient across the spin source.

This sophisticated method was vetoed, so numerical evaluation of 28 discrete rings based on uniform spacing were adjusted to reduce the overall gradient. This forms the $B_z$ field. Uniform transverse fields based on a cylindrical geometry are produced by cosine windings defined in Appendix D of Ref. [114]. The angle of the $j^{th}$ wire in a quadrant is given by

$$\phi_j = \arcsin \left[ \frac{1}{N} \left( j - \frac{1}{2} \right) \right] \quad (6.10)$$

where $N = 7$ for the $B_y$ coil and $N = 4$ for the $B_x$ coil. The specifications of each coil are listed in Table 6.1.

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7This technique has been implemented by a local company [Twinleaf]
The coils are hand wound on grooves in a 11.5 inch long 3.190 inch outer diameter G-7 tube (Fig. 6.4). The walls are 0.25 inch thick to allow windings of all three coils at different depths from 22 gauge wire. The $B_z$ coil contains trigonal windings: two wires laid side by side with the third centered on top for each ring. Wires connecting the individual rings form a twisted pair to reduce their effect on the homogeneity. The form has two 0.5 inch holes for air intake and outtake of the forced hot air heater. Loops avoiding these holes are considered in the model. The coil form is insulated with polyamide foam and polyamide (Kapton) tape to reach an outer diameter of 5 inches and is supported by two G-10 posts.

6.2.4 Performance

The AFP sweep is produced by a National Instruments PCI-6229 multifunction card and digitally controlled by a computer. The signal is digitized at 833 kHz, and a simple low pass filter at 100 kHz smooths discontinuities. In a holding field of $B_z = 7.8$ gauss, a sweep from 12-36 kHz with $\tau = 80$ ms, $\tau_1 = 30$ ms, and $B_{rf} = 0.5$ gauss produces a repeatable reversal with losses less than $2.5 \times 10^{-6}$ per flip. This is evaluated by measuring the losses over 3000 consecutive reversals. The amplitude $B_{rf}$ is limited by the Pasco Scientific 9587C amplifier and the inductance of the $B_y$ coil. The high efficiency of the AFP is attributed to the uniformity of the magnetic fields. Despite close placement to the magnetic shields of the comagnetometer where magnetic flux is concentrated, the reversals still maintain a high efficiency without significant improvement from external gradient compensation. This result surpasses a loss rate of $2 \times 10^{-5}$ per flip reported in Ref. [115] by an order of magnitude.
Figure 6.4: a) Magnetic field coils neatly wound on the G-7 coil form. b) Oven with supporting posts. A groove at each end supports circular pyrex windows (not AR coated) and seals with a Viton o-ring. c) Three dimensional rendering of the coil form design in SolidWorks.
6.3 $^3$He Polarimetry

The strength of the anomalous interaction is proportional to the number of polarized spins in the source. An accurate measurement of the $^3$He polarization can be implemented using a polarization-dependant frequency shift in the alkali Zeeman resonance as described in Ref. [110]. Effectively, the K atoms are used as a magnetometer to measure the magnetization of $^3$He in real-time.

6.3.1 Electron Parametric Resonance

The $4S_{1/2}$ ground state splits into two hyperfine manifolds, $F = 1$ and $F = 2$. The energy of each of the $m_F$ levels in the $F = I \pm \frac{1}{2}$ state in a magnetic field of magnitude $B$ is described by the Breit-Rabi equation [44]

$$E(B, m_F) = -\frac{\hbar \nu_{HFS}}{2(2I + 1)} - g_I \mu_n B m_F \pm \frac{\hbar \nu_{HFS}}{2} \sqrt{1 + \frac{4m_F x_B}{2I + 1} + x_B^2}$$

(6.11)

where $x_B = g_I \mu_B B / (\hbar \nu_{HFS})$, $\nu_{HFS} = 461.7$ MHz, $g_I = 0.3914/I$ [16], and $\mu_n$ is the nuclear magneton. These levels are pictured in Fig 2.6.

Circularly polarized photons pump the atoms to the end states with maximum projection of angular momentum, $|F = 2, m_F = \pm 2\rangle$. An RF magnetic field depopulates the end state by driving atoms to the nearby $|F = 2, m_F = \pm 1\rangle$ Zeeman sublevel. These atoms are able to absorb photons which results in a dip in the transmission of the pump light through the cell (Fig. 6.5). Exciting this transition is called electron parametric resonance (EPR).

6.3.2 Polarization Measurement

An AFP flip reverses the magnetization of $^3$He and the local magnetic field experienced by the K atoms. This corresponds to a shift in the EPR resonance of twice the
magnetization. The frequency shift is related to the polarization $P$ as

$$\Delta \nu = \frac{8\pi}{3} \frac{d\nu(F, m_F, \pm)}{dB} \kappa_{\text{He}} n_b P \quad (6.12)$$

where $\nu(F, m_F, \pm)$ is the frequency of the EPR transition $|F, m_F\rangle \rightarrow |F, m_F \pm 1\rangle$ [170]. Due to the cylindrical geometry of the cell, the enhancement factor is modified to

$$\kappa = \kappa_0 + \frac{1}{2} - \frac{3}{2} \left( 1 + \frac{a}{l} - \sqrt{1 + \left(\frac{a}{l}\right)^2} \right) \quad (6.13)$$

where $a$ is the radius and $l$ is the length of the cylindrical shell as reported in Ref. [104]. $\kappa_0$ is as listed in Eq. 2.54 and produces a value of $\kappa = 6.09$ for our cell geometry at 190°C.

The nonlinear part of the Breit-Rabi splitting implies that the EPR frequency will be different depending on the handedness of the pump light. Figure 6.5b illustrates the measured frequency difference as a function of magnetic field. The curve is determined by Eq. 6.11 with no free parameters. Typical shifts from the $^3$He are 50 kHz, so this is a significant effect. In a field of 7.68 gauss with a polarization of 100%, shifts of

Figure 6.5: a) Transmission dip from a frequency sweep across the EPR resonance. b) Difference in EPR resonance for opposite circular polarizations of the pump light as a function of magnetic field.
Figure 6.6: Three complete reversals of the $^3\text{He}$ without reversing the handedness of the pump light easily determines the polarization value. This is followed by several reversals of both the $^3\text{He}$ and the pump handedness. The kink in the leading edge is from an imperfect shift by the analog boost. This illustration is completed with a repetition of the starting reversals.

$127.9\ \text{kHz}$ or $119.4\ \text{kHz}$ are expected for spins in the $m_F = -2$ or $m_F = +2$ state. In the case of Ref. [110], the nonlinearity is only a small systematic effect where the hyperfine splitting in Rb is $\nu_{\text{HFS}}^{\text{Rb}} = 1012\ \text{MHz}$.

A single rectangular loop of wire $5\ \text{cm}$ wide extends along the length of the cell. This is driven by a voltage controlled oscillator on a Wavetek 166 function generator. A lock-in amplifier applies a $0.7\ \text{kHz}$ modulation at $250\ \text{Hz}$ to the EPR frequency. The output of the lock-in creates a derivative signal for feedback from a simple analog PID loop. A relay shorts the integral feedback during a reversal of the spins coordinated with a reversal of the handedness of the pump light. The opposite Zeeman transition is separated by several hundred kHz, so an analog signal provides a boost of the dc feedback output to reduce the distance to the peak (Fig. 6.6). The loop quickly re-engages to follow the EPR frequency. The frequency of the EPR is then monitored by a counter on the PCI-6229 card.
6.4 Magnetic Field Compensation

The magnetic dipole interaction provides a known coupling between the spins in the spin source and the comagnetometer. A 20% polarization produces a local field of 6 mgauss. An estimated shielding of $10^3$ from the comagnetometer and $10^5$ from the shields reduces the strength of the interaction. Given the 48.7 cm distance between the spin source and the comagnetometer cell, the interaction is estimated at 0.05 aT. The spin source is designed to be surrounded by a $\mu$-metal magnetic shield to suppress the magnetic field; however, a solenoid is placed around the cell to provide active cancelation of the magnetic field during reversals (Fig. 6.3a).

6.4.1 Compensation Coil

A continuous solenoid of 28 gauge wire extends the length of the cell. The magnetic field of the coil mimics the surface current of a uniformly magnetized object [116]. The solenoid closely approximates the field despite the 4 mm thick cell walls and the curved face of the cell. Shim coils were considered to compensate for these differences but were not implemented.

6.4.2 EPR Consideration

During the experiment, compensation of the $^3$He magnetization contributes to the field experienced by the K vapor. This compensation field does not cancel the EPR frequency shifts since the enhancement factor $\kappa \neq 1$. This is demonstrated in Fig. 6.7. For effective polarization measurements, the handedness of the pump light as well as the current in the compensation coil are maintained constant under AFP reversals. The known field applied by the compensation coil is accounted for in Eq. 6.11 in the EPR measurement.
Figure 6.7: Measured EPR frequency over several AFP flips without a reversal of the pump polarization. After 40 s, the magnetic field compensation is engaged and later disengaged after 110 s. Due to the enhancement factor $\kappa \neq 1$, there is a nonzero EPR shift during a reversal of both the $^3$He and the coil current. The spikes are an artifact of shorting the capacitor for the integral feedback of the PID.

6.4.3 Performance

A measurement of the polarization described in Section 6.3.1 determines the appropriate current to apply to the coil with a measured calibration of 30.1 mgauss/mA. This reduces the correlated magnetic field by a factor of 10 as monitored by a fluxgate placed near the cell. The coil is also useful in determining the magnetic field leakage. Reversals of a large current through the coil mimics the uncompensated field of the spin source. A correlation with a magnetic field and the comagnetometer limits the magnetic field leakage to $4 \times 10^{-3}$ aT.

6.5 Spin Source Operation

Operation of the spin source follows the same scheme as reversals of the CPT-II apparatus described in Chapter 5. Historically, this implementation inspired the control sequence in CPT-II. The spin source and the comagnetometer are electrically isolated and powered by separate lab ac phases. A separate computer handles all of the data.
acquisition and control of the spin source. Most of the electronics and computer control of the spin source reside in the next room with several BNC and an ac line connections passing through a hole in the wall. The two computers are synchronized using Dimension 4 software and a navy timeserver connection. A trigger from the comagnetometer to begin flipping sequences is disconnected electrically through an optocoupled signal.

6.5.1 Flipping Sequence

The spins and compensation coil current are reversed every 2.8 s with synchronous control of the liquid crystal waveplate and feedback boost. The fluxgate and EPR frequency are continuously monitored and recorded to a file. The spin source also maintains the state of the spins by switching binary values saved in a file indicating the spin state. This is particularly useful if there is a power failure as the last value will always be saved.

A typical reversal proceeds as follows: The AFP parameters are loaded and the waveform is calculated. A digital channel controls the relay to short the capacitor on the integral feedback of the PID. The AFP waveform is generated while analog levels of the feedback boost and compensating coil are smoothly transitioned at resonance. A digital channel switches the state of the liquid crystal waveplate at resonance and is also read on an analog input as a marker of the state in the files. At the end of the AFP waveform, the capacitor is unshorted for the PID to reacquire the EPR peak. In addition, a digital channel marks the time during which the AFP is executed and recorded on an analog channel as a marker of the flip.

After a 200 s record with over 60 reversals, the comagnetometer pauses to perform zeroing routines. During this time, the $^3$He spins are reversed without reversals of other parameters to perform a polarization measurement. Based on the measured polarization, the current for the compensation coil is determined. Occasionally, the
feedback will fail to lock to the EPR peak potentially disrupting an entire record with the wrong current to the compensation coil. To account for this, the previous ten polarization values are saved and compared against the most recent value. If there is a discrepancy by more than 2.5 standard deviations of the recent value and the mean of the previous values, the recent value is replaced by the mean. This has led to robust operation with few failures.

6.5.2 Long-Range Spin-Dependant Forces Search

A brief summary of the results of the long-range spin-dependent forces search will be summarized here. Familiar techniques using the 3-point overlapping string analysis provide the reversal correlated amplitude of the comagnetometer signal with the orientation of the spin source. Specific details regarding the CPT-I comagnetometer conditions and the analysis are already well-explained in Refs. [42, 43] and will not be repeated.

