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Inflation with variable Ω

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Abstract

We propose a simple version of chaotic inflation which leads to a division of the universe into infinitely many open universes with all possible values of Ω from 1 to 0.

Flatness of the universe ($\Omega = 1$) for a long time has been considered as one of the most definite predictions of the inflationary theory. However, what will happen if ten years from now it will be shown beyond any reasonable doubt that our universe is open or closed? Would it mean that inflationary theory is wrong?

Apparently, many experts do not think so. So far we do not have any alternative solution to the homogeneity, isotropy, horizon and monopole problems. Inflationary theory provides a natural mechanism for generation of perturbations necessary for galaxy formation. It would be much better to modify this scenario so as to produce $\Omega \neq 1$ instead of simply giving up and living without any consistent cosmological theory at all.

Indeed, it is possible to have $\Omega \neq 1$ in inflationary cosmology. It is especially simple in the case of a closed universe, $\Omega > 1$. For example one can consider a particular version of the chaotic inflation scenario [1] with the effective potential

$$V(\phi) = \tfrac{1}{2}m^2\phi^2 \exp\left(\frac{\phi}{CM_P}\right)^2 . \tag{1}$$

Potentials of a similar type often appear in supergravity. In this theory inflation occurs only in the interval $\frac{1}{2}M_P \lesssim \phi \lesssim CM_P$. The most natural way to realize inflationary scenario in this theory is to assume that the universe was created "from nothing" with the field ϕ in the interval $\frac{1}{2}M_P \lesssim \phi \lesssim CM_P$. According to [2], the probability of this process is suppressed by

$$P \sim \exp\left(-\frac{3M_P^4}{8V(\phi)}\right) . \tag{2}$$

Therefore the maximum of the probability appears near the upper range of values of the field ϕ for which inflation is possible, i.e. at $\phi_0 \sim CM_P$. The probability of such an event will be so strongly suppressed that the universe will be formed almost ideally homogeneous and spherically symmetric. As pointed out in [3], this solves the homogeneity, isotropy and horizon problems even before inflation really takes over. Then the size of the newly born universe in this model expands by the factor $\exp(2\pi\phi_0^2 M_P^{-2}) \sim \exp(2\pi C^2)$ during the stage of inflation [4]. If $C \gtrsim 3$, i.e. if $\phi_0 \gtrsim 3M_P \sim 3.6 \times 10^{19}$ GeV, the universe expands more than e^{60} times, and it becomes very flat. Meanwhile, for $C \ll 3$ the universe always remains "underinflated" and very curved, with $\Omega \gg 1$. We em-

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phasize again that in this particular model "underinflation" does not lead to any problems with homogeneity and isotropy. The only problem with this model is that in order to obtain Ω in the interval between 1 and 2 at the present time one should have the constant C to be fixed somewhere near $C = 3$ with an accuracy of few percent. This is a fine-tuning, which does not sound very attractive. However, it is important to realize that we are not talking about an exponentially good precision; accuracy of few percent is good enough.

It is much more complicated to obtain an open universe, which would be "underinflated" and still homogeneous, see, e.g. [5]. However, it also proves to be possible. The basic idea goes back to the papers by Coleman and De Luccia [6] and by Gott [7] who pointed out that the space inside a bubble formed during the false vacuum decay looks like an open universe. It is amazing that one can hide an infinitely large universe inside one bubble! (Of course, the trick works only if the bubbles do not collide, but this condition can be easily satisfied if the probability of bubble formation is exponentially small.) This mechanism was further explored in the papers by other authors, notably by Sasaki, Tanaka, Yamamoto and Yokoyama [8]. However, as a result of the tunneling one would obtain an open universe which would be almost completely empty, with infinitesimally small Ω.

A significant progress in this direction has been achieved recently, when Bucher, Goldhaber and Turok suggested that the interior of the bubble after tunneling should continue expanding exponentially, due to additional stage of inflation, just like in the new or chaotic inflation scenario [9]. Then the first stage of inflation and the spherical symmetry of the created bubble take care of the large scale homogeneity and isotropy, whereas the total size of the universe grows after the tunneling due to the second stage. This stage is largely responsible for density perturbations produced during inflation. The theory of these density perturbations and the corresponding cosmogony was developed by several authors even before this particular mechanism was suggested, but this mechanism adds some new distinctive features to the large scale part of the spectrum [10].

