

# Faraday Rotation

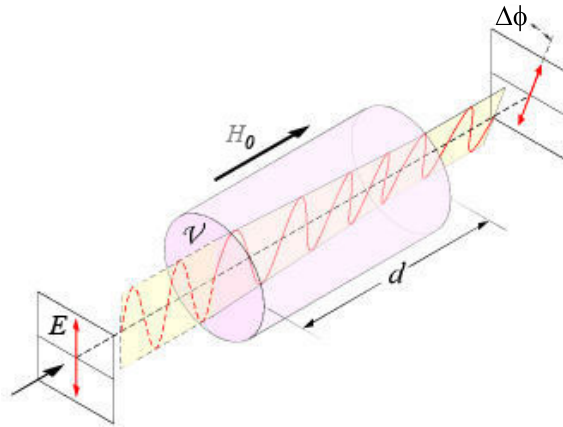
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## 1 Problem

A nonconducting cylinder of length  $d$  is immersed in a uniform, static, external magnetic field  $\mathbf{H}_0$  parallel to its axis, as shown below. A linearly polarized electromagnetic wave (typically a beam of light) with electric field  $\mathbf{E} = E_i e^{i(kz - \omega t)} \hat{\mathbf{x}}$  is incident on one end of the cylinder, where  $\omega = 2\pi\nu$  is the angular frequency of the wave, and  $k = \omega/c = 2\pi/\lambda$  is the wave number in vacuum, and  $c$  is the speed of light. Deduce the small angle  $\Delta\phi$  by which the plane of polarization of the transmitted wave is rotated with respect to that of the incident wave.



Give separate (classical) discussions for polarizable media and for magnetic media.

This effect was discovered by Faraday in 1845 [1] and was the first clear evidence for electromagnetic effects on the propagation of light. For an extensive bibliography through 1967, see [2].

## 2 Solution

### 2.1 Microscopic Analysis for a Polarizable Gaseous Medium

We first give an analysis for a gaseous medium in which the index of refraction is near unity.<sup>1</sup> Here, we ignore any magnetization of the medium.

The sense of the analysis is that there is a different index of refraction for left- and right-handed circularly polarized waves that propagate parallel to the external magnetic field. Then, the left- and right-handed components of a linearly polarized wave accumulate a phase difference as they traverse the medium, such that the direction of linear polarization changes with time/distance.

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<sup>1</sup>This section follows sec. 20 of [3]. See also [4]. Compare the case of propagation of waves along magnetic field lines in the Earth's ionosphere [5].

We follow the usual microscopic analysis of the index of refraction of a polarizable medium by deducing the electric dipole moment  $\mathbf{p} = -e \mathbf{x}$  on an electron of charge  $-e$  and mass  $m$  that is bound to the origin by a spring of constant  $K = m\omega_0^2$  under the influence of the external magnetic field  $\mathbf{B}_0 = \mathbf{H}_0 = H_0 \hat{\mathbf{z}}$  and a weak electromagnetic wave with (transverse) electric field  $\mathbf{E}_w e^{i(kz-\omega t)}$  and  $B_w = E_w \ll B_0$ . We suppose that the velocity of the electron in this field is small compared to the speed of light, so that the magnetic field of the wave does not influence the motion of the electron. Then, the equation of motion of the electron is

$$m\ddot{\mathbf{x}} = -m\omega_0^2 \mathbf{x} - e \left( \mathbf{E}_w e^{i(kz-\omega t)} + \frac{\mathbf{v}}{c} \times H_0 \hat{\mathbf{z}} \right). \quad (1)$$

We henceforth assume that the electron remains close to its rest position, such that  $z$  in eq. (1) can be regarded as a constant. Using the trial solution  $\mathbf{x} = \mathbf{x}_0 e^{i(kz-\omega t)}$ , we find

$$(\omega_0^2 - \omega^2) \mathbf{x}_0 - \frac{i\omega e H_0}{mc} \mathbf{x}_0 \times \hat{\mathbf{z}} = -\frac{e}{m} \mathbf{E}_w, \quad (2)$$

which implies that the displacement  $\mathbf{x}_0$  is in the  $x$ - $y$  plane.

