Stress and Momentum in a Capacitor That Moves with Constant Velocity

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1 Problem

Consider a parallel-plate capacitor whose plates are held apart by a nonconducting slab of unit (relative) dielectric constant and unit (relative) magnetic permeability.[1] Discuss the energy, momentum and stress in this (isolated) system when at rest and when moving with constant velocity parallel or perpendicular to the electric field.

Fringe-field effects can be ignored. The velocity can be large or small compared to the speed of light.

2 Solution

This problem is concerned with the relativistic transformation of properties of the capacitor. It represents a macroscopic application of the concepts of "Poincaré stresses" [2] that were introduced into classical models of the electron. Versions of this problem have also appeared in [3, 4, 5].

We suppose that the capacitor supports a uniform electric field $\mathbf{E}^\star = E^\star\,\hat{\mathbf{z}}$ between its plates, in its rest frame. *Taking the (relative) dielectric constant* ϵ *of the material between the plates to be unity, the electric displacement is given by* $\mathbf{D}^\star = \mathbf{E}^\star$ *(in Gaussian units). The macroscopic magnetic fields vanish in the capacitors rest frame,* $\mathbf{B}^\star = \mathbf{H}^\star = 0$, *noting that the relative permeability is* $\mu = 1$.

Associated with these electromagnetic fields is the 4-dimensional, macroscopic (symmetric) electromagnetic energy-momentum-stress tensor (secs. 32-33 of [6], sec. 12.10B of [7]),

$$\mathsf{T}^{\mu\nu}_{EM} = \left(\begin{array}{c|c} u_{EM} & c\,\mathbf{p}_{EM} \\ \hline c\,\mathbf{p}_{EM} & -T^{ij}_{EM} \end{array} \right), \tag{1}$$

where indices μ and ν take on values 0, 1, 2, 3, spatial indices i and j take on values 1, 2, 3, u_{EM} is the electromagnetic field energy density,

$$u_{EM} = \frac{E^2 + B^2}{4\pi}\,, \tag{2}$$

c is the speed of light in vacuum, $\mathbf{p}_{EM}$ is the electromagnetic momentum density,

$$\mathbf{p}_{EM} = \frac{\mathbf{E}\times\mathbf{B}}{4\pi c}\,, \tag{3}$$

[1] The use of unit dielectric constant and unit permeability avoids entering into the interesting controversy as to the so-called Abraham and Minkowski forms of the energy-momentum-stress tensor [1].

and T^{ij}_{EM} is the 3-dimensional electromagnetic stress tensor

$$T^{ij}_{EM} = \frac{E_i E_j + B_i B_j}{4\pi} - \frac{E^2 + B^2}{8\pi}\,. \tag{4}$$

In the rest frame of the capacitor the electromagnetic energy-momentum-stress tensor has components

$$\mathsf{T}^{\star\mu\nu}_{EM} = \left(\begin{array}{c|ccc} \frac{E^{\star 2}}{8\pi} & & \mathbf{0} & \\ \hline & \frac{E^{\star 2}}{8\pi} & & \\ \mathbf{0} & & \frac{E^{\star 2}}{8\pi} & \\ & & & -\frac{E^{\star 2}}{8\pi} \end{array}\right), \tag{5}$$

in the region between the capacitor plates. The nonzero diagonal elements $\mathsf{T}^{\star 11}$, $\mathsf{T}^{\star 22}$ and $\mathsf{T}^{\star 33}$, indicate that there are internal electric forces on the material between the capacitor plates. If the isolated capacitor is to be at rest, there must be internal mechanical stresses that are equal and opposite to the electromagnetic ones. Thus, we are led to consider also the mechanical energy-momentum stress tensor,

$$\mathsf{T}^{\mu\nu}_{\rm mech} = \left(\begin{array}{c|c} u_{\rm mech} & c\,\mathbf{p}_{\rm mech} \\ \hline c\,\mathbf{p}_{\rm mech} & -T^{ij}_{\rm mech} \end{array}\right), \tag{6}$$

where the mechanical energy density is $u_{\rm mech} = \rho_m c^2$, the mass density when there is no electric field in the capacitor is ρ_m, the density of mechanical momentum is $\mathbf{p}_{\rm mech}$, and $T^{ij}_{\rm mech}$ is the 3-dimensional mechanical stress tensor.