Data collected over a month sets a limit on an anomalous spin-dependent force of $0.05 \pm 0.56 \text{ aT}$ with a reduced $\chi^2$ of 0.87. This improves upon the previous limit by a factor of 500 [98]. In addition, it is equivalent to measuring a frequency shift in the $^3\text{He}$ precession of less than 18 pHz and represents the highest energy resolution of any experiment. Systematic effects are checked by collecting several runs with the spins in the spin source and the comagnetometer reversed with respect to the electronic levels of each system. These collections give consistent results (Fig. 6.8). The measurements within each run form a gaussian distribution. In addition, correlated amplitudes are also binned by sidereal day to set limits on Lorentz-violating spin potentials. The structure of $^3\text{He}$ in Eq. 5.10 gives the sensitivity of the measurement to both the neutron and the proton.

Several different spin-dependant potentials correspond to the existence of newly proposed particles. For example, the coupling $g_p$ of a pseudoscalar boson $\phi$ with mass
Figure 6.8: Spin-correlated measurement of $\beta_y^n$ for the spin source aligned in the y-direction. Each point represents the average correlation over approximately one day. Up and down triangles indicate the orientation of the spin source while the filled and empty triangles indicate the orientation of the comagnetometer. Top Inset: Within each run, the correlation from 200 s records follows a gaussian distribution. Bottom Inset: Correlations plotted and averaged modulo one sidereal day shows no significant variation. Image Credit [42].

$m$ to a fermion $\psi$ with mass $M_n$ can occur from either a Yukawa or derivative form

$$\mathcal{L}^{Yuk} = -i g_p \bar{\psi} \gamma^5 \psi \phi \quad \text{or} \quad \mathcal{L}^{Der} = \frac{g_p}{2M_n} \bar{\psi} \gamma_\mu \gamma^5 \psi \partial^\mu \phi \quad (6.14)$$

where both terms lead to the same single boson exchange potential [117]

$$V(\hat{r}, \vec{S}_1, \vec{S}_2) = \frac{g_p^2}{16 \pi M_n^2} \left[ \vec{S}_1 \cdot \vec{S}_2 \left( \frac{m^2}{r^2} + \frac{1}{r^3} \right) - (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) \left( \frac{m^2}{r^2} + \frac{3m}{r^2} + \frac{3}{r^3} \right) \right] e^{-mr} \quad (6.15)$$

where $r$ represents the distance between the spins with $\hbar = c = 1$. This particular potential was originally derived in the context of one-pion exchange where the mass of the pion is much less than the mass of other more strongly interacting particles [118] and reduces to the familiar dipole-dipole potential for $m = 0$. In addition, the dynamics of a Goldstone boson $\pi$ associated with spontaneous breaking of Lorentz
symmetry have been suggested \[119\]. This boson would couple to fermions as

\[
\mathcal{L} = \frac{1}{F} \bar{\psi} \gamma^\mu \gamma^5 \psi \partial_\mu \pi + \frac{M_\pi^2}{F} \bar{\psi} \gamma^0 \gamma^5 \psi
\]

(6.16)

to leading order and have an unusual quadratic dispersion relationship \(\omega = k^2/M_\pi\).

The first term leads to a \(1/r\) potential and the second term leads to a \(\vec{S} \cdot \vec{v}\) interaction where \(\vec{v}\) is the velocity with respect to a preferred frame. The velocity-dependant interaction is constrained by considering a sidereal variation in the spin-spin interaction. Further discussion of these potentials and others are left to Refs. \[42, 43\].

### 6.5.3 Future Improvements

The spin-dependant forces measurement can be improved through the increased energy sensitivity of a K-Rb-\(^{21}\)Ne comagnetometer \[97\]. Likewise, the inverse distance scaling of the potentials means that the smaller shields of the CPT-II comagnetometer lead to an additional improvement in sensitivity. The existing spin source would need to be modified to operate in a vacuum. A likely first step would be a replacement of the forced hot air heater with a twisted wire resistive heating scheme. The external cavity laser would likely need to be outside the bell jar to allow for occasional adjustments to the cavity. In addition, it is believed that the \(^3\)He polarization is limited by the laser power, so additional lasers will likely lead to continued improvements in the spin source.
Chapter 7

Conclusion

This thesis sets new limits on a Lorentz- and CPT-violating field coupling to the spin of the neutron. The equatorial components of the proposed Lorentz-violating field are measured to be $\tilde{b}_n^X = (0.1 \pm 1.6) \times 10^{-33}$ GeV and $\tilde{b}_n^Y = (2.5 \pm 1.6) \times 10^{-33}$ GeV. This can be interpreted as a limit on $|\tilde{b}_n^\perp| < 3.7 \times 10^{-33}$ GeV, improving the previous bound by a factor of 30. This represents the highest energy resolution of any spin anisotropy measurement.

This measurement was accomplished through the use of a K-\textsuperscript{3}He comagnetometer on a rotating platform for lock-in modulation of the Lorentz- and CPT-violating field on a timescale much shorter than a sidereal day. The rotating platform introduces several background effects that have been investigated in this work. New and more robust calibration schemes have been presented along with observation of novel features of the K-\textsuperscript{3}He comagnetometer. Several improvements to the experiment are proposed, most notably placement at the South Pole for a suppression of Earth’s rotation as a fixed background signal.

Furthermore, a hyper-polarized \textsuperscript{3}He nuclear spin source was designed, constructed, and used in combination with the K-\textsuperscript{3}He comagnetometer to perform a search for long-range spin-spin interaction from newly proposed particles. The spin source achieves
$9 \times 10^{21}$ polarized nuclear spins that can be reversed frequently with losses below $2.5 \times 10^{-6}$ per flip. This search improves previous bounds by a factor of 500 and achieves an energy sensitivity of 18 pHz. This is the highest energy resolution of any experiment.

Each of these experiments will benefit from the improved energy sensitivity of the newly developed K-Rb-$^{21}$Ne comagnetometer [97]. The alkali-metal noble-gas comagnetometer has achieved energy sensitivity of $10^{-34}$ GeV [12], and future improvements are expected to reach the level of $10^{-36}$ GeV where effects suppressed by two orders of the Planck mass can be explored.
Appendix A

Time Conventions

Lorentz violation searches present measurements in sidereal time [17]. The length of a sidereal day is defined as the time it takes a point on the earth to rotate 360° with respect to the celestial frame. A sidereal day contains 86164.09 s and differs from the 86400 s of a diurnal day from the combination of orbital and rotational motion of the Earth with respect to an inertial frame. As a convention, all results are reported in local sidereal time (LST).

The algorithm to calculate the sidereal time has already been implemented in the first generation experiment and described in Ref. [28]. The calculation converts the computer Unix time to solar days from January 1, 2000. These solar days are then converted to sidereal days since January 1, 2000, to match the convention. These times give the Greenwich Mean Sidereal Time (GMST) and are the values of “Sidereal Day” reported in this thesis. The GMST values have been verified with a known astronomy timeserver, and the rest of the calculation is confirmed to be correct

GMST refers to the position of Greenwich, England, while the experiment occurs in Princeton, NJ. The data is analyzed in LST to account for this difference. LST

\footnote{Automatic updates to the local time on the computer from daylight savings time will cause a phase shift in the recorded times, so this feature is disabled in all computers involved in experiments.}
Figure A.1: a) Transformation of Celestial Frame to Lab Frame as given in [17]. b) Celestial Coordinate System. Image Credit: [28].

and GMST differ by a correction in the longitude as

$$LST = GMST + \lambda/360 \tag{A.1}$$

where $\lambda$ is longitude of the observer on Earth. Princeton, NJ is at $74.652^\circ$ W longitude, so this correction is $-0.207$.

Measurements are reported in the sun-centered celestial equatorial frame (XYZ) that is approximately inertial over thousands of years [17]. Axes are defined by convention that is well established in the field of astronomy [120]. This coordinate system assigns $\hat{X}$ to have a declination and right ascension of $0^\circ$ in the equatorial plane of the Earth (Fig. A.1b). $\hat{Y}$ has a declination of $0^\circ$ and right ascension of $90^\circ$. $\hat{Z}$ is oriented along the Earth’s rotation axis, and these directions form an orthogonal basis. For an observer at $0^\circ$ longitude at the equator, the $\hat{X}$ direction is directly overhead on an integer sidereal day.
In terms of the SME, the authors of Ref. [17] define the following coordinate transformation\(^2\)

\[
\begin{pmatrix}
\hat{n}_x \\
\hat{n}_y \\
\hat{n}_z 
\end{pmatrix} =
\begin{pmatrix}
\cos \chi \cos \Omega t & \cos \chi \sin \Omega t & -\sin \chi \\
\sin \Omega t & \cos \Omega t & 0 \\
\sin \chi \cos \Omega t & \sin \chi \sin \Omega t & \cos \chi 
\end{pmatrix}
\begin{pmatrix}
\hat{X} \\
\hat{Y} \\
\hat{Z} 
\end{pmatrix}
\tag{A.2}
\]

as seen in Fig. A.1a. Within this convention, the quantization axis is along \(\hat{n}_z\). The rotating comagnetometer is sensitive to fields interacting along its \(y\)-axis in the horizontal plane. This axis can also be reoriented by rotation of the apparatus. We define the following transformation to further rotate our coordinate system.

\[
\begin{pmatrix}
\hat{z} \\
\hat{x} \\
\hat{y} 
\end{pmatrix} =
\begin{pmatrix}
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1 \\
\cos \theta & \sin \theta & 0 
\end{pmatrix}
\begin{pmatrix}
\hat{n}_x \\
\hat{n}_y \\
\hat{n}_z 
\end{pmatrix}
\tag{A.3}
\]

where \(\theta\) is the angle of rotation lock-in amplifier from the south position as defined in Appendix B. Under such a rotation, sensitivity to the Lorentz- and \(CPT\)-violating field \(\tilde{b}\) becomes

\[
\hat{y} = \tilde{b}_X (\cos \theta \cos \chi \cos \Omega t - \sin \theta \sin \Omega t) + \tilde{b}_Y (\cos \theta \cos \chi \sin \Omega t + \sin \theta \cos \Omega t) - \tilde{b}_Z \cos \theta \sin \chi.
\tag{A.4}
\]

Rotations of \(\theta\) determine the sensitivity of the reversal correlated amplitude for reversals along the N-S and E-W axes.

\(^2\)Note that \(\chi\) is used as the latitude elsewhere in the thesis where here \(\chi - \pi/2\) determines the latitude.
Appendix B

Local Coordinate System

The CPT-II comagnetometer uses a rotating platform for measurement of possible Lorentz-violating fields on timescales shorter than one sidereal day. The sensitive y-axis of the magnetometer is oriented horizontally such that rotations of the platform about the vertical z-axis reorient this direction with respect to the celestial frame. The absolute orientation of the apparatus is a factor in determining the projection of the measurement within the SME, as it was in the CPT-I experiment. However, reorientation of the apparatus with respect to Earth’s rotation axis provides a significant Earth-fixed background response. Measurements of spatial anisotropy rely on relative changes in this signal; however, a correct measurement of the gyroscope signal provides an important check of systematic effects on our signal. Here, we discuss the determination of the location of CPT-II with respect to Earth’s rotation axis and possible variations.

B.1 Laboratory Location

The comagnetometer is located at 40° 20′ 43″ N (40.345° N) and 74° 39′ 7″ W (74.652° W) in the basement of Jadwin Hall at Princeton University in Princeton, NJ. Since the y-direction of the CPT-II comagnetometer is mounted horizontally, the
Figure B.1: Projection of the sensitive y-axis along Earth’s rotation axis provides the magnitude of response.

The magnitude of Earth’s rotation response is given by the overlap of the y-direction and Earth’s rotation axis (Fig. B.1). From Eq. 3.43, we expect the response to be

$$\frac{\Omega_\oplus}{\gamma_n} \left(1 - \frac{Q\gamma_n}{\gamma_e}\right) \cos \chi \simeq 271 \text{ fT}$$

in effective magnetic units where $\Omega_\oplus = \frac{2\pi}{(86164.09 \text{ s})}$ for one sidereal day. This is the amplitude of a sine wave with clear maximum along the N-S axis of the lab (Fig. 3.16).