For any vector  $\mathbf{A}$  that is transverse to the  $z$ -axis we can write

$$\begin{aligned} \mathbf{A} &= A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} = A_x \left( \frac{\hat{\mathbf{x}} + i\hat{\mathbf{y}}}{2} + \frac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{2} \right) - iA_y \left( \frac{\hat{\mathbf{x}} + i\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{2} \right) \\ &= \frac{A_x - iA_y}{\sqrt{2}} \frac{\hat{\mathbf{x}} + i\hat{\mathbf{y}}}{\sqrt{2}} + \frac{A_x + iA_y}{\sqrt{2}} \frac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{\sqrt{2}} \equiv A_- \hat{\mathbf{e}}_+ + A_+ \hat{\mathbf{e}}_-, \end{aligned} \quad (3)$$

where

$$A_{\pm} = \frac{A_x \pm iA_y}{\sqrt{2}} \quad \text{and} \quad \hat{\mathbf{e}}_{\pm} = \frac{\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}}{\sqrt{2}}. \quad (4)$$

Then,

$$\hat{\mathbf{e}}_{\pm} \times \hat{\mathbf{z}} = \frac{\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}}{\sqrt{2}} \times \hat{\mathbf{z}} = \frac{-\hat{\mathbf{y}} \pm i\hat{\mathbf{x}}}{\sqrt{2}} = \pm i \frac{\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}}{\sqrt{2}} = \pm i \hat{\mathbf{e}}_{\pm}. \quad (5)$$

The equation of motion (2) can now be written

$$(\omega_0^2 - \omega^2) (x_{0-} \hat{\mathbf{e}}_+ + x_{0+} \hat{\mathbf{e}}_-) + \omega\omega_H (x_{0-} \hat{\mathbf{e}}_+ - x_{0+} \hat{\mathbf{e}}_-) = -\frac{e}{m} (E_{w-} \hat{\mathbf{e}}_+ + E_{w+} \hat{\mathbf{e}}_-), \quad (6)$$

where

$$\omega_H = \frac{eH_0}{mc} \quad (7)$$

is the Larmor (cyclotron) frequency of an electron in the static magnetic field  $H_0$ .

The equation of motion (6) in the  $\hat{\mathbf{e}}_{\pm}$  basis does not mix the components  $x_{0\pm}$  (which is why we chose to use that basis), so we immediately find that

$$x_{0\pm} = -\frac{e}{m} \frac{E_{w\pm}}{\omega_0^2 - \omega^2 \mp \omega\omega_H}. \quad (8)$$

The resulting electric polarization  $\mathbf{P}_w$  of the medium, with number density  $N$  of electrons, is

$$\mathbf{P}_w = -Nex = -Ne(x_{0-} \hat{\mathbf{e}}_+ + x_{0+} \hat{\mathbf{e}}_-) \equiv P_{w-} \hat{\mathbf{e}}_+ + P_{w+} \hat{\mathbf{e}}_-, \quad (9)$$

where

$$P_{w\pm} = \frac{Ne^2}{m} \frac{E_{w\pm} e^{i(kz-\omega t)}}{\omega_0^2 - \omega^2 \mp \omega\omega_H}. \quad (10)$$

The electric displacement  $\mathbf{D}_w$  of the wave is

$$\begin{aligned} \mathbf{D}_w &= \mathbf{E}_w + 4\pi\mathbf{P}_w = [(E_{w-} + 4\pi P_-) \hat{\mathbf{e}}_+ + (E_{w+} + 4\pi P_+) \hat{\mathbf{e}}_-] e^{i(kz-\omega t)} \\ &\equiv (\epsilon_- E_{w-} \hat{\mathbf{e}}_+ + \epsilon_+ E_{w+} \hat{\mathbf{e}}_-) e^{i(kz-\omega t)}, \end{aligned} \quad (11)$$

where we introduce two dielectric constants  $\epsilon_{\pm}$  according to

$$\epsilon_{\pm} = 1 + \frac{4\pi Ne^2/m}{\omega_0^2 - \omega^2 \mp \omega\omega_H} = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 \mp \omega\omega_H}, \quad (12)$$