We infer that in the rest frame of the isolated capacitor, the mechanical energy-momentum-stress tensor has components

$$\mathsf{T}^{\star\mu\nu}_{\rm mech} = \left(\begin{array}{c|ccc} \rho^\star_m c^2 & & \mathbf{0} & \\ \hline & -\frac{E^{\star 2}}{8\pi} & & \\ \mathbf{0} & & -\frac{E^{\star 2}}{8\pi} & \\ & & & \frac{E^{\star 2}}{8\pi} \end{array}\right), \tag{7}$$

The total energy-momentum-stress tensor is the sum of the electromagnetic and mechanical tensors (1) and (6). In the rest frame of the capacitor the total energy-momentum-stress tensor has components

$$\mathsf{T}^{\star\mu\nu}_{\rm total} = \mathsf{T}^{\star\mu\nu}_{EM} + \mathsf{T}^{\star\mu\nu}_{\rm mech} = \left(\begin{array}{c|c} \rho^\star_m c^2 + \frac{E^{\star 2}}{8\pi} & \mathbf{0} \\ \hline \mathbf{0} & 0 \end{array}\right), \tag{8}$$

We interpret the component $\mathsf{T}^{\star 00}$ as implying the total mass density in the rest frame to be

$$\rho^\star_{\rm total} = \rho^\star_m + \frac{E^2}{8\pi c^2}\,. \tag{9}$$

2.1 The Capacitor Has Velocity $\mathbf{v} \parallel \mathbf{E}^\star$

In a frame where the capacitor has constant velocity $\mathbf{v} = v\,\hat{\mathbf{z}}$, the electric and magnetic fields between its plates are given by the transformation

$$\mathbf{E}_\parallel = \mathbf{E}_\parallel^\star = E^\star\,\hat{\mathbf{z}}, \tag{10}$$

$$\mathbf{E}_\perp = \gamma(\mathbf{E}_\perp^\star - \frac{\mathbf{v}}{c} \times \mathbf{B}^\star) = 0, \tag{11}$$

$$\mathbf{B}_\parallel = \mathbf{B}_\parallel^\star = 0, \tag{12}$$

$$\mathbf{B}_\perp = \gamma(\mathbf{B}_\perp^\star + \frac{\mathbf{v}}{c} \times \mathbf{E}^\star) = 0, \tag{13}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$. That is, the electromagnetic fields have the same values inside the capacitor in its rest frame and in frames where the capacitor moves with velocity $\mathbf{v}$ parallel to $\mathbf{E}$. Hence, the electromagnetic energy-momentum-stress tensor has the same component values in all such frames,

$$\mathsf{T}^{\mu\nu}_{EM} = \mathsf{T}^{\star\mu\nu}_{EM}. \tag{14}$$

It is useful to confirm this result via a Lorentz transformation of the stress tensor. The transformation L_z from the rest frame to a frame in which the capacitor has velocity $v\,\hat{\mathbf{z}}$ can be expressed in tensor form as

$$\mathsf{L}_z^{\mu\nu} = \left(\begin{array}{c|ccc} \gamma & 0 & 0 & \gamma\beta \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{array} \right), \tag{15}$$

where $\beta = v/c$. Then the transform of a tensor,

$$\mathsf{T}^{\star\mu\nu} = \left(\begin{array}{c|ccc} \mathsf{T}^{\star 00} & 0 & 0 & 0 \\ \hline 0 & \mathsf{T}^{\star 11} & 0 & 0 \\ 0 & 0 & \mathsf{T}^{\star 22} & 0 \\ 0 & 0 & 0 & \mathsf{T}^{\star 33} \end{array} \right), \tag{16}$$