Early measurements showed the phase shifted 20-30° W and the response amplitudes 20-30% larger than predicted. A systematic study of the location and orientation of the lab was initiated to determine if these were real effects. Ultimately, this observation would result in identification of the probe beam background effects (Section 4.1), rotational sensitivity to the x-direction from nonzero $B_z$ and $L_z$ fields (Section 4.4), and an overly precise determination in the location of the laboratory.
B.1.1 Jadwin Hall Exterior

An orthorectified aerial photograph (Fig. B.2) provides a useful measurement of Jadwin Hall with respect to latitude and longitude. This image was taken as part of the New Jersey state aerial survey conducted between February-April 2002. Image G10A12 was taken from an altitude of 9600 feet above mean terrain and combined with simultaneous GPS data to determine its geospatial location. The image has been corrected from original photo negative to account for camera lens distortion, vertical displacement, film shrinkage, atmospheric refraction, and variations in aircraft altitude and orientation. Figure B.2 has an accuracy of ±4.0 feet at the 95% confidence level. More details are available in the extensive metadata from the survey.

The latitude and longitude in Fig. B.2 are displayed in the North American Datum of 1983 (NAD 83) New Jersey State Plane coordinate system. Because the Earth is not a sphere, cartographers use a best fit ellipsoid. The NAD 83 geodetic reference system is a datum, a system of reference, optimized for accurate representation of North America and also serves as the legal horizontal coordinate system for the United States government. The origin of the ellipsoid is 2 m from the center of the Earth and considered stable such that North America should not experience horizontal motion with respect to NAD 83 coordinates. The State Plane coordinate system is a developable surface, meaning that the curved surface is not stretched or torn when represented on a flat plane. The New Jersey State plane uses the surface of a cylinder with the axis along a central longitude line to project the state onto a flat plane. The distortions are largest to the east and west far from the center longitude. Despite these effects, the convergence of the longitude lines in New Jersey State Plane Coordinates is in good agreement with the rotation axis of the Earth and can be confirmed using a standard geographic information system (GIS) software tool, ArcGIS.

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1Orthorectified images from state mapping surveys are freely available through the Geographic Information Systems (GIS) section of the New Jersey Department of Environmental Protection (NJDEP) website.
Figure B.2: Aerial image of Jadwin Hall orthorectified to NAD83 State plane coordinates (2002).
A line drawn along the western edge of Jadwin Hall in Fig. B.2 determines a heading of 29.5° W. Furthermore, a check of the latitude and longitude of two points along this edge in ArcGIS gives a heading\(^1\) of 29.7° W. A similar result is achieved for confirmation using Google Maps\(^\text{TM}\) aerial photo. Google uses images set in the World Geodetic System of 1984 (WGS 84) coordinate system. There are slight differences between WGS 84 and NAD 83 in latitude and longitude, but the heading remains the same to within our measurement precision. WGS 84 was developed by the US Department of Defense specifically for global coverage rather than focusing on North America.

We considered a commercial Global Positioning System (GPS) measurement along the outside of the building. This type of measurement is well suited for measurements where a clean line of sight is simultaneously available to at least four satellites. The GPS system relies on precise timing of the microwave signal between the receiver and satellites to accurately determine the distance. Any reflection off the building or construction equipment generally littering the outside of area could bias any attempted measurement. Also, an accurate determination of the heading depends on a long baseline which is generally unavailable due to construction equipment surrounding the building.

To circumvent these issues, we asked the Princeton University Facilities Department to independently investigate the orientation of Jadwin Hall (Fig. B.3). Using known benchmarks around the campus, Facilities Engineers surveyed the western edge of the building using a theodolite. The hash marks indicate points that were surveyed. This method determines an orientation of 30.35° W of Grid North. Grid North is subtly different from True North\(^2\). True North is towards the North Pole as determined by the Earth’s rotation axis. Grid North is determined by the Universal

\(^2\)One should be careful to determine the distances on the surface of the sphere since latitude and longitude are a measure of angle not distance.

\(^3\)There is a third “North” of interest, Magnetic North. See section B.1.4.
Figure B.3: Princeton University Facilities Survey of Jadwin Hall. The hash marks indicate survey points. Grid North in this area is 12’ E of True North where we identify the North Pole.
Transverse Mercator (UTM) grid. In the vicinity of Princeton, NJ, Grid North is 12’ E of True North implying that survey indicates that Jadwin lies 30.15° E of True North. The 0.6° discrepancy with the aerial photos is smaller than uncertainties with construction of the CPT-II equipment rack.

B.1.2 Jadwin Hall Interior

Figure B.4 shows the basement floorplan of Jawdin Hall where the experiment is located in room B17. The north arrow in the lower right gives a heading of 35° in disagreement with the aerial photos. It is impossible to determine how the civil engineer determined this angle. For a map such as this, it is a common practice for surveyors to give only a general sense of a northerly direction. Also, surveyors will often lay out a “Local North” arbitrarily for the job site. Therefore, we reject this arrow for any precise determination of direction. We assume that the interior walls of Jadwin are parallel to the exterior walls. We could request a survey of the lab with reference to a known outdoor benchmark much like surveying an underground mine. We lack motivation to embark on this project and choose to assume parallel interior and exterior walls.

B.1.3 Comagnetometer Orientation

The axes of the comagnetometer are determined by the laser light fields. The sensitive direction is a vector mutually orthogonal to propagation of the pump and probe beams. One could determine this direction by removing optics and projecting the probe beam across the room. This is most useful after optimization of comagnetometer noise before taking data; however, it is at this point that the experimenter is most loath to remove any optics. The propagation direction of the pump and probe beams are constrained by the placement of the vacuum tubes entering the shields. It is difficult to precisely measure the propagation direction, though centering the
Figure B.4: Floor plan of the basement of Jadwin Hall at Princeton University. The experiment is located in room B17.
beam to within 1 mm on both the windows gives alignment within 0.2°. This is much smaller than the absolute uncertainty in the placement of the windows with respect to the optical breadboard, the orientation of the floating vibration isolation platform, and the construction of the equipment rack. The absolute orientation of all of these pieces can be determined with progressively smaller uncertainties for increasing measurement effort. However, the experiments described in this thesis rely on relative changes with respect to the comagnetometer, so these measurements were not pursued.

In addition to the alignment of the light fields with the frame, it is likely the magnetic field coils are not precisely lined up with the with the rest of the apparatus. Mixing of field orientations with respect to the comagnetometer axes can cause small offsets in the various zeroing routines as a modulation of $B_y$ can become a modulation of $B_y + \theta B_x$ where $\theta$ is the misalignment angle. Our control software allows for mixing of one magnetic field basis into the light-field basis of up to a 5° misalignment. We have not found a significant discrepancy to warrant an investigation of these effects.

### B.1.4 Magnetic North

Magnetic North is irrelevant in determining the response of the magnetometer. The comagnetometer inherits its sensitivity from a SERF magnetometer, but is not sensitive to magnetic fields. The self-compensation and magnetic shields reduce the influence of Earth’s magnetic field to 0.5 fT. Furthermore, the large Helmholtz coils surrounding the experiment further suppress Earth’s magnetic field two orders of magnitude. The magnetic North Pole lies in northern Canada and is distinct from the Earth’s rotation axis. The magnetic declination, the difference between True North and Magnetic North, in Princeton, New Jersey is 12.6° W of True North in 2010. Due to magnetic pole drift, it drifts at 1’ E per year.
Figure B.5: Overhead view of the CPT-II apparatus in room B17.
B.2 Coordinate Frame Variations

This discussion so far has determined the spatial relationship between the experiment and a fixed axis. It is interesting to consider the changes in Earth’s rotation vector because it is known to vary. There are three degrees of freedom; the axial component corresponds to the rate of rotation while the transverse components correspond to polar motion. Most civil works and military construction projects do not require reference to polar drift or tectonic plate motion [121], therefore, we expect these effects to be small. These variations are not captured in the NAD 83 datum referenced in the preceding sections, but are included in the International Terrestrial Reference Frame (ITRF). The origin of the ITRF is at the center of the Earth and it is continuously updated by Very Long Baseline Interferometry (VLBI) measurements. Several intense VLBI campaigns have captured high resolution changes to the Earth’s rotation vector [122], so these parameters are well-known.

B.2.1 Polar Motion

Polar motion is often reported in units of milliarcseconds (mas) of wobble. The comagnetometer is most sensitive to polar motion when oriented along the E-W axis. For a 1 aT response of the comagnetometer, the required polar motion would be 1000 mas or a 30 m shift of the pole. The largest measured oscillations are at the level of 100 mas (Fig. B.6). These are a combination of a forced annual wobble and the Chandler Wobble [123, 124]. Any irregularly shaped solid body rotating about an axis misaligned with its figure axis, axis of symmetry, will exhibit Chandler Wobble. The Earth has a measured Chandler Wobble period of 433 days that is continuously driven by a combination of atmospheric and oceanic pressure fluctuations. The tides produce diurnal and semidiurnal variations in the polar motion on a timescale relevant to Lorentz violation searches; however, the largest of these are 0.5 milliarcseconds
peak-peak [125]. The long timescales and small scale of polar motion preclude any relevant impact on the comagnetometer signal.

### B.2.2 Length of Day

Regular revolutions of the Earth have long been used as a means of keeping time referred to as Universal Time (UT). A correction to account for polar motion (Sec-
tion \( B.2.1 \) gives the standard UT1 found in the literature \[125\]. The variations in UT1 are monitored by comparing rotations of the Earth against Ephemeris Time following the motion of the solar system or Atomic Time kept by atomic clocks. It seems confusing, but the literature will sometimes report rotation rate changes in the length-of-day (LOD). For a 1 aT change in the comagnetometer signal, the LOD would have to change by 0.3 s. The largest observed LOD changes are from El Niño and the Southern Oscillation (ENSO) (Fig. \( B.6 \)). These changes have a magnitude 500 \( \mu \)s and occur on timescales of years \[126\]. The tides also influence the rotation of the Earth through conservation of angular momentum of the total Earth-ocean system \[127\]. Variations in the length of day from the tides are on the order of 100 \( \mu \)s peak-peak \[125\].

B.2.3 Earthquakes

On February 27, 2010, a magnitude 8.8 earthquake struck in Chile while the experiment was running on sidereal day 3721. This event was large enough to modify both Earth’s rotation rate and its axis of rotation. This is a consequence of conservation of angular momentum under a modification of Earth’s inertia tensor. Based on the 1.26 \( \mu \)s change in the period of one day and the 2.7 milliarcsecond shift in the rotation axis reported in the Jet Propulsion Laboratory press release \[128\], we would require sensitivity of 3 zT/\( \sqrt{\text{Hz}} \) to observe this effect. It is no surprise that no change in our signal was observed.

B.2.4 Comment

Variations in Earth’s rotation do not influence the state of the art comagnetometer performance and are not expected to in the near future. The comagnetometer’s rotational sensitivity extends to an inertial frame, so it is interesting to consider rotations beyond the Earth? In defining Earth’s rotation rate as the inverse of a
sidereal day, we already account for the Earth’s rotation about the Sun. But what about the Sun’s rotation about our galaxy? This has a period of approximately 225 million terrestrial years \([129]\) called a cosmic year, an exceptionally small rotation. Larger scale, more sensitive ring laser gyroscopes have just begun to observe polar motion from tides \([130]\), and can safely constrain other possible rotations from an inertial frame. It is interesting to note that the observed “tilt-over mode” \((K_1)\) in Earth’s polar motion has a period of precisely one sidereal day. If the comagnetometer ever reaches this level of sensitivity, the tilt-over mode will be major systematic in future Lorentz violation searches.

B.3 Rotation Conventions

Positive rotations of the platform correspond to “spin up” under the right hand rule convention. Almost all of the data has been taken with rotations in the positive direction. We identify the “home” or zero degree position for the rotary encoder when the edge of the platform holding the vertically mounted SRS lockin amplifier is lined up by eye with the baseplate (Fig. B.5). The lock-in amplifier is a convenient reference to a side of the apparatus carrying the y-axis of the comagnetometer. We assume that the baseplate is perpendicular to the leg facing the command center. The angle of this leg can be consistently measured with respect to the floor tiles. The floor tiles seem to be reasonably well aligned with the interior walls of room B17. The starting south position is typically a \(+ (16 - 18)^\circ\) rotation from the home position.