and we recall that

$$\omega_p = \sqrt{\frac{4\pi Ne^2}{m}} \quad (13)$$

is the plasma frequency of the medium. Corresponding to the dielectric constants (12) are two indices of refraction,<sup>2</sup>

$$n_{\pm} = \sqrt{\epsilon_{\pm}} \approx 1 + \frac{\omega_p^2}{2(\omega_0^2 - \omega^2 \mp \omega\omega_H)}, \quad (14)$$

where the approximations hold for gaseous media where  $n_{\pm} \approx 1$ , and two wave numbers

$$k_{\pm} = \frac{\omega n_{\pm}}{c}. \quad (15)$$

Hence, we learn that the two wave components  $E_{w-} \hat{\mathbf{e}}_+$  and  $E_{w+} \hat{\mathbf{e}}_-$  propagate along the  $z$ -direction with different velocities, and the wave function  $\mathbf{E}_w$  should actually be written

$$\mathbf{E}_w = E_{w-} e^{i(k_- z - \omega t)} \hat{\mathbf{e}}_+ + E_{w+} e^{i(k_+ z - \omega t)} \hat{\mathbf{e}}_-, \quad (16)$$

and similarly for the fields  $\mathbf{D}_w$  and  $\mathbf{P}_w$ . The wave  $E_{w-} e^{i(k_- z - \omega t)} \hat{\mathbf{e}}_+$  is designated as **left-handed circularly polarized**, and from eqs.(12) and (15) we see that for frequencies  $\omega < \omega_0$  (as holds at optical frequencies in typical media) the velocity of this wave is smaller than that of the right-handed circularly polarized wave  $E_{w+} e^{i(k_+ z - \omega t)} \hat{\mathbf{e}}_-$ .

Turning at last to the particular context of this problem, we suppose that the medium extends from  $z = 0$  to  $L$ , and that the wave enters the medium with linear polarization in the  $x$ -direction. That is, at  $z = 0$  the electric field is  $\mathbf{E}_w(z = 0) = E_w e^{-i\omega t} \hat{\mathbf{x}}$ . In this case we see from eq. (4) that  $E_{w+}(z = 0) = E_{w-}(z = 0) = E_w/\sqrt{2}$ . Then, according to eq. (16) the waveform at the exit of the medium is

$$\mathbf{E}_w(z = L) = \frac{E_w}{\sqrt{2}} \left( e^{ik_- L} \hat{\mathbf{e}}_+ + e^{ik_+ L} \hat{\mathbf{e}}_- \right) e^{-i\omega t}$$

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<sup>2</sup>In principle, there is a contribution to the index of refraction  $n = \sqrt{\epsilon\mu}$  due to the diamagnetic permeability  $\mu$  of the medium. In the present model, the driven atomic electrons are associated with magnetic moments  $-(e/2c)(x_{0+}v_+ - x_{0-}v_-) = -(e\omega/2c)(x_{0+}^2 - x_{0-}^2) \propto B_\omega^2$ , so the diamagnetic permeability differs from unity by a small, nonlinear correction that we ignore.

$$\begin{aligned}
&= \frac{E_w}{2} \left[ \left( e^{ik_-L} + e^{ik_+L} \right) \hat{\mathbf{x}} + i \left( e^{ik_-L} - e^{ik_+L} \right) \hat{\mathbf{y}} \right] e^{-i\omega t} \\
&= \frac{E_w}{2} \left[ \left( e^{i(k_- - k_+)L/2} + e^{-i(k_- - k_+)L/2} \right) \hat{\mathbf{x}} + i \left( e^{i(k_- - k_+)L/2} - e^{-i(k_- - k_+)L/2} \right) \hat{\mathbf{y}} \right] \\
&\quad e^{i(k_- + k_+)L/2} e^{-i\omega t} \\
&= E_w \left( \cos \frac{(k_- - k_+)L}{2} \hat{\mathbf{x}} - \sin \frac{(k_- - k_+)L}{2} \hat{\mathbf{y}} \right) e^{i[(k_- + k_+)/2 - k]L} e^{i(kL - \omega t)} \\
&= E_w \left( \cos \frac{\Delta n \omega L}{2c} \hat{\mathbf{x}} + \sin \frac{\Delta n \omega L}{2c} \hat{\mathbf{y}} \right) e^{i[kL - \omega t + (n_{\text{ave}} - 1)\omega L/c]}, \tag{17}
\end{aligned}$$