Unfortunately, just as in the closed-universe case discussed above, the realization of this scenario suggested by Bucher, Goldhaber and Turok [9] requires fine tuning. The best model suggested by them recently [11] was the chaotic inflation scenario with the potential $V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{3}\alpha\phi^3 + \frac{1}{4}\lambda\phi^4$ [12]. In order to get an open inflationary universe in this model it was necessary to adjust its parameters in such a way as to ensure that the tunneling occurs to the point $\phi \sim 3M_P$ with the accuracy of few percent. Also, the tunneling should occur to the part of the potential which almost does not change it slope during inflation at smaller ϕ, since otherwise one does not obtain scale-invariant density perturbations. One of the necessary conditions was that the barrier should be very narrow. Indeed, if $V'' \ll H^2$ at the barrier, then the tunneling occurs to its top, as in the Hawking-Moss case [13]; see [3] for the interpretation of this regime. If this happens, the large scale density perturbations become huge, $\delta\rho/\rho \sim H^2/\dot{\phi} > 1$, since $\dot{\phi} = 0$ at the maximum. In order to avoid this problem the authors assumed that the field ϕ had a nonminimal kinetic term. Thus the model gradually became not only fine-tuned, but also rather complicated. It does not discredit the whole idea, it is nice to have at least one working realization of the scenario outlined in [9], but with all these complications it becomes very tempting to find a more natural realization of the chaotic inflation scenario which would give inflation with $\Omega < 1$.

In this paper we will try to do it. The model is so simple that its description will take less space than this long introduction. Let us consider a model of two noninteracting scalar fields, ϕ and σ, with the effective potential

$$V(\phi, \sigma) = \frac{1}{2}m^2\phi^2 + V(\sigma) . \tag{3}$$

Here ϕ is a weakly interacting inflaton field, and σ, for example, can be the field responsible for the symmetry breaking in GUTs. We will assume that $V(\sigma)$ has a minimum at $\sigma = 0$, just as in the old inflationary theory. The shape of the potential can be, e.g., $\frac{1}{2}M^2\sigma^2 - \alpha M\sigma^3 + \frac{1}{4}\lambda\sigma^4$, but it is not essential; no fine tuning of the shape of this potential will be required.

Note that so far we did not make any unreasonable complications to the standard chaotic inflation scenario; at large ϕ inflation is driven by the field ϕ, and the GUT potential is necessary in the theory anyway. In order to obtain density perturbations of the necessary amplitude the mass m of the scalar field ϕ should be of the order of $10^{-6}M_P \sim 10^{13}$ GeV [4].

Inflation begins at $V(\phi, \sigma) \sim M_P^4$. At this stage fluctuations of both fields are very strong, and the universe enters the stage of self-reproduction, which finishes (for the field ϕ) only when it becomes smaller than $M_P\sqrt{M_P}/m$ and the energy density drops down to $mM_P^3 \sim 10^{-6}M_P^4$ [4]. Quantum fluctuations of the field σ in some parts of the universe put it directly to the minimum of $V(\sigma)$, but in some other parts the scalar field σ appears in the local minimum of $V(\sigma)$ at $\sigma = 0$. Since the energy density in such domains will be greater, their volume will grow with the greater speed, and therefore they will be especially important for us. One may worry that all domains with $\sigma = 0$ will tunnel to the minimum of $V(\sigma)$ at the stage when the field ϕ was very large and quantum fluctuations of the both fields were large too. However, this decay can be easily suppressed if one introduces a small interaction $g^2\phi^2\sigma^2$ between these two fields, which stabilized the state with $\sigma = 0$ at large ϕ. Note that even when the field ϕ drops down to $\phi = 0$, inflation in the domains with $\sigma = 0$ continues, being supported by the false vacuum energy $V(\sigma = 0)$, until the field σ tunnels to the minimum of $V(\sigma)$.