Over several weeks, the legs tend to rotate \(\sim 1^\circ\). This is because the entire apparatus rests on the floor. We have chosen not to secure the legs to the foundation of the building. In the event of an emergency stop, the shaft locks, and we have observed the entire apparatus to spin up to \(10^\circ\) from a hard stop. If the equipment were secured to the floor, this energy would be absorbed by the gears and shaft,
possibly damaging a critical piece of the rotation mechanism. We choose instead to keep the area around the experiment clear of debris and dissipate the energy through friction with the floor.
Appendix C

Comagnetometer Toy Model

There is a disconnect between the descriptive picture presented in Section 3.1.1 and the steady state expansion of the Bloch equations presented in Section 3.2. It is not immediately clear that the two are related. This appendix calculates the response of an alkali magnetometer to an applied magnetic field if a compensating spin species is present to cancel this field. This is a toy model treatment of the comagnetometer behavior. While this treatment is heuristic, it is helpful to visualize how the nuclear spins compensate for magnetic field changes from the geometry. Hopefully, this presentation will provide intuition to the reader as well as a meaningful insight to the question “Why is this term in the Bloch equations solution?”.

C.1 Steady State Compensation

The geometrical approach to the comagnetometer behavior ignores spin-exchange effects, pumping rates, and the finite magnetization of the electron spins. The treatment in Section 3.2 can quantify these effects but requires a tour de force of algebraic manipulation. The following treatment will not capture all of the fine details, but provides a simple road to the general structure of the solution.
Figure C.1: The $^3$He follows the total field around the compensation point. This leaves the uncompensated fields $\Delta B_z$ and $\Delta B_\perp$.

C.1.1 Derivation

We decouple the electron and nuclear spins to consider an alkali-metal magnetometer superimposed on top of the hyper-polarized noble-gas and enforce that the noble-gas spins follow an adiabatic shift in the total field. We consider the response from the reaction of the magnetometer to an incomplete cancelation of the applied field. The magnetometer response to an applied fields is given by Eq. 2.48 and we find it convenient to regroup the constants such that

$$P^e_x = \frac{P^e_z \gamma_e}{R_{tot}} \left[ \frac{B_y + \gamma_e R_{tot} B_x B_z}{1 + (\gamma_e R_{tot})^2 (B_x^2 + B_y^2 + B_z^2)} \right]. \quad (C.1)$$

Here, we have the framework for the numerator and denominator described earlier where we can expand the denominator for $B_i \ll R_{tot}/\gamma_e = 20 \mu$gauss. It is necessary to determine which field the magnetometer will react to.

It is simplest to consider imperfect compensation by the noble-gas spins in two dimensions. We consider an applied deviation from the compensation point as $\delta B_z$ and $\delta B_\perp$ (Fig. C.1). The $^3$He aligns to this direction, and the uncompensated field is
given by

\[ \Delta B_z^{2d} = B_c^a + \delta B_z - B_c^a \cos \theta \]  
\[ \Delta B_{\perp}^{2d} = \delta B_{\perp} - B_c^a \sin \theta \]  

where the approximation \(-B^n \approx B_c^a\) since \(|B^c| \ll |B^n|\). Both \(\sin \theta\) and \(\cos \theta\) can be rewritten in terms of fields through the geometry of Fig. C.1 as

\[ \Delta B_z^{2d} = B_c^a + \delta B_z - \frac{B_c^a + \delta B_z}{\sqrt{\delta B_z^2 + (1 + \delta B_z)^2}} \]  
\[ \Delta B_{\perp}^{2d} = \delta B_{\perp} - \frac{\delta B_{\perp}}{\sqrt{\delta B_{\perp}^2 + (B_c^a + \delta B_z)^2}}. \]  

The applied deviation \(\delta B_i\) is reduced at first order, and the remnant shift of \(\Delta B_i\) depends on \(\delta B_{\perp}, \delta B_z,\) and \(B_c^a\). The geometry is mildly more complicated in three dimensions where these equations simplify to

\[ \Delta B_x = B_x - \frac{B_x}{\sqrt{B_x^2 + B_y^2 + (B_c^a + B_z)^2}} \]  
\[ \Delta B_y = B_y - \frac{B_y}{\sqrt{B_x^2 + B_y^2 + (B_c^a + B_z)^2}} \]  
\[ \Delta B_z = (B_c^a + B_z) - \frac{(B_c^a + B_z)}{\sqrt{B_x^2 + B_y^2 + (B_c^a + B_z)^2}} \]  

where it has become convenient to drop the \(\delta B_i\) notation and refer to the deviation from the compensation point of a field \(B_i\). We would ultimately like to include the effects of anomalous fields and for shorthand introduce the definitions

\[ \vec{B}^e \equiv \vec{\beta}^e + \vec{L} - \frac{\vec{\Omega}Q(P^e)}{\gamma^e} \]  
\[ \vec{B}^n \equiv \vec{\beta}^n - \frac{\vec{\Omega}}{\gamma^n} \]  

where it has become convenient to drop the \(\delta B_i\) notation and refer to the deviation from the compensation point of a field \(B_i\). We would ultimately like to include the effects of anomalous fields and for shorthand introduce the definitions

\[ \vec{B}^e \equiv \vec{\beta}^e + \vec{L} - \frac{\vec{\Omega}Q(P^e)}{\gamma^e} \]  
\[ \vec{B}^n \equiv \vec{\beta}^n - \frac{\vec{\Omega}}{\gamma^n} \]  

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for generalized anomalous electron and nuclear couplings respectively. The distinction between electron and nuclear couplings to rotations is distinguished by the appropriate gyromagnetic ratio. We introduce the appropriate anomalous field to Eq. C.1 and Eqs. C.6-C.8 as follows

\[
P_{ex} = \frac{P_{ex} \gamma_e}{\gamma_e} \left[ \frac{B_y + B^\alpha_y + \frac{\gamma_e}{R_{tot}} (B_x + B^\alpha_x) (B_z + B^\alpha_z)}{1 + \left( \frac{\gamma_e}{R_{tot}} \right)^2 \left( (B_x + B^\alpha_x)^2 + (B_y + B^\alpha_y)^2 + (B_z + B^\alpha_z)^2 \right)} \right]
\]

(C.11)

\[
\Delta B_x = B_x - \frac{B_x + B^\alpha_x}{\sqrt{(B_x + B^\alpha_x)^2 + (B_y + B^\alpha_y)^2 + (B_z + B^\alpha_z)^2}}
\]

(C.12)

\[
\Delta B_y = B_y - \frac{B_y + B^\alpha_y}{\sqrt{(B_x + B^\alpha_x)^2 + (B_y + B^\alpha_y)^2 + (B_z + B^\alpha_z)^2}}
\]

(C.13)

\[
\Delta B_z = (B^\alpha_z + B_z) - \frac{(B^\alpha_z + B_z + B^\alpha_z)}{\sqrt{(B_x + B^\alpha_x)^2 + (B_y + B^\alpha_y)^2 + (B_z + B^\alpha_z)^2}}
\]

(C.14)

where we consider anomalous fields to influence the correction to the fields and not the definition of the compensation point, \( B^\alpha_c = -B^\alpha_n \).

Replacement of \( \Delta B_i \) in Eqs. C.12-C.14 for \( B_i \) in Eq. C.11 easily provides a familiar leading order response

\[
P_{ex} = \frac{P_{ex} \gamma_e}{R_{tot}} \left( \beta_y - \beta^\alpha_y + L_y + \Omega_y \left( \frac{1}{\gamma_n} - \frac{Q(P^\alpha)}{\gamma_e} \right) \right)
\]

(C.15)

in the difference between electron and nuclear couplings. We can already start to see why our Eq. 3.43 has a \( B_y B_z \) term and \( B_x B^2_z \) from expansion of Eqs. C.12-C.14. Further expansion of this expression to arbitrary orders can start to produce terms listed in Section 3.2. For completeness of the argument, we expand all the terms to an arbitrary third order product in fields to show the correlation between these expressions and those produced earlier by a different means.

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C.1.2 Steady State Equation Lists

The following sections provide lists of all the terms from the expansion in the geometric method. We denote each of these as $T(X)$ for “toy model” where $X$ is the variable of interest. This should distinguish the responses from our signal $S(X)$ which includes spin-exchange effects, pumping rates, and finite electron magnetization from the Bloch equations. Terms have been ranked from large to small according to the parameters listed in Table 3.2 and the modifications in Section 3.2.9 to maintain terms for all components of the $\vec{\beta}^e$ and $\vec{\beta}^n$ fields.

Short of ranking terms, the lists constitute raw output. Since limits on $\vec{\beta}^e$ and $\vec{\beta}^n$ are already stringent, products of these components have been disregarded. Similarly, $T(X)$ considers coefficients of terms proportional to $X$, but does not explicitly include terms proportional to $X^2$. We therefore produce dependencies on products of magnetic fields to complete some of the higher order picture. As before, we cannot simply add all the dependencies together since this would double count some terms and not others.

Algebraically grouping terms as in Section 3.2 has not been taken at this point. This author has found the output in raw form very helpful to directly view throughout the course of the work in this thesis. It seems useful to look through the list and compare the relevant suppression of terms by $\gamma_c/R_{tot}$ or $B_c$. Many of the terms beyond the leading terms have so far been too small to observe, but in estimating their effects, this author has gained a lot of intuition. Circling and assembling terms in these lists has can develop intuition beyond the neatly presented\textsuperscript{1} effects in Chapter 3. These lists also provide a sanity check in carefully expanding the denominator in Section 3.2. All denominators have been expanded, so we must be careful when comparing $T(X)$

\textsuperscript{1}Here, we present the complete list of terms from the expansion up to third order. Yes, it’s a mess, but I believe it deserves to be reproduced in an appendix.
and $S(X)$. They also enable the estimation of terms under different comagnetometer conditions without sorting through a complicated Mathematica notebook once again.

It is important to keep in mind that field expansions as a product with two distinct parameters have been performed here, $R_{tot}/\gamma_c \approx 20 \text{ \mu gauss}$ and $B_c \approx 3 \text{ \mu gauss}$. These expressions only hold for the smallest field deviations where the product is much less than unity.

### C.1.3 Primary Magnetic Field Dependence

Expansions for the magnetic fields match the expressions presented earlier with the prominent $B_y B_z$ and $B_x B_z^2$ terms. There is no linear dependence on $B_x$ here since this effect originated with spin-exchange terms.

\[
T(B_z) = \frac{P_{z}^{e} e_{z}}{R_{tot}} B_{z} \left( \frac{\gamma_{e} L_{x}}{R_{tot}} - \frac{\gamma_{e} \beta_{x}^{n}}{R_{tot}} + \frac{\gamma_{e} \gamma_{c}^{z}}{R_{tot}} + \frac{B_{y}}{B_{c}} + \frac{\gamma_{e} \Omega_{x}}{\gamma_{n} R_{tot}} - \frac{2 \gamma_{e}^{2} L_{y} L_{z}}{R_{tot}^{2}} + \frac{2 \gamma_{e}^{2} L_{z} \beta_{y}^{n}}{R_{tot}^{2}} - \frac{2 \gamma_{e}^{2} L_{y} \beta_{z}^{n}}{R_{tot}^{2}} + \frac{B_{x} \gamma_{c}^{z}}{B_{c} R_{tot}} + \frac{2 \gamma_{e}^{2} \Omega_{y} \beta_{z}^{n}}{B_{c} R_{tot}^{2}} - \frac{2 \gamma_{e} \gamma_{c}^{z} \Omega_{x}}{B_{c} \gamma_{n} R_{tot}} - \frac{2 \gamma_{e} \gamma_{c}^{z} \Omega_{y}}{B_{c} \gamma_{n} R_{tot}} - \frac{2 \gamma_{e} \gamma_{c}^{z} \Omega_{x}}{B_{c} \gamma_{n} R_{tot}} - \frac{2 \gamma_{e} \gamma_{c}^{z} \Omega_{y}}{B_{c} \gamma_{n} R_{tot}} \right)
\]

(C.16)
Combinations of two magnetic fields capture the higher order terms missed in Section C.1.3. As expected, \( B_z^2 \) has a \( B_x \) dependence.