that is diagonal in the rest frame is given by

$$\mathsf{T}^{\mu\nu} = (\mathsf{L}_z \mathsf{T}^\star \mathsf{L}_z)^{\mu\nu} = \left(\begin{array}{c|ccc} \gamma^2\mathsf{T}^{\star 00} + \gamma^2\beta^2\mathsf{T}^{\star 33} & 0 & 0 & \gamma^2\beta(\mathsf{T}^{\star 00} + \mathsf{T}^{\star 33}) \\ \hline 0 & \mathsf{T}^{\star 11} & 0 & 0 \\ 0 & 0 & \mathsf{T}^{\star 22} & 0 \\ \gamma^2\beta(\mathsf{T}^{\star 00} + \mathsf{T}^{\star 33}) & 0 & 0 & \gamma^2\beta^2\mathsf{T}^{\star 00} + \gamma^2\mathsf{T}^{\star 33} \end{array} \right). \tag{17}$$

In particular, the transformation of $\mathsf{T}^{\star\mu\nu}_{EM}$, eq. (5), is

$$\mathsf{T}^{\mu\nu}_{EM} = \frac{E^{\star 2}}{8\pi}\left(\begin{array}{c|ccc} \gamma^2(1-\beta^2) & 0 & 0 & \gamma^2\beta(1-1) \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma^2\beta(1-1) & 0 & 0 & -\gamma^2(1-\beta^2) \end{array}\right) = \frac{E^{\star 2}}{8\pi}\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \mathsf{T}^{\star\mu\nu}_{EM}, \tag{18}$$

as found above.

Similarly, the transformation of the mechanical stress tensor $\mathsf{T}^{\star\mu\nu}_{\rm mech}$, eq. (7), is

$$\mathsf{T}^{\mu\nu}_{\rm mech} = \left(\begin{array}{c|ccc} \gamma^2\rho^\star_m c^2 + \gamma^2\beta^2\frac{E^{\star 2}}{8\pi} & 0 & 0 & \gamma^2\beta\left(\rho^\star_m c^2 + \frac{E^{\star 2}}{8\pi}\right) \\ \hline 0 & -\frac{E^{\star 2}}{8\pi} & 0 & 0 \\ 0 & 0 & -\frac{E^{\star 2}}{8\pi} & 0 \\ \gamma^2\beta\left(\rho^\star_m c^2 + \frac{E^{\star 2}}{8\pi}\right) & 0 & 0 & \gamma^2\beta^2\rho^\star_m c^2 + \gamma^2\frac{E^{\star 2}}{8\pi} \end{array}\right), \tag{19}$$

the transformation of the total stress tensor $\mathsf{T}^{\star\mu\nu}_{\rm total}$, eq. (8), is

$$\mathsf{T}^{\mu\nu}_{\rm total} = \left(\begin{array}{c|ccc} \gamma^2\rho^\star_{\rm total}c^2 & 0 & 0 & \gamma^2\beta\rho^\star_{\rm total}c^2 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \gamma^2\beta\rho^\star_{\rm total}c^2 & 0 & 0 & \gamma^2\beta^2\rho^\star_{\rm total}c^2 \end{array}\right) = \mathsf{T}^{\mu\nu}_{\rm EM} + \mathsf{T}^{\mu\nu}_{\rm mech}, \tag{20}$$

with the total mass density $\rho^\star_{\rm total}$ given by eq. (9).

One noteworthy feature of eq (20) is the nonzero value of $\mathsf{T}^{33}_{\rm total}$. We recall that the purely spatial components of a stress-energy-momentum tensor have the dual interpretation as the momentum-flux tensor. In the present case, the flux of momentum is in the z direction, with magnitude equal to the momentum density times v, namely $(\gamma\rho^\star_{\rm total}v)\cdot v = \gamma^2\beta^2\rho^\star_{\rm total}c^2 = \mathsf{T}^{33}_{\rm total}$.