Here comes the main idea of our scenario. Because the fields σ and ϕ do not interact with each other, and the dependence of the probability of tunneling on the vacuum energy at the GUT scale is negligibly small [6], tunneling to the minimum of $V(\sigma)$ may occur with equal probability at all sufficiently small values of the field ϕ. The parameters of the bubbles of the field σ are determined by the mass scale corresponding to the effective potential $V(\sigma)$. This mass scale in our model is much greater than m. Thus the duration of tunneling in the Euclidean "time" is much smaller than m^{-1}. Therefore the field ϕ practically does not change its value during the tunneling. Note, that if the probability of decay at a given ϕ is small enough, then it does not destroy the whole vacuum state $\sigma = 0$ [14]; the bubbles of the new phase are produced all the way when the field ϕ rolls down to $\phi = 0$. In this process the universe becomes filled with (nonoverlapping) bubbles immersed in the false vacuum state with $\sigma = 0$. Interior of each of these bubbles represents an open universe. However, these bubbles contain *different* values of the field ϕ, depending on the value of this field at the moment when the bubble formation occurred. If the field ϕ inside a bubble is smaller than $3M_P$, then the universe inside this bubble will have a vanishingly small Ω, at the age 10^{10} years after the end of inflation it will be practically empty, and life of our type could not exist there. If the field ϕ is much greater than $3M_P$, the universe inside the bubble will be almost exactly flat, $\Omega = 1$, as in the simplest version of the chaotic inflation scenario. It is important, however, that *in an eternally existing self-reproducing universe there will be infinitely many universes containing any particular value of Ω, from $\Omega = 0$ to $\Omega = 1$.*

Of course, one can argue that we did not solve the problem of fine tuning, we just transformed it into the fact that only a very small percentage of all universes will have, say, $0.2 < \Omega < 0.3$. However, first of all, we achieved our goal in a very simple theory, which does not require any artificial potential bending and nonminimal kinetic terms. Then, there may be some reasons why it is preferable for us to live in a universe with a small (but not vanishingly small) Ω. Indeed, the total volume of the bubbles with $\Omega = 1$ grows at a much smaller rate after the phase transition. Thus, the later the phase transition happen, the more volume we get. This emphasizes the universes with small Ω. On the other hand, we cannot live in empty universes when Ω is too small. The percentage of the universes with different Ω can be strongly influenced by introducing a small coupling $g^2\phi^2\sigma^2$, which stabilizes the state $\sigma = 0$ for large ϕ. The tunneling becomes possible only for sufficiently small ϕ. This suppresses the number of bubbles with $\Omega = 1$.

We do not want to pursue this line of arguments any further. Comparison of volumes of different universes in the context of a theory of a self-reproducing inflationary universe is a very ambiguous task, since in this case we must compare infinities [15]. If we would know how to solve the problem of measure in quantum cosmology, perhaps we would be able to obtain something similar to an open universe without any first order phase transitions [16]. In the meantime, it is already encouraging that in our scenario there are infinitely many inflationary universes with any particular value of Ω. It may happen that the only way to find out whether we live in one of them is to make observations.