\[
T(B_y B_z) = \frac{P_{e}^{c}\gamma_{e}B_{y}}{R_{tot}} B_{z} \left( 1 - \frac{2\beta_{z}^{n}}{B_{c}} \right) \tag{C.19}
\]

\[
T(B_x B_z) = \frac{P_{e}^{c}\gamma_{e}B_{x}}{R_{tot}} B_{z} \left( \frac{\gamma_{e}\Omega_{y}}{R_{tot}} - \frac{\gamma_{e}\Omega_{y}}{R_{tot}} B_{c} \frac{\gamma_{e}}{B_{c}n} + \frac{\beta_{z}^{n}}{B_{c}} \frac{\gamma_{e}}{B_{c}n} - \frac{\gamma_{e}\Omega_{y}}{R_{tot}} \frac{\gamma_{e}}{B_{c}n} \right) \tag{C.20}
\]

\[
T(B_y B_x) = \frac{P_{e}^{c}\gamma_{e}B_{y}}{R_{tot}} B_{x} \left( \frac{\beta_{x}^{n}}{B_{c}} - \frac{\Omega_{x}}{B_{c}n} \right) \tag{C.21}
\]
\[ T(B_y^2) = \frac{P_e \gamma_e B_y^2}{R_{tot} R_{tot}} \left( \frac{L_x}{2B_c} - \frac{\beta_n^e}{2B_c} + \frac{\beta_x^e}{2B_c} + \frac{\Omega_y}{2B_c \gamma_n} + \frac{3R_{tot} \beta_n^y}{2B_c^2 \gamma_e} - \frac{3R_{tot} \Omega_y}{2B_c^2 \gamma_e \gamma_n} - \frac{Q \Omega_x}{2B_c \gamma_e} \right) \] (C.22)

\[ T(B_x^2) = \frac{P_e \gamma_e B_x^2}{R_{tot} R_{tot}} \left( \frac{L_x}{2B_c} - \frac{\beta_n^e}{2B_c} + \frac{\beta_x^e}{2B_c} + \frac{B_y R_{tot}}{2B_c^2 \gamma_e} \right. \\
\left. + \frac{\Omega_x}{2B_c \gamma_n} + \frac{R_{tot} \beta_y^n}{2B_c^2 \gamma_e} - \frac{R_{tot} \Omega_y}{2B_c^2 \gamma_e \gamma_n} - \frac{Q \Omega_x}{2B_c \gamma_e} \right) \] (C.23)

\[ T(B_z B_z) = \frac{P_e \gamma_e}{R_{tot}} \frac{B_x B_z}{B_c} \left( \frac{\gamma_e L_x}{R_{tot}} + \frac{\gamma_e \beta_z^n}{R_{tot}} + \frac{\gamma_e \beta_z^e}{R_{tot}} - \frac{\gamma_e \Omega_z}{\gamma_n R_{tot}} - \frac{Q \Omega_z}{R_{tot}} \right) \] (C.24)
\[ T(\Omega_y) = \frac{P_c\gamma_x}{R_{\text{tot}}\gamma_n} \left( 1 - \frac{Q\gamma_n}{\gamma_x} - \frac{B_z^2\gamma_x^2}{R_{\text{tot}}^2} - \frac{2B_zL_z\gamma_x^2}{R_{\text{tot}}^2} - \frac{B_z}{B_c} - \frac{3L_y^2\gamma_x^2}{R_{\text{tot}}^2} - \frac{2\beta^rB_z\gamma_x^2}{R_{\text{tot}}^2} \right) \]

\[ - \frac{L_z^2\gamma_x^2}{R_{\text{tot}}^2} - \frac{L_z^2\gamma_x^2}{R_{\text{tot}}^2} + \frac{Q^2 \gamma_e \gamma_n}{R_{\text{tot}}^2} \]

\[ - \frac{2\beta^rL_z\gamma_x^2}{R_{\text{tot}}^2} + \frac{2\beta^nL_z\gamma_x^2}{R_{\text{tot}}^2} - \frac{2\beta^eL_z\gamma_x^2}{R_{\text{tot}}^2} - \frac{\gamma_x^2}{R_{\text{tot}}^2} + \frac{2Q \gamma_y L_z \gamma_x \gamma_n}{R_{\text{tot}}^2} \]

\[ - \frac{B_y L_x \gamma_x}{B_c R_n} + \frac{\Omega_x}{B_c \gamma_n} - \frac{2L_x\gamma_x^2\Omega_x}{R_{\text{tot}}^2 \gamma_n} + \frac{3Q^2 L_y^2 \gamma_x}{R_{\text{tot}}^2} + \frac{2Q^2 \gamma_y B_z \gamma_x \gamma_n}{R_{\text{tot}}^2} \]

\[ - \frac{3B_y^2}{2B_c^2} + \frac{\beta^r B_y \gamma_e}{B_c R_{\text{tot}}} - \frac{\beta^e B_y \gamma_e}{B_c R_{\text{tot}}} + \frac{QL_x^2 \gamma_x}{R_{\text{tot}}^2} + \frac{QL_y^2 \gamma_y}{R_{\text{tot}}^2} + \frac{B_y^2}{R_{\text{tot}}^2} \]

\[ - \frac{6Q \gamma_y^2 \gamma_x^2 \gamma_n}{R_{\text{tot}}^2} + \frac{6Q \gamma_y^2 \gamma_x^2 \gamma_n}{R_{\text{tot}}^2} + \frac{2Q \gamma_x \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{2 \gamma_x^2 \gamma_y^2}{R_{\text{tot}}^2} \]

\[ - \frac{2 \beta^r \gamma_x^2 \Omega_x}{R_{\text{tot}}^2 \gamma_n} + \frac{\gamma_x^2}{R_{\text{tot}}^2 \gamma_n} - \frac{2 \beta^r \gamma_x^2 \Omega_x}{R_{\text{tot}}^2 \gamma_n} + \frac{2Q \gamma_y \gamma_x \gamma_n}{R_{\text{tot}}^2} + \frac{2 \gamma_x^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} \]

\[ - \frac{2 \gamma_x^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{2 \beta^r \gamma_x^2 \Omega_x}{R_{\text{tot}}^2 \gamma_n} - \frac{2Q \gamma_y \gamma_x \gamma_n}{R_{\text{tot}}^2} + \frac{2 \gamma_x^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} \]

\[ - \frac{3 \beta_y B_y}{B_c R_{\text{tot}} \gamma_n} - \frac{\beta^r B_y \gamma_e}{B_c R_{\text{tot}}} + \frac{Q^2 B_y \gamma_n}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} \]

\[ + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} - \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} - \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} \]

\[ - \frac{2Q \gamma_y B_y}{B_c R_{\text{tot}}} - \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} - \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} + \frac{Q \gamma_y B_y}{B_c R_{\text{tot}}} \]

\[ - \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} - \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} + \frac{Q^2 \gamma_y \gamma_n}{R_{\text{tot}}^2} \]
\begin{align}
T(\Omega_x) &= \frac{P^e \gamma_x \Omega_x}{R_{\text{tot}} \gamma_x} \left( B_y \gamma_x + \frac{L_y \gamma_x}{R_{\text{tot}}} - \frac{B_z \gamma_x}{R_{\text{tot}}} - \frac{Q B_x \gamma_x}{R_{\text{tot}}} - \frac{2 B_z L_y \gamma_x}{R_{\text{tot}}} - \frac{Q L_y \gamma_x}{R_{\text{tot}}} - \frac{\beta_x \gamma_x}{R_{\text{tot}}} - \frac{2 B_z \gamma_x}{R_{\text{tot}}} - \frac{2 B_z L_y \gamma_x^2}{R_{\text{tot}}} - \frac{2 B_z \gamma_x^2}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x^2}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \gamma_y}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{tot}}} - \frac{2 \beta_x \gamma_x \Omega_x}{R_{\text{ tot}}}
\right) 
\end{align}
C.1.6 Lightshifts

The $L_x L_z$ term appears as well as the first order dependance of $L_y$.

\[
T(L_x) = \frac{P_x^\varepsilon \gamma_x}{R_{tot}^2} \left( \frac{\gamma_x L_x}{R_{tot}} - \frac{\gamma_x \beta_x^e}{R_{tot}} + \frac{\gamma_x \beta_x^c}{R_{tot}} - \frac{2 B_x \gamma_x^2 L_y}{R_{tot}^2} + \frac{\gamma_x \Omega_x}{R_{tot}} + \frac{2 B_x \gamma_x^2 \beta_y^2}{R_{tot}^2} \right)
- \frac{2 B_x \gamma_x \beta_y^2}{R_{tot}^2} + \frac{B_x \gamma_x \Omega_x}{B_c R_{tot}} - \frac{2 B_x \gamma_x^2 \beta_y^e}{R_{tot}^2} - \frac{\gamma_x \Omega_x}{B_c \gamma_x R_{tot}} + \frac{B_x \gamma_x \Omega_x}{B_c R_{tot}^2} + \frac{2 Q \gamma_x \Omega_y \beta_y^2}{R_{tot}^2}
- \frac{2 Q \gamma_x \Omega_y \beta_y^e}{R_{tot}^2} + \frac{2 B_x \gamma_x \Omega_x}{B_c \gamma_x R_{tot}^2} - \frac{2 B_x \gamma_x \Omega_x}{B_c \gamma_x R_{tot}^2} + \frac{2 Q \gamma_x \Omega_y \beta_y^2}{R_{tot}^2}
\]

\[
T(L_x) = \frac{P_x^\varepsilon \gamma_x}{R_{tot}^2} \left( \frac{\gamma_x L_x}{R_{tot}} + \frac{\gamma_x \beta_x^e}{R_{tot}} + \frac{\gamma_x \beta_x^c}{R_{tot}} - \frac{2 B_x \gamma_x^2 L_y}{R_{tot}^2} + \frac{\gamma_x \Omega_x}{R_{tot}} + \frac{B_x \gamma_x \beta_y^2}{R_{tot}^2} \right)
- \frac{2 B_x \gamma_x \beta_y^2}{R_{tot}^2} - \frac{\gamma_x \Omega_x}{B_c \gamma_x R_{tot}} - \frac{2 B_x \gamma_x \Omega_x}{B_c \gamma_x R_{tot}^2} + \frac{2 Q \gamma_x \Omega_y \beta_y^2}{R_{tot}^2}
- \frac{2 Q \gamma_x \Omega_y \beta_y^e}{R_{tot}^2} - \frac{2 B_x \gamma_x \Omega_x}{B_c \gamma_x R_{tot}^2} + \frac{2 Q \gamma_x \Omega_y \beta_y^2}{R_{tot}^2}
\]

(C.28)

(C.29)
\[ T(L_y) = \frac{P^e \gamma_e}{R_{\text{tot}}} L_y \left( 1 - \frac{B_z \gamma_e^2}{R_{\text{tot}}^2} - \frac{2B_z \gamma_e^2 L_z}{R_{\text{tot}}^2} - \frac{2B_z \gamma_e^2 \beta_z^e}{R_{\text{tot}}^2} - \frac{2B_z \gamma_e^2 \beta_x^e}{R_{\text{tot}}^2} - \frac{\gamma_e^2 L_z^2}{R_{\text{tot}}^2} - \frac{\gamma_e^2 L_x^2}{R_{\text{tot}}^2} \right) \]

\[ - \frac{2\gamma_e^2 L_z \beta_z^e}{R_{\text{tot}}^2} + \frac{2\gamma_e^2 L_x \beta_x^e}{R_{\text{tot}}^2} - \frac{2\gamma_e^2 L_z \beta_x^e}{R_{\text{tot}}^2} + \frac{6\gamma_e^2 \Omega_y \beta_y^e}{R_{\text{tot}}^2} \]

\[ - \frac{\gamma_n R_{\text{tot}}^2}{R_{\text{tot}}^2} \frac{2Q \gamma_e \Omega_z}{R_{\text{tot}}^2} + \frac{2Q \gamma_e L_z \Omega_x}{R_{\text{tot}}^2} - \frac{\gamma_n R_{\text{tot}}^2}{R_{\text{tot}}^2} \frac{2Q \gamma_e \Omega_x \beta_x^e}{R_{\text{tot}}^2} + \frac{6Q \gamma_e \Omega_y \beta_y^e}{R_{\text{tot}}^2} \]

\[ + \frac{2Q \gamma_e \Omega_z \beta_z^e}{R_{\text{tot}}^2} + \frac{6Q \gamma_e \Omega_x^2}{R_{\text{tot}}^2} - \frac{2Q \gamma_e \Omega_x \beta_x^e}{R_{\text{tot}}^2} + \frac{2Q \gamma_e \Omega_x^2}{R_{\text{tot}}^2} + \frac{2Q \gamma_e \Omega_x \beta_x^e}{R_{\text{tot}}^2} - \frac{3Q^2 \Omega_y^2}{R_{\text{tot}}^2} - \frac{Q^2 \Omega_z^2}{R_{\text{tot}}^2} - \frac{Q^2 \Omega_x^2}{R_{\text{tot}}^2} \] \hspace{1cm} (C.30)