Turning to the component $\mathsf{T}^{00}_{\rm total}$, we note that the mass of the material between the capacitor plates, when moving with velocity $\mathbf{v}$, is larger than its rest mass by the factor γ. However, a moving volume element is smaller by the factor $1/\gamma$ than when that element is at rest. Hence, the mass density ρ_m is larger by a factor of γ^2 for the moving capacitor than when at rest,

$$\rho_m = \gamma^2\rho^\star_m. \tag{21}$$

Thus, the component $\mathsf{T}^{00}_{\rm total}$ transforms as expected for a mass density.

Furthermore, the four components

$$(\mathsf{T}^{00}_{\rm total}, \mathsf{T}^{01}_{\rm total}, \mathsf{T}^{02}_{\rm total}, \mathsf{T}^{03}_{\rm total}) \tag{22}$$

of the total energy-momentum-stress tensor transform as an energy-momentum-density 4-vector, although this is not the case for the sets of components

$$(\mathsf{T}^{00}_{EM}, \mathsf{T}^{01}_{EM}, \mathsf{T}^{02}_{EM}, \mathsf{T}^{03}_{EM}) \qquad \text{or} \qquad (\mathsf{T}^{00}_{\rm mech}, \mathsf{T}^{01}_{\rm mech}, \mathsf{T}^{02}_{\rm mech}, \mathsf{T}^{03}_{\rm mech}) \tag{23}$$

separately. This illustrates a general result that within volumes that contain both electromagnetic fields and matter, the concepts of electromagnetic momentum density, $\mathsf{T}^{0i}_{EM}/c = \mathbf{E} \times \mathbf{B}/4\pi c$, and mechanical momentum density, $\mathsf{T}^{0i}_{\rm mech}/c$, are not consistent with being components of a energy-momentum 4-vector; only the total momentum density, $\mathsf{T}^{0i}_{\rm total}/c$, is satisfactory in this respect. The great utility of the concept of "electromagnetic" momentum in matter-free regions leads us to attach similar significance to it in systems containing matter. However, this often results in difficulties in the interpretation of the "mechanical" part of the momentum.

For a capacitor moving with $\mathbf{v} \parallel \mathbf{E}$ there is no electromagnetic field momentum density ($\mathsf{T}^{0i}_{EM} = 0$), and the mechanical momentum density is given from eq. (19) as

$$\mathbf{p}_{\rm mech} = \gamma^2 \left(\rho^\star_m + \frac{E^{\star 2}}{8\pi c^2} \right) \mathbf{v} = \rho_{\rm total} \mathbf{v}. \tag{24}$$

This suggests that we consider the term $E^{\star 2}/8\pi c^2$ to be part of the mechanical mass density of the capacitor in the frame in which the latter has velocity $\mathbf{v} \parallel \mathbf{E}$. However, the component $\mathsf{T}^{00}_{\rm mech}$ of eq. (19) suggests that we interpret the mechanical mass density to be

$$\boldsymbol{\rho}_{\rm mech} = \gamma^2 \left(\rho^\star_m + \beta^2 \frac{E^{\star 2}}{8\pi c^2} \right) \neq \rho_{\rm total}. \tag{25}$$

That is, the interpretation of the mechanical energy-momentum-stress tensor in terms of mechanical mass and momentum densities leads us to the unsatisfactory relation that

$$\mathbf{p}_{\rm mech} \neq \boldsymbol{\rho}_{\rm mech} \mathbf{v}. \tag{26}$$

One attitude towards this difficulty is to de-emphasize eqs. (25)-(26) and to focus instead on the possible meaning of the mechanical momentum density (24). This approach is associated with the terminology "hidden" mechanical momentum. See, for example, [8].

For example, we do not interpret the vector $\gamma^2 E^{\star 2}\mathbf{v}/8\pi c^2$ that appears in eq. (24) as an electromagnetic field momentum density. We could, however, describe this term as a "hidden" mechanical momentum density. An "explanation" of this "hidden" mechanical momentum density could then be based on eq. (17); namely that since this term can be expressed as $\gamma^2 \beta \mathsf{T}^{\star 33}$, we associate it with the mechanical stress in the capacitor.[2]

[2] Note that mechanical stress is a microscopic manifestation of electromagnetic forces that are not described by the macroscopic fields **E** and **B** considered here. These microscopic electromagnetic fields in matter could be described as "hidden". Then, saying that the "hidden" mechanical momentum is associated with mechanical stress further associates "hidden" mechanical momentum with "hidden" electromagnetic fields.