Some words of caution are in order here. The bubbles produced in our scenario are not *exactly* open universes. Indeed, in the models discussed in [6]–[9] the time of reheating (and the temperature of the universe after the reheating) was synchronized with the value of the scalar field inside the bubble. In our case the situation is very similar, but not exactly [17]. Suppose that the Hubble constant induced by $V(0)$ is much greater than the Hubble constant related to the energy density of the scalar field ϕ. Then the speed of rolling of the scalar field ϕ sharply increases inside the bubble. Thus, in our case the field σ synchronizes the motion of the field ϕ, and then the hypersurface of a constant field ϕ determines the hypersurface of a constant temperature. In the models where the rolling of the field ϕ can occur only inside the bubble (we will discuss such a model shortly) the synchronization is precise, and everything goes as in the models of Refs. [6]- [9]. However, in our simple model the scalar field ϕ moves down outside the bubble as well, even though it does it very slowly. Thus, synchronization of motion of the fields σ and ϕ is not precise; hypersurface of a constant σ ceases to be a hypersurface of a constant density. For example, suppose that the field ϕ has taken some value ϕ_0 near the bubble wall when the bubble was just formed. Then the bubble expands, and during this time the field ϕ outside the wall decreases, as $\exp\left(-m^2t/3H(0)\right)$, where $H(0) = \sqrt{8\pi V(0)/3M_P^2}$ [4]. At the moment when the bubble expands e^{60} times, the field ϕ in the region just reached by the bubble wall decreases to $\phi_0 \exp\left(-20m^2/H^2(0)\right)$ from its original value ϕ_0. The universe inside the bubble is a homogeneous open universe only if this change is negligibly small. This may not be a real problem. Indeed, let us assume that $V(0) = M^4$, where $M = 10^{17}$ GeV. In this case $H(0) = 1.7 \times 10^{15}$ GeV, and for $m = 10^{13}$ GeV one obtains $20m^2/H^2(0) \sim 10^{-4}$. In such a case a typical degree of distortion of the picture of a homogeneous open universe is very small.

Still this issue deserves careful investigation. When the bubble wall continues expanding even further, the scalar field outside of it eventually drops down to zero. Then there will be no new matter created near the wall. Instead of infinitely large homogeneous open universes we are obtaining spherically symmetric islands of a size much greater than the size of the observable part of our universe. We do not know whether this unusual picture is a curse or a blessing for our model. Is it possible to consider different parts of the same exponentially large island as domains of different "effective" Ω? Can we attribute some part of the dipole anisotropy of the microwave background radiation to the possibility that we live somewhere outside of the center of such island?

Another potential problem associated with this model is the possibility that the density perturbations on the horizon scale can appear larger than expected. Indeed, the Hubble constant before the tunneling in our model was much greater than the Hubble constant after the tunneling. This may lead to very large density perturbations on the scale comparable to the size of the bubble. Again, this may not be a real problem, since in the new coordinate system, in which the interior of the bubble looks like an open universe, the distance from us to the bubble walls is infinite. However, to be on a safe side it would be nice to have a model where we do not have any problems with synchronization and with the large jumps of the Hubble constant. This can be achieved by a generalization (simplification) of our model (3):

$$V(\phi,\sigma) = \tfrac{1}{2}g^2\phi^2\sigma^2 + V(\sigma) . \tag{4}$$

We eliminated the massive term of the field ϕ and added explicitly the interaction $\frac{1}{2}g^2\phi^2\sigma^2$, which, as we have mentioned already, is desirable for stabilization of the state $\sigma = 0$ at large ϕ. Note that in this model the line $\sigma = 0$ is a flat direction in the (ϕ,σ) plane. At large ϕ the only minimum of the effective potential with respect to σ is at the line $\sigma = 0$. To give a particular example, one can take $V(\sigma) = \frac{1}{2}M^2\sigma^2 - \alpha M\sigma^3 + \frac{1}{4}\lambda\sigma^4 + V_0$. Here V_0 is a constant which is added to ensure that $V(\phi,\sigma) = 0$ at the absolute minimum of $V(\phi,\sigma)$. In this case the minimum of the potential $V(\phi,\sigma)$ at $\sigma \neq 0$ is deeper than the minimum at $\sigma = 0$ only for $\phi < \phi_c$, where $\phi_c = \frac{M}{g}\sqrt{(2\alpha^2/\lambda) - 1}$. This minimum for $\phi = \phi_c$ appears at $\sigma = \sigma_c = 2\alpha M/\lambda$.

The bubble formation becomes possible only for $\phi < \phi_c$. After the tunneling the field ϕ acquires an effective mass $m = g\sigma$ and begins to move towards $\phi = 0$, which provides the mechanism for the second stage of inflation inside the bubble. In this scenario evolution of the scalar field ϕ is exactly synchronized with the evolution of the field σ, and the universe inside the bubble appears to be open.