### C.1.7 Anomalous Fields

Expansions for the anomalous fields are in agreement with Sections 3.2.4 and 3.2.9

\[ T(\beta_y^e) = \frac{P^e \gamma_e}{R_{\text{tot}}} \beta_y^e \left( 1 - \frac{B_z \gamma_e^2}{R_{\text{tot}}^2} - \frac{2B_z \gamma_e^2 L_z}{R_{\text{tot}}^2} - \frac{3\gamma_e^2 L_y^2}{R_{\text{tot}}^2} - \frac{\gamma_e^2 L_z^2}{R_{\text{tot}}^2} - \frac{\gamma_e^2 L_x^2}{R_{\text{tot}}^2} \right) \]

\[ - \frac{6\gamma_e^2 L_y \Omega_y}{R_{\text{tot}}^2} - \frac{\gamma_n R_{\text{tot}}^2}{R_{\text{tot}}^2} \frac{2Q \gamma_e L_z \Omega_x}{R_{\text{tot}}^2} - \frac{3\gamma_n R_{\text{tot}}^2}{R_{\text{tot}}^2} + \frac{6Q \gamma_e L_y \Omega_y}{R_{\text{tot}}^2} \]

\[ + \frac{2Q \gamma_e \Omega_z}{R_{\text{tot}}^2} - \frac{\gamma_n R_{\text{tot}}^2}{R_{\text{tot}}^2} \frac{2Q \gamma_e L_z \Omega_x}{R_{\text{tot}}^2} + \frac{6Q \gamma_e \Omega_y^2}{R_{\text{tot}}^2} + \frac{2Q \gamma_e \Omega_x^2}{R_{\text{tot}}^2} \]

\[ - \frac{3Q^2 \Omega_y^2}{R_{\text{tot}}^2} - \frac{Q^2 \Omega_z^2}{R_{\text{tot}}^2} - \frac{Q^2 \Omega_x^2}{R_{\text{tot}}^2} \] \hspace{1cm} (C.31)
\[ T(\beta^n_y) = \frac{P^n_{\gamma e} \beta^n_y}{R_{tot}} \left( -1 + \frac{B^2_{\gamma e}}{R^2_{tot}} + \frac{2B_{\gamma e}^2 L_{yz}}{R^2_{tot}} + \frac{B_{\gamma e}}{B_c} + \frac{3\gamma_{\gamma e} L_{y}^2}{R^2_{tot}} + \frac{\gamma_{\gamma e} L_{y}^2}{R^2_{tot}} + \frac{\gamma_{\gamma e} L_{x}^2}{R^2_{tot}} \right) \]

\[ + \frac{6\gamma_{\gamma e}^2 L_{y} L_{x}}{\gamma_n R^2_{tot}} + \frac{6\gamma_{\gamma e}^2 L_{y} L_{x}}{\gamma_n R^2_{tot}} + \frac{2\gamma_{\gamma e}^2 L_{x} \Omega_{x}}{\gamma_n R^2_{tot}} + \frac{3B_{\gamma e}^2 R_{tot}}{2B_c^2} - \frac{B_{\gamma e}^2}{B_c^2} - \frac{2QB_{\gamma e}^2 \gamma_{\gamma e} \Omega_{x}}{R^2_{tot}} \]

\[ + \frac{B_{\gamma e}^2}{2B_c^2} + \frac{3\gamma_{\gamma e}^2 Q_{\gamma e}}{\gamma_n R^2_{tot}} - \frac{6Q_{\gamma e} L_{y} \Omega_{y}}{\gamma_n R^2_{tot}} \]

\[ + \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} + \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} - \frac{2B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} L_{x} \Omega_{y}}{B_c \gamma_n R_{tot}} - \frac{\gamma_{\gamma e} L_{x} \Omega_{y}}{B_c \gamma_n R_{tot}} \]

\[ + \frac{Q_{\gamma e}^2 Q_{\gamma e}^2}{B_{\gamma e} \gamma_n R_{tot}} + \frac{Q_{\gamma e}^2 Q_{\gamma e}^2}{B_{\gamma e} \gamma_n R_{tot}} + \frac{Q_{\gamma e}^2 Q_{\gamma e}^2}{B_{\gamma e} \gamma_n R_{tot}} + \frac{Q_{\gamma e}^2 Q_{\gamma e}^2}{B_{\gamma e} \gamma_n R_{tot}} \]  

\[ (C.32) \]

\[ T(\beta^n_x) = \frac{P^n_{\gamma e} \beta^n_x}{R_{tot}} \left( -\frac{B_{\gamma e}}{R_{tot}} + \frac{\gamma_{\gamma e} L_{x}}{R_{tot}} + \frac{2\gamma_{\gamma e}^2 L_{x} \Omega_{x}}{R_{tot}} - \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} + \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} - \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} - \frac{B_{\gamma e} \gamma_{\gamma e}}{B_c \gamma_n R_{tot}} \right) \]

\[ - \frac{Q \Omega_{x}}{R_{tot}} - \frac{2\gamma_{\gamma e} L_{y} \Omega_{x}}{\gamma_n R^2_{tot}} + \frac{B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{\gamma_n R^2_{tot}} + \frac{B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{\gamma_n R^2_{tot}} + \frac{2\gamma_{\gamma e}^2 \Omega_{x} \Omega_{y}}{R^2_{tot}} + \frac{2\gamma_{\gamma e}^2 \Omega_{x} \Omega_{y}}{R^2_{tot}} \]

\[ + \frac{2B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{\gamma_n R^2_{tot}} + \frac{4\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x} \Omega_{y}}{\gamma_n R^2_{tot}} \]

\[ + \frac{\gamma_{\gamma e}^2 \Omega_{y}^2}{2B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e}^2 \Omega_{x}^2}{2B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e}^2 \Omega_{x}^2}{2B_c \gamma_n R_{tot}} \]  

\[ (C.33) \]

\[ T(\beta^n_x) = \frac{P^n_{\gamma e} \beta^n_x}{B_{tot}} \left( -\frac{B_{\gamma e}}{B_{tot}} + \frac{\gamma_{\gamma e} L_{x}}{B_{tot}} + \frac{2\gamma_{\gamma e}^2 L_{x} \Omega_{x}}{B_{tot}} + \frac{2\gamma_{\gamma e}^2 L_{x} \Omega_{x}}{B_{tot}} \right) \]

\[ + \frac{B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} + \frac{B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} + \frac{2B_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \Omega_{x}^2}{2B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \Omega_{x}^2}{2B_c \gamma_n R_{tot}} \]

\[ - \frac{2\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} - \frac{\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} - \frac{\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x}}{B_c \gamma_n R_{tot}} \]

\[ - \frac{Q \Omega_{x} \Omega_{y}}{R_{tot}} \]

\[ + \frac{4\gamma_{\gamma e} \gamma_{\gamma e} \Omega_{x} \Omega_{y}}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \Omega_{x}^2}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \Omega_{x}^2}{B_c \gamma_n R_{tot}} + \frac{\gamma_{\gamma e} \Omega_{x}^2}{B_c \gamma_n R_{tot}} \]  

\[ (C.34) \]

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\[ T(\beta_e^z) = \frac{P_e^{\gamma} \beta_e^z}{R_{tot}} \left[ \gamma_e L_y - \frac{2 B_z \gamma_e^2 L_y}{R_{tot}^2} + \frac{\gamma_e \Omega_x}{R_{tot}} - \frac{2 \gamma_e^2 L_y L_z}{R_{tot}^2} + \frac{B_x B_z \gamma_e}{B_c R_{tot}} - \frac{2 B_z \gamma_e^2 \Omega_y}{\gamma_n R_{tot}^2} \right. \\
- \left. \frac{2 \gamma_e^2 L_y \Omega_y}{\gamma_n R_{tot}^2} + \frac{Q \Omega_x}{R_{tot}} + \frac{2 Q B_z \gamma_e \Omega_y}{R_{tot}^2} - \frac{B_x \gamma_e \Omega_z}{B_c R_{tot}^2} - \frac{B \gamma_e \Omega_x}{B_c \gamma_n R_{tot}} \\
+ \frac{2 Q \gamma_e L_y \Omega_y}{R_{tot}^2} + \frac{2 Q \gamma_e \Omega_y B_z}{R_{tot}^2} + \frac{\gamma_e \Omega_x \Omega_z}{\gamma_n R_{tot}^2} - \frac{B_c \gamma_e \Omega_x}{B_c \gamma_n R_{tot}} - \frac{B \gamma_e \Omega_y \Omega_z}{\gamma_n R_{tot}^2} - \frac{B \gamma_e \Omega_x \Omega_z}{\gamma_n R_{tot}^2} - \frac{2 Q \gamma_e B_z}{B_c \gamma_n R_{tot}} \right) \] (C.35)

\[ T(\beta_n^z) = \frac{P_e^{\gamma} \beta_n^z}{R_{tot}} \left[ \gamma_e L_y - \frac{2 B_z \gamma_e^2 L_y}{R_{tot}^2} + \frac{\gamma_e \Omega_x}{R_{tot}} - \frac{2 \gamma_e^2 L_y L_z}{R_{tot}^2} + \frac{B_x B_z \gamma_e}{B_c R_{tot}} - \frac{2 B_z \gamma_e^2 \Omega_y}{\gamma_n R_{tot}^2} \right. \\
- \left. \frac{2 \gamma_e^2 L_y \Omega_y}{\gamma_n R_{tot}^2} + \frac{Q \Omega_x}{R_{tot}} + \frac{2 Q B_z \gamma_e \Omega_y}{R_{tot}^2} - \frac{B_x \gamma_e \Omega_z}{B_c R_{tot}^2} - \frac{B \gamma_e \Omega_x}{B_c \gamma_n R_{tot}} \\
+ \frac{2 Q \gamma_e L_y \Omega_y}{R_{tot}^2} + \frac{2 Q \gamma_e \Omega_y B_z}{R_{tot}^2} + \frac{\gamma_e \Omega_x \Omega_z}{\gamma_n R_{tot}^2} - \frac{B_c \gamma_e \Omega_x}{B_c \gamma_n R_{tot}} - \frac{B \gamma_e \Omega_y \Omega_z}{\gamma_n R_{tot}^2} - \frac{B \gamma_e \Omega_x \Omega_z}{\gamma_n R_{tot}^2} - \frac{2 Q \gamma_e B_z}{B_c \gamma_n R_{tot}} \right) \] (C.36)

C.1.8 Signal

Sorting through expressions Eq. [C.16] to [C.36] to select the leading terms produces,

\[ T_L = \frac{P_e^{\gamma} \gamma_e}{R_{tot}} \left[ \beta_y - \beta_n^m + \frac{\Omega_y}{\gamma_n} - \frac{\gamma_e \Omega_y B_z}{B_c} + \frac{\gamma_e \Omega_e}{R_{tot} \gamma_n} \right. \\
+ \left. \frac{\gamma_e}{R_{tot}} \left( B_x (B_z + L_z) \right) - L_y \frac{\gamma_e B_z}{R_{tot} \gamma_n} - \frac{\gamma_e \Omega_y B_z^2}{R_{tot} \gamma_n} \right] \] (C.37)

which is precisely the same as Eq. 3.43 produced by a solution to the complete Bloch equations that disregards pumping rates and a finite electron magnetization.

