Air Hockey Bowling

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
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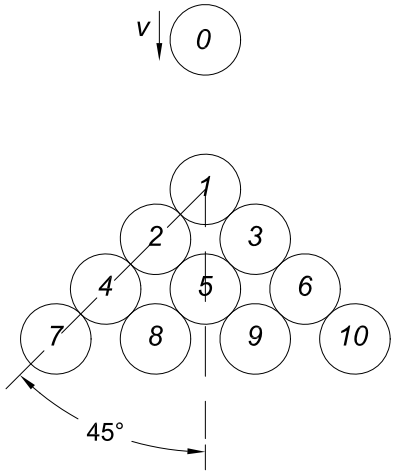
1 Problem

In a game of "bowling" with circular pucks on an air table, 10 pucks are arranged (at rest) in a triangular pattern as shown. The 11th puck is launched with velocity v along the symmetry axis of the triangle. All 11 pucks have the same mass. The pucks can be assumed not to move during the brief, elastic collisions.



Just after all the collisions have taken place, which pucks are in motion, and with what velocities?

2 Solution

2.1 Collision between Pucks 0 and 1

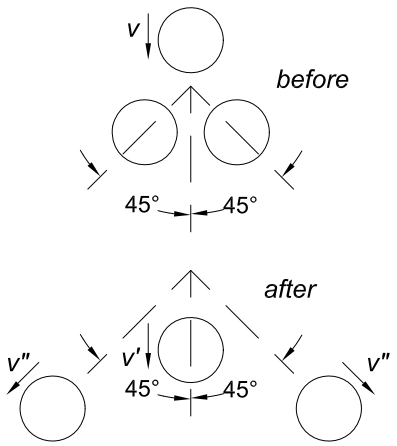
In a head-on elastic collisions between equal mass pucks 0 and 1 with puck 1 intially at rest, puck 1 takes on the initial velocity v of the moving puck, 0, and puck 0 ends up at rest.

2.2 Collision between Pucks 1, 2 and 3

In this collision puck 1 ends up with speed v' along the direction of initial motion of puck 0, and pucks 2 and 3 each end up with speed v'' while moving at angle 45° with respect that direction, since the contact force between puck 0 and pucks 1 and 2 is exerted along their line of centers.

Conservation of momentum (along the vertical direction on the page) requires that

$$v = v' + 2v'' \cos 45^\circ = v' + \sqrt{2}v''. \tag{1}$$



Conservation of energy (in the elastic collision of equal-mass pucks) requires that

$$v^2 = v'^2 + 2v''^2. \tag{2}$$

From eq. (1), we write $v' = v - \sqrt{2}v''$, and insert the square of this into eq. (2) to find

$$v^2 = [v^2 - 2\sqrt{2}vv'' + 2v''^2] + 2v''^2 = v^2 - 2\sqrt{2}vv'' + 4v''^2, \tag{3}$$

$$v'' = \frac{\sqrt{2}v}{2}, \tag{4}$$

$$v' = v - \sqrt{2}v'' = 0. \tag{5}$$

2.3 Subsequent Collisions

Because pucks 2 and 3 move at 45° to the initial direction of puck 0 they do not collide with puck 5. Puck 2 collides with puck 4 in a head-on elastic collision between equal-mass pucks, such that puck 2 comes to rest and puck 4 takes on velocity v'' at 45° to the initial direction of puck 0. Similarly, puck 3 collides with puck 6 in a head-on elastic collision between equal-mass pucks, such that puck 3 comes to rest and puck 6 takes on velocity v'' at 45° to the initial direction of puck 0.

Then, puck 4 collides with puck 7 in a head-on elastic collision between equal-mass pucks, such that puck 4 comes to rest and puck 7 takes on velocity v'' at 45° to the initial direction of puck 0. Similarly, puck 6 collides with puck 10 in a head-on elastic collision between equal-mass pucks, such that puck 6 comes to rest and puck 10 takes on velocity v'' at 45° to the initial direction of puck 0.

Pucks 8 and 9 remain at rest.

In the final configuration, only pucks 7 and 10 are in motion, each with speed $v'' = \sqrt{2}v/2$ at $\pm 45°$ to the initial direction of puck 0.