Effective mass of the field ϕ at the minimum of $V(\phi,\sigma)$ with $\phi = \phi_c$, $\sigma = \sigma_c = 2\alpha M/\lambda$ is $m = g\sigma_c = 2g\alpha M/\lambda$. With a decrease of the field ϕ its effective mass at the minimum of $V(\phi,\sigma)$ will grow, but not significantly. Let us consider, e.g., the theory with $\lambda = \alpha^2 = 10^{-2}$, and $M \sim 5 \times 10^{15}$ GeV, which seems quite natural. In this case it can be shown that the effective mass m is equal to $2gM$ at $\phi = \phi_c$, and then it grows by only 25% when the field ϕ changes all the way down from ϕ_c to $\phi = 0$. As we already mentioned, in order to obtain the proper amplitude of density perturbations one should have $m \sim 10^{13}$ GeV. In our case $m \sim 10^{13}$ GeV for $g \sim 10^{-4}$, which gives $\phi_c \sim 5 \times 10^{19}$ GeV $\sim 4M_P$. The bubble formation becomes possible only for $\phi < \phi_c$. If it happens in the interval $4M_P > \phi > 3M_P$, we obtain a flat universe. If it appears at $\phi < 3M_P$, we obtain an open universe. Depending on the initial value of the field ϕ, we can obtain all possible values of Ω, from $\Omega = 1$ to $\Omega = 0$. The value of the Hubble constant at the minimum with $\sigma \neq 0$ at $\phi = 3M_P$ in our model does not differ much from the value of the Hubble constant before the bubble formation. Therefore we do not expect any specific problems with the large scale density perturbations in this model. Note also that the probability of tunneling at large ϕ is very small since the depth of the minimum at $\phi \sim \phi_c$, $\sigma \sim \sigma_c$ does not differ much from the depth of the minimum at $\sigma = 0$. Therefore the number of flat universes produced by this mechanism will be strongly suppressed as compared with the number of open universes. Meanwhile, life of our type is impossible in empty universes with $\Omega \ll 1$. This may provide us with a tentative explanation of the small value of Ω in the context of our model.

As we have seen, the models which can give rise to an open inflationary universe are very simple, but they lead to a rather complicated dynamics. Therefore they deserve thorough investigation. The main purpose of this paper was to show that there exists a wide class of models which can describe an open inflationary universe. It is still necessary to find out which of these models could describe observational data in a better way. Note, that there is no need to have an extremely small coupling constant $\lambda \sim 10^{-13}$ in our model. Instead of it we have a small coupling $g = 10^{-4}$. Thus at least in this aspect we are *reducing* the level of fine tuning required in the simplest inflationary models with $\Omega = 1$. We will return to the discussion of these and some other models with $\Omega \neq 1$ in the forthcoming publication [17].

We would like to conclude this article with some general remarks. Fifteen years ago many different cosmological models (HDM, CDM, $\Omega = 1$, $\Omega \ll 1$, etc.) could describe all observational data reasonably well. The main criterion for a good theory was its beauty and naturalness. Right now it becomes more and more complicated to explain all observational data. In such a situation cosmologists should remember that the standard theory of electroweak interactions contains about twenty free parameters which so far did not find an adequate theoretical explanation. Some of these parameters may appear rather unnatural. The best example is the coupling constant of the electron to the Higgs field, which is 2×10^{-6}. It is a pretty unnatural number which is fine-tuned in such a way as to make the electron 2000 lighter than the proton. We do not have any reason to expect that the cosmological theory will be simpler than that. It is important, however, that the electroweak theory is based on two fundamental principles: gauge invariance and spontaneous symmetry breaking. As far as these principles hold, we can adjust our parameters and wait until they get their interpretation in a context of a more general theory. It seems that inflation provides a very good guiding principle for constructing internally consistent cosmological models. The simplest versions of inflationary theory predict a universe with $\Omega = 1$. However, it is very encouraging that this theory, if needed, can be versatile enough to include models with all possible values of Ω, without forcing us to give up all advantages of inflationary cosmology. This is an important point which often escapes attention of those who tries to compare predictions of inflationary cosmology with observational data. At the present moment inflation is the only mechanism known to us that could produce a large homogeneous universe with $\Omega \neq 1$.

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