2.2 The Capacitor Has Velocity $\mathbf{v} \perp \mathbf{E}^\star$

In a frame where the capacitor has constant velocity $\mathbf{v} = v\,\hat{\mathbf{x}}$, the electric and magnetic fields between its plates are given by the transformation

$$\mathbf{E}_\parallel = \mathbf{E}^\star_\parallel = 0, \tag{27}$$

$$\mathbf{E}_\perp = \gamma(\mathbf{E}^\star_\perp - \frac{\mathbf{v}}{c} \times \mathbf{B}^\star) = \gamma E^\star\,\hat{\mathbf{z}}, \tag{28}$$

$$\mathbf{B}_\parallel = \mathbf{B}^\star_\parallel = 0, \tag{29}$$

$$\mathbf{B}_\perp = \gamma(\mathbf{B}^\star_\perp + \frac{\mathbf{v}}{c} \times \mathbf{E}^\star) = -\gamma\frac{v}{c}E^\star\,\hat{\mathbf{y}}. \tag{30}$$

Using eqs. (1)-(4) together with eqs. (27)-(30), the electromagnetic energy-momentum-stress tensor of the moving capacitor is

$$\mathsf{T}^{\mu\nu}_{EM} = \frac{E^{\star 2}}{8\pi}\left(\begin{array}{c|ccc} \gamma^2(1+\beta^2) & 2\gamma^2\beta & 0 & 0 \\ \hline 2\gamma^2\beta & \gamma^2(1+\beta^2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right). \tag{31}$$

We confirm this result using the Lorentz transformation L_x from the rest frame to a frame in which the capacitor has velocity $v\,\hat{\mathbf{x}}$,

$$\mathsf{L}^{\mu\nu}_x = \left(\begin{array}{c|ccc} \gamma & \gamma\beta & 0 & 0 \\ \hline \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right). \tag{32}$$

Then, the transform of a tensor (16) that is diagonal in the rest frame is given by

$$\mathsf{T}^{\mu\nu} = (\mathsf{L}_x\mathsf{T}^\star\mathsf{L}_x)^{\mu\nu} = \left(\begin{array}{c|ccc} \gamma^2\mathsf{T}^{\star 00} + \gamma^2\beta^2\mathsf{T}^{\star 11} & \gamma^2\beta(\mathsf{T}^{\star 00} + \mathsf{T}^{\star 11}) & 0 & 0 \\ \hline \gamma^2\beta(\mathsf{T}^{\star 00} + \mathsf{T}^{\star 11}) & \gamma^2\beta^2\mathsf{T}^{\star 00} + \gamma^2\mathsf{T}^{\star 11} & 0 & 0 \\ 0 & 0 & \mathsf{T}^{\star 22} & 0 \\ 0 & 0 & 0 & \mathsf{T}^{\star 33} \end{array}\right), \tag{33}$$

In particular, the transformation of $\mathsf{T}^{\star\mu\nu}_{EM}$, eq. (5), is

$$\mathsf{T}^{\mu\nu}_{EM} = \frac{E^{\star 2}}{8\pi}\left(\begin{array}{c|ccc} \gamma^2(1+\beta^2) & 2\gamma^2\beta & 0 & 0 \\ \hline 2\gamma^2\beta & \gamma^2(1+\beta^2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right), \tag{34}$$

as found above.