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Appendix D

Lorentz Violation Summary

This appendix provides details on each of the data sets that comprises the 143 days of data collected for the Lorentz violation search in Chapter 5. Each of the files is labeled by the GMST value near the start of the run. The tables are broken in two halves to distinguish between the summer and winter collections.

Table D.1 provides information regarding the selection of data in each record. The winter collection implemented the probe background measurements (Section 4.1.7) referred to as Secondary data. The effective noise bandwidth (ENBW) is calculated by comparing the average spectrum over 60 selections and follows the procedure described in Section E.2. A butterworth filter with a 2 Hz cutoff, 5 Hz stopband, 3 dB passband ripple, and 60 dB stopband attenuation precedes the ENBW calculation to remove oscillations at the compensation frequency. It is curious to note that the effective noise bandwidth is larger for the background measurements than for the comagnetometer signal.

Table D.2 provides details on the rate of sampling the comagnetometer signal and the lock-in bandwidth on these measurements. Notes regarding significant changes to the operation of the apparatus or interrupted operation are listed. Datasets 3541.74-3553.04 and 3669.09-3685.43 were taken sequentially with only a brief pause to create
a new file and resume operation. The data is collected at 200 Hz and averaged down
to the frequency reported as the sample rate. There is a minor technical issue in
analyzing the data where the entire dataset must be loaded into on contiguous block
of memory. For data saved at 40 Hz, this means that analysis is simplest if a new file
is created every week. This aligns well with a regular performance check on a weekly
basis.

Table D.3 provides the time duration of each file as well as the particular rotation
scheme applied. Most of the rotations occur with 22 s intervals except for file 3720.51.
Likewise, rotations in this file occur at the “slow” rotation speed. All files except file
3553.04 start in the south location and the direction of rotations was reversed at
file 3754.58. These changes were made to check against lingering systematic effects;
however, not enough data was collected for a realistic comparison. The orientation of
the $^3$He spin provides an important such check. The extra rotation column describes
the switching from the N-S to E-W axes. For example, file 3536.96 starts in the
“south” position and rotates 8 times with an extra rotation of $+90^\circ$. This implies
that the positions are [south, north, south, north, south, north, south, north, west]
before the next minor zeroing event. The minor zeroing events follow the ordering
[south, west, north, east, south,...].

The length of the files in the winter are significantly longer than those in the sum-
mer. This is due to a computer upgrade in the fall of 2009. The original computer
did not have enough system resources to average the signal to low sample rates. In
addition, this computer would experience intermittent DAQ errors that would halt
operation when the hardware lost timing. Simply clicking “OK” on the screen would
resume normal operation. These errors were intermittent enough that vigilance in
monitoring the experiment was required as it was unclear at the time what caused
the errors. Small gaps exist in the data when these issues were identified right away.
Larger gaps initiated the start of a new file. These problems where eventually identi-
fied as a flaw in the PCI bridge allowing two PCI cards to be used in a single PCI slot on the compact computer. A new computer for second half of the data set with two PCI slots, more memory, and faster CPU solved this issue that has not returned.
<table>
<thead>
<tr>
<th>File Number</th>
<th>Primary Window (s)</th>
<th>Primary Offset (s)</th>
<th>ENBW (Hz)</th>
<th>Secondary Window (s)</th>
<th>Secondary Offset (s)</th>
<th>Secondary ENBW (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3492.86</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3495.44</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3498.72</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3500.97</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3504.05</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3506.63</td>
<td>3</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3510.80</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3513.63</td>
<td>7</td>
<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3518.31</td>
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<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3519.56</td>
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<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3524.54</td>
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<td>0</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3536.96</td>
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<td>0</td>
<td>1.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3541.74</td>
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<td>0</td>
<td>1.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3544.79</td>
<td>7</td>
<td>0</td>
<td>1.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3548.96</td>
<td>7</td>
<td>0</td>
<td>1.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3553.04</td>
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<td>1.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3551.18</td>
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<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3566.54</td>
<td>3</td>
<td>-5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
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<td>3663.19</td>
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<td>2</td>
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<td>4.5</td>
</tr>
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<td>3666.47</td>
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<td>-5</td>
<td>1.25</td>
<td>2</td>
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<td>4</td>
</tr>
<tr>
<td>3669.09</td>
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<td>-5</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3677.13</td>
<td>3</td>
<td>-5</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3685.43</td>
<td>3</td>
<td>-5</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3692.34</td>
<td>3</td>
<td>-5</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3711.32</td>
<td>3</td>
<td>-5.3</td>
<td>1.5</td>
<td>2</td>
<td>-0.3</td>
<td>6</td>
</tr>
<tr>
<td>3720.51</td>
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<td>-5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>3734.60</td>
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<td>-5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
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<td>-5</td>
<td>2.12</td>
<td>2</td>
<td>0</td>
<td>8.95</td>
</tr>
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<td>3754.58</td>
<td>3</td>
<td>-5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>3756.33</td>
<td>3</td>
<td>-5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Table D.1: Secondary data was implemented for the second half of data collection. File 3510.80 has no ENBW since it is dropped from the data set.
Table D.2: Listing of datasets from the Lorentz violation search with notes on relevant events.

<table>
<thead>
<tr>
<th>File Number</th>
<th>Sample Rate (s)</th>
<th>Lock-in Rate (τ (ms))</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>3492.86</td>
<td>130</td>
<td>3</td>
<td>Noisy traditional calibration improved</td>
</tr>
<tr>
<td>3495.44</td>
<td>130</td>
<td>3</td>
<td>DAQ Error</td>
</tr>
<tr>
<td>3498.72</td>
<td>130</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3500.97</td>
<td>130</td>
<td>3</td>
<td>Early drifts</td>
</tr>
<tr>
<td>3504.05</td>
<td>130</td>
<td>3</td>
<td>Gap in data</td>
</tr>
<tr>
<td>3506.63</td>
<td>80</td>
<td>3</td>
<td>Bad file</td>
</tr>
<tr>
<td>3510.80</td>
<td>80</td>
<td>3</td>
<td>Lock-in knob rotates to second harmonic</td>
</tr>
<tr>
<td>3513.63</td>
<td>80</td>
<td>3</td>
<td>Giorgos repairs with duct tape and restarts</td>
</tr>
<tr>
<td>3518.31</td>
<td>80</td>
<td>3</td>
<td>Mike start with large pump wavelength offset</td>
</tr>
<tr>
<td>3519.56</td>
<td>80</td>
<td>3</td>
<td>Virus scanner interrrups; large one day oscillations</td>
</tr>
<tr>
<td>3524.54</td>
<td>80</td>
<td>3</td>
<td>Major zeroing to 7 hour intervals; E-W bifurcation</td>
</tr>
<tr>
<td>3536.96</td>
<td>80</td>
<td>3</td>
<td>2 hour gap; E-W discontinuity</td>
</tr>
<tr>
<td>3541.74</td>
<td>80</td>
<td>3</td>
<td>(1/4) Wavemeter fails, linear interpolation corrects</td>
</tr>
<tr>
<td>3544.79</td>
<td>80</td>
<td>3</td>
<td>(2/4)</td>
</tr>
<tr>
<td>3548.96</td>
<td>80</td>
<td>3</td>
<td>(3/4)</td>
</tr>
<tr>
<td>3553.04</td>
<td>80</td>
<td>3</td>
<td>(4/4)</td>
</tr>
<tr>
<td>3651.18</td>
<td>40</td>
<td>100</td>
<td>Christmas run; major zeroing remotely from MA; multiple wavemeter and encoder failures</td>
</tr>
<tr>
<td>3656.54</td>
<td>40</td>
<td>100</td>
<td>Mike start</td>
</tr>
<tr>
<td>3663.19</td>
<td>40</td>
<td>100</td>
<td>Reduce medium data rate from 10 Hz to 2.5 Hz</td>
</tr>
<tr>
<td>3666.47</td>
<td>40</td>
<td>100</td>
<td>Computer crash; three records without fast data</td>
</tr>
<tr>
<td>3669.09</td>
<td>40</td>
<td>100</td>
<td>(1/3) Princeton OIT turns off internet</td>
</tr>
<tr>
<td>3677.13</td>
<td>40</td>
<td>100</td>
<td>(2/3)</td>
</tr>
<tr>
<td>3685.43</td>
<td>40</td>
<td>100</td>
<td>(3/3) Unknown PC reboot terminates longest run</td>
</tr>
<tr>
<td>3692.34</td>
<td>40</td>
<td>100</td>
<td>HB$_z$ in zeroing; all B$_y$B$_z$ slopes starting at 3693.38</td>
</tr>
<tr>
<td>3711.32</td>
<td>40</td>
<td>100</td>
<td>Tilt sensor 3; spectrum degraded; wavemeter crash; adjust triggers from clock synchronization error</td>
</tr>
<tr>
<td>3720.51</td>
<td>40</td>
<td>100</td>
<td>Slow speed; wavemeter failure at start; stress plate disabled; rest measurements; major zero south only</td>
</tr>
<tr>
<td>3734.60</td>
<td>50</td>
<td>100</td>
<td>Loose contact causes heater drift; polarization drift gives large E-W error bars; wavemeter gaps</td>
</tr>
<tr>
<td>3741.32</td>
<td>40</td>
<td>100</td>
<td>Stem connectors repair</td>
</tr>
<tr>
<td>3754.58</td>
<td>40</td>
<td>100</td>
<td>Reverse rotations; temperature oscillations increase</td>
</tr>
<tr>
<td>3756.33</td>
<td>40</td>
<td>100</td>
<td>Stabilized temperature</td>
</tr>
<tr>
<td>File Number</td>
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<td>Interval Number</td>
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Table D.3: Summary of parameters relevant to the rotation of the apparatus.
Appendix E

Signal Analysis

The results presented in this thesis depend on a careful data analysis and understanding the uncertainty in the measurement. Many of these ideas are well-known and can be referenced from a variety of resources [131]. The technical details contributing to the measurements in this thesis are described here.

E.1 Uncertainty

In a real measurement, not all recorded values are precisely the same and are subject to random variations. The size of these random variations leads to the statistical uncertainty in the measurement. This section outlines the general methods for determining statistical uncertainty. In what follows, the systematic uncertainty, a bias in the measurement is neglected. There is a clear prescription for treatment of statistical uncertainties whereas treatment of systematic uncertainty is handled on a case by case basis.
E.1.1 Independent Measurements

Given $N$ independent, equally-weighted measurements of a quantity $x$ subject to random variations, the best estimate of $x$ is the central value or mean

$$\bar{x} = \sum_{i=1}^{N} \frac{x_i}{N}$$  \hspace{1cm} (E.1)

where $x_i$ is the value of each measurement. The variance of this distribution is

$$\sigma_x^2 = \sum_{i=1}^{N} \frac{(\bar{x} - x_i)^2}{N - 1}$$  \hspace{1cm} (E.2)

where the approximation $N - 1 \approx N$ is common when $N$ is large. We can represent the uncertainty in the measurement as the standard deviation of the mean

$$\sigma_x = \frac{\sigma_x}{\sqrt{N}}$$  \hspace{1cm} (E.3)

which can been seen to decrease as the square root of the number of measurements. $\sigma_x$ is sometimes referred to as the standard error or standard error of the mean.

E.1.2 Weighted Average and Uncertainty

If there is uncertainty associated with each measurement, the average of these quantities should reflect the relative precision of the contributing values. Given $N$ independent measurements of the same quantity $x_i$ with associated uncertainties $\sigma_i$, the weighted average of the $x_i$ measurements is defined as

$$\langle x \rangle = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}$$  \hspace{1cm} (E.4)
where the weights are defined as $w_i = 1/\sigma_i^2$. The uncertainty in the weighted average is

$$\sigma_{\langle x \rangle} = \frac{1}{\sqrt{\sum_{i=1}^{N} w_i}}.$$  \hfill (E.5)

The values with the smallest $\sigma_i$ carry the largest statistical weight.

### E.1.3 Reduced $\chi^2$

The uncertainty $\sigma_i$ in measurement $x_i$ often represents the best estimate of the uncertainty in the measurement. It is important to check the relationship between the reported $\sigma_i$ and the Gaussian distribution of $x_i$ that we expect. The quantity $\chi^2$ defined as

$$\chi^2 = \sum_{i=1}^{N} \frac{(\langle x \rangle - x_i)^2}{\sigma_i^2}$$  \hfill (E.6)

is useful in performing this check. It is often more standard to consider the reduced chi-squared

$$\tilde{\chi}^2 = \chi^2 / d$$  \hfill (E.7)

where $d$ is the number of degrees of freedom given by $N - c$ where $c$ is the number of constraints in the model. A $\tilde{\chi}^2$ that is close to 1 is often a good check that the error bars are representative of the scatter. If $\tilde{\chi}^2 \neq 1$, this indicates that the scatter in the data are larger than the representative error bars. Similarly, if $\tilde{\chi}^2 < 1$, the scatter in the data is smaller than the representative error bars.

If $\tilde{\chi}^2$ is not close to 1, reinvestigation of how the initial $\sigma_i$ are determined is recommended because this typically indicates a more appropriate estimate is required. In some cases, it is reasonable to readjust $\sigma_i$ by the $\tilde{\chi}^2$ of the distribution such that

$$\sigma'_i = \sigma_i \sqrt{\tilde{\chi}^2}.$$  \hfill (E.8)
If experimental parameters are changing over the timescale of many measurements but are relatively stable over an individual measurement, $\chi^2$ will be large. Correcting by the reduced chi-squared will capture this variation, though it may be more worthwhile to stabilize these changing parameters if they are known. Caution is recommended here as this is not always the most appropriate representation of the uncertainty and this step should be carefully justified. A more careful estimate of the initial uncertainties can result in a better estimate of the uncertainty.

### E.1.4 Error Propagation

A parameter of interest is often a function of several variables. If we are interested in the uncertainty in function $q(x, y, ...z)$ and measurements of $x \pm \delta x$, $y \pm \delta y$, $z \pm \delta z$ are available, then

$$
\delta q(x, y, ...z) = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + \left(\frac{\partial q}{\partial y} \delta y\right)^2 + ... \left(\frac{\partial q}{\partial z} \delta z\right)^2}.
$$

(E.9)

In the case that $q = x + y$, this expression generalizes to the well-known result of adding uncertainties in quadrature

$$
\delta q = \sqrt{(\delta x)^2 + (\delta y)^2}.
$$

(E.10)

Similarly, the case of $q = xy$ generates the expression

$$
\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}
$$

(E.11)

which is the sum of the fractional uncertainties in quadrature.
E.2 Over-Sampled Measurements

The definition of the standard deviation of the mean in Eq. E.3 relies on the assumption that each measured point is independent. In recording analog signals, the sample frequency is at least twice the bandwidth to avoid aliasing. If the bandwidth is \( f \), then a signal recorded at \( 2f \) or \( 10f \) should have the same uncertainty; oversampling the data faster than the correlations does not decrease the uncertainty. The effective noise bandwidth\(^1\) (ENBW) of the filter will correct for over-sampling if the characteristics of the filter are known. Equation [E.3] becomes

\[
\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N \times \text{ENBW}/(f_s/2)}} \tag{E.12}
\]

where \( f_s \) is the sample rate. The number of independent points is modified by the ratio of the effective noise bandwidth to the Nyquist frequency, \( f_s/2 \).

The comagnetometer measurements in this thesis involves a SR830 lock-in amplifier. The predicted ENBW for this device with the slope of the low pass filter set to 24 dB/octave is \( 5/(64\tau) \) where \( \tau \) is the lock-in time constant \(^2\) [132]. Practically, the lock-in signal is digitally recorded from the analog output of the lock-in by a data acquisition card. Higher frequency components beyond the lock-in bandwidth are reintroduced to the signal in the form of white noise from analog to digital conversion. Therefore, it is important to be able to determine the effective noise bandwidth of an arbitrary signal\(^2\).

The ENBW is defined as the bandwidth of an ideal brick filter that passes the same power as the non-ideal filter. By taking the fourier transform of the data in the

\(^1\)Sometimes called the noise bandwidth.
\(^2\)This point can be circumvented by digitally recording the data from the lock-in, however a fast and robust bit rate along with synchronization to other analog channels introduces some other technical hurdles.
time domain, this can be written as

$$\text{ENBW} = \frac{1}{|A(0)|^2} \int_0^\infty |A(f)|^2 df$$  \hspace{1cm} (E.13)

where $A(f)$ is the Fourier component of the signal at frequency $f$ [133]. This operation turns an arbitrary spectrum into that of the idealized brick case with a constant and a sharp cut-off in the power spectrum. The effective noise bandwidth is the frequency of the cut-off (Fig. E.1) and predicts the value of $5/(64\tau)$ for the 24 dB/octave low pass filter on the lock-in. In the case of a discrete signal, Eq. E.13 becomes

$$\text{ENBW} = \frac{\sum_{k=0}^N |A(f_k)|^2 f_k}{N|A(f_0)|^2 f_s}$$  \hspace{1cm} (E.14)

where $N = f_s/(2T)+1$, $T$ is the measurement interval, and $k$ is the discrete frequency component in the Fourier transform.

### E.3 String Analysis

In the experiments described in this thesis, we search for changes in the signal with respect to a binary change in either the spin source or apparatus orientation. We could simply compare the difference in pairs of points in either state to determine the
correlated amplitude. This method is highly susceptible to drifts in the signal over many reversals. Instead, we follow the string analysis techniques first introduced in a neutron electric dipole moment search [134]. This technique removes systematic shifts in measurements of the reversal correlated amplitude. We exclusively use a 3-point overlapping string analysis and discuss those details. Generalization to higher order strings is available in [135].

Consider a sequence of regularly sampled measurements subject to a linear drift where the parameter of interest \( a \) is reversed

\[
y_i = x_0 + a_i \\
y_{i+1} = x_0 - a_i + bt \\
y_{i+2} = x_0 + a_i + 2bt \\
....
\]

where \( y_i \) is the \( i \)th measurement, \( x_0 \) is the starting offset, and \( b \) is the slope of linear drift in time \( t \). We assume that the measurements are short compared to the time between measurements such that the drift does not significantly affect the uncertainty \( \sigma_i \) in measurement \( y_i \). Given these assumptions, the following combination of measurements

\[
a_i = \frac{(-1)^{i+1}}{4}(y_i - 2y_{i+1} + y_{i+2}) \quad \text{(E.15)}
\]

form a string which removes the linear drift. This amplitude can be calculated for additional consecutive measurements. The uncertainty in \( a_i \) is given by

\[
\sigma_{a_i} = \frac{1}{4} \sqrt{\sigma_i^2 + 4\sigma_{i+1}^2 + \sigma_{i+2}^2}. \quad \text{(E.16)}
\]

These uncertainties are independent and add as in Eq. E.10. The coefficient on the middle term is important since this element has been counted twice. Naturally, this
technique can be extended to arbitrary order and written in compact form using binomial coefficients to remove higher order drifts \[135\].

Each string represents a measurement of \(a\). The selection of a particular string is arbitrary, so we consider all possible 3-point strings and allow them to overlap. Typically, one also finds better statistical uncertainty using overlapping strings. The weighted average of the strings over a measurement interval can be expressed as

\[
\langle a \rangle = \frac{\sum_{i=1}^{N-2} a_i / (\sigma_{a_i})^2}{\sum_{i=1}^{N-2} 1 / (\sigma_{a_i})^2}
\]  
(E.17)

where \(N\) is the number of measurements. The uncertainty of \(\langle a \rangle\) is given by

\[
\sigma_{\langle a \rangle} = \frac{f_o}{\sqrt{\sum_{i=1}^{N-2} 1 / (\sigma_{a_i})^2}} \sqrt{\chi^2}
\]  
(E.18)

where \(f_o = 4/\sqrt{6}\) is a statistical weight correction for the 3-point overlapping strings. The reduced \(\chi^2\) is defined much like in Section \[E.1.3\] as

\[
\tilde{\chi}^2 = \frac{1}{N-2} \sum_{i=1}^{L-2} \frac{(a_i - \langle a \rangle)^2}{\sigma_{a_i}^2}.
\]  
(E.19)

Explicitly rescaling Eq. \[E.18\] by \(\sqrt{\tilde{\chi}^2}\) serves to help match the uncertainty to the scatter in the string points. This technique removes a linear drift the in the \(y_i\) measurements and over short intervals can suppress quadratic drift. It is straightforward to generalize this technique to higher order strings to remove higher order drifts \[135\].

\textbf{E.3.1 String Analysis Refinement}

The string analysis described in Section \[E.3\] works very well for long strings \(N \gg 10\) such as those in an EDM experiment \[135\]. Due to the zeroing schedule of the comagnetometer, measurement intervals are limited to \(\sim 250\) s limiting 8-16 reversals
in the case of apparatus rotations in CPT-II. We refine the string analysis to more accurately represent measurements in the low reversal limit.

We reject the correction by the square root of the $\tilde{\chi}^2$ parameter. This value is often less than 1 and serves to shrink the uncertainty. This is not from a flaw in our data. Even for white gaussian noise, it takes up to $\sim 30$ measured points for the distribution of $\tilde{\chi}^2$ to be peaked near 1 as expected. In addition, measurements with $N \sim 10$ do not provide strong enough statistics for this correction to be representative of the measurement interval.

We also need to correct the mysterious $f_o$ parameter for short strings. This correctly accounts for uneven weighting of the string points in the string analysis to what one might get from a naive ordering of the points. In the naive sense, one would compute the differences in successive measurements and find the weighted average ignoring any linear drift

$$\langle a_n \rangle = \frac{\sum_{j=1}^{N/2} (y_{2j-1} - y_{2j})/(\sigma_{2j-1}^2 + \sigma_{2j}^2)}{2 \sum_{j=1}^{N/2} 1/(\sigma_{2j-1}^2 + \sigma_{2j}^2)} \quad (E.20)$$

where $N \in 2\mathbb{Z}$ for simplicity in comparing pairs of points with the corresponding uncertainty

$$\sigma_{\langle a_n \rangle} = \frac{1}{\sqrt{\sum_{j=1}^{N/2} 4/(\sigma_{2j-1}^2 + \sigma_{2j}^2)}} = \frac{\sigma_o}{\sqrt{N}} \quad (E.21)$$

where the last step assumes that each $\sigma_i = \sigma_o$. This is the statistical precision to which we know $\langle a_n \rangle$. Since we have the same amount of information but chose to group it differently in the calculation of $\langle a \rangle$, the uncertainty should be the same. Clever grouping in the string analysis removes a systematic bias of linear drift and should retain the same number of degrees of freedom. In determining $\sigma_{\langle a \rangle}$ and ignoring the
adjustment by $\sqrt{\chi^2}$,

$$\sigma_{(a)} = \frac{f_o}{\sqrt{\sum_{i=1}^{N-2} \left( \frac{4^2}{\sigma_i^2 + 4\sigma_{i+1}^2 + \sigma_{i+2}^2} \right)}} = \frac{f_o \sqrt{6}}{4} \frac{\sigma_o}{\sqrt{N-2}} \quad (E.22)$$

where again in the last step we assume $\sigma_i = \sigma_o$. We clearly observe the appearance of $4/\sqrt{6}$. A comparison of Eq. E.21 and E.22 indicates that

$$f_o = \frac{4}{\sqrt{6}} \sqrt{\frac{N-2}{N}} \quad (E.23)$$

is a more appropriate assignment. In the limit $N \gg 1$, $\sqrt{1 - 2/N} \approx 1$ and is ignored in Ref. [135]. In the case of $N = 10$ common in the Lorentz violation experiment, this leads to a 10% correction in the uncertainty which is significant in the propagation of uncertainty. We could wonder if the above argument is somewhat oversimplified. Starting with Eq. E.17 and using Eq. E.11, the result simplifies to the above argument for overlapping strings.

The modification to $f_o$ just described was not applied in the analysis of the Lorentz violation experiment described in Chapter [3]. When the least squares fit to sidereal amplitudes is applied, the uncertainties in the fit parameters are rescaled by the $\sqrt{\chi^2}$ of the fit. Since the modification is the same throughout the entire file, this overall scaling factor isn’t necessary. If different length measurement intervals were used in a file, this correction would be necessary to restore the relative weights of the uncertainties.
Bibliography


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[28] Chilean quake may have shortened Earth days. NASA Jet Propulsion Laboratory Press Release, March 2010.

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