Similarly, the transformation of the mechanical stress tensor $\mathsf{T}^{\star\mu\nu}_{\rm mech}$, eq. (7), is

$$\mathsf{T}^{\mu\nu}_{\rm mech} = \left(\begin{array}{c|ccc} \gamma^2 \rho^\star_m c^2 - \gamma^2\beta^2 \frac{E^{\star 2}}{8\pi} & \gamma^2\beta\left(\rho^\star_m c^2 - \frac{E^{\star 2}}{8\pi}\right) & 0 & 0 \\ \hline \gamma^2\beta\left(\rho^\star_m c^2 - \frac{E^{\star 2}}{8\pi}\right) & \gamma^2\beta^2\rho^\star_m c^2 - \gamma^2\frac{E^{\star 2}}{8\pi} & 0 & 0 \\ 0 & 0 & -\frac{E^{\star 2}}{8\pi} & 0 \\ 0 & 0 & 0 & \frac{E^{\star 2}}{8\pi} \end{array} \right), \tag{35}$$

the transformation of the total stress tensor $\mathsf{T}^{\star\mu\nu}_{\rm total}$, eq. (8), is

$$\mathsf{T}^{\mu\nu}_{\rm total} = \left(\begin{array}{c|ccc} \gamma^2 \rho^\star_{\rm total} c^2 & \gamma^2\beta\rho^\star_{\rm total} c^2 & 0 & 0 \\ \hline \gamma^2\beta\rho^\star_{\rm total} c^2 & \gamma^2\beta^2\rho^\star_{\rm total} c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) = \mathsf{T}^{\mu\nu}_{\rm EM} + \mathsf{T}^{\mu\nu}_{\rm mech}, \tag{36}$$

with the total mass density $\rho^\star_{\rm total}$ again given by eq. (9).[3]

From the component $\mathsf{T}^{00}_{\rm mech}$ we infer that the mechanical mass density of a capacitor with velocity $\mathbf{v}$ perpendicular to $\mathbf{E}$ is given by

$$\rho_{\rm mech} = \gamma^2 \left(\rho^\star_m - \beta^2 \frac{E^{\star 2}}{8\pi c^2} \right). \tag{37}$$

Note that for small velocity, $v \ll c$, and large electric field strength $E^\star$ it is possible that the mechanical mass density $\rho_{\rm mech}$ for the moving capacitor is smaller than $\rho^\star_m$ for a capacitor at rest.

The mechanical momentum density for the moving capacitor follows from eq.(35) as

$$\mathbf{p}_{\rm mech} = \gamma^2 \left(\rho^\star_m - \frac{E^{\star 2}}{8\pi c^2} \right) \mathbf{v} \neq \rho_{\rm mech}\mathbf{v}. \tag{38}$$

Nonetheless, we could designate the term $-\gamma^2 E^{\star 2}\mathbf{v}/8\pi c^2$ as the "hidden" mechanical momentum density for a capacitor with velocity $\mathbf{v} \perp \mathbf{E}$. This momentum density opposes the "ordinary" mechanical momentum density $\gamma\rho^\star\mathbf{v}$, because the mechanical mass density of the moving capacitor is smaller than its "ordinary" mass density $\gamma^2\rho^\star_m$.

[3]The top row of eq. (36) shows that the flow of total energy between the capacitor plates equals the energy density there times the velocity of the capacitor. However, the top rows of eqs. (34)-(35) indicate that this is not true separately for the electromagnetic and mechanical energies. If the dielectric between the capacitor plates does not cover the entire area of the plates, such that there are regions in which the only form of energy density is electromagnetic, then the flow of energy here is not simply the energy density times the bulk velocity of the capacitor. Since this is a steady-state example, there must be a closed circulation of energy flow, which therefore includes some flow opposite to the direction of motion of the capacitor. To account for this circulation, we must consider both the fringe field of the capacitor, and the mechanical energy density inside the (stressed) plates of the capacitor [4]. This more complicated variant has counterintuitive aspects also encountered is examples such as the belt drive considered by Taylor and Wheeler [9, 10].

However, we must remain aware that partitioning the total momentum into "electromagnetic" and "mechanical" pieces is ambiguous [1], so that further characterization of pieces of the mechanical momentum will not be satisfactory in all respects.

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