where

$$\Delta n = n_+ - n_- \approx \frac{\omega \omega_H \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2}, \quad n_{\text{ave}} = \frac{n_+ + n_-}{2} \approx 1 + \frac{(\omega_0^2 - \omega^2) \omega_p^2}{2[(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2]}, \tag{18}$$

and the approximations hold for gaseous media where  $n_{\pm} \approx 1$ .

The wave in vacuum for  $z > L$  has a phase retardation  $\delta = (n_{\text{ave}} - 1)\omega L/c$  as is typical for wave propagation through a dielectric medium of length  $L$  at velocity  $c/n_{\text{ave}}$ ; and the wave is linearly polarized at angle

$$\Delta\phi = \frac{\Delta n \omega L}{2c} \approx \frac{e}{2mc^2} \frac{\omega^2 \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2} H_0 L \equiv V H_0 L, \tag{19}$$

with respect to the  $x$ -axis, where

$$V \approx \frac{e}{2mc^2} \frac{\omega^2 \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2} \tag{20}$$

is called the Verdet constant. This change in angle of the polarization of the wave is the Faraday rotation.

The wave takes time  $\Delta t = Ln_{\text{ave}}/c$  to traverse the medium of length  $L$ , so the direction of polarization of the wave inside the medium precesses at rate<sup>3</sup>

$$\Omega = \frac{\Delta\phi}{\Delta t} = \frac{\Delta n}{2n_{\text{ave}}} \omega = \frac{n_+ - n_-}{n_+ + n_-} \omega \approx \frac{\omega^2 \omega_p^2}{2[(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2] + \omega_p^2(\omega_0^2 - \omega^2)} \omega_H. \tag{21}$$

In general, the Faraday rotation is very small, except when the wave frequency  $\omega$  is close to a natural frequency  $\omega_0$  of the polarizable medium.

## 2.2 Microscopic Analysis for a Magnetic Medium

Many important examples of Faraday rotation occur in magnetic rather than dielectric media. Classical models of magnetic media are less satisfactory than those for dielectric media. Here we give a model for the Faraday effect in magnetic media which is fairly plausible, following Becquerel [7]. See also [8].

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<sup>3</sup>If the external magnetic field  $\mathbf{H}_0$  is in the  $-z$ -direction, then  $H_0$ ,  $\omega_H$  and  $\Omega$  are negative. This implies that vectors  $\mathbf{H}_0$  and  $\mathbf{\Omega}$  are in the same direction in all cases. An elaborate discussion of this factoid is given in [6].

We suppose that the bulk magnetization density  $\mathbf{M} = N\boldsymbol{\mu}$  of the (nonconducting) medium is due to a distribution of individual magnetic moments  $\boldsymbol{\mu}$  at  $N$  sites per unit volume. These magnetic moments have fixed locations inside the medium, but the direction of the moment is affected by a magnetic field  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$  according to the torque equation

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = \boldsymbol{\mu} \times \mathbf{H}, \quad (22)$$

where  $\mathbf{L} = -\boldsymbol{\mu}/\Gamma$  is the angular momentum associated with a magnetic moment  $\boldsymbol{\mu}$ . We recall that  $\Gamma = e/2mc$  in case the magnetic moment is due to orbital motion of an electron, while  $\Gamma = e/mc$  for the intrinsic (spin) magnetic moment of an electron. The torque equation (22) can be rewritten as

$$\frac{d\boldsymbol{\mu}}{dt} = \Gamma\mathbf{H} \times \boldsymbol{\mu} = \boldsymbol{\omega}_H \times \boldsymbol{\mu}, \quad (23)$$

which implies that the magnetic moment  $\boldsymbol{\mu}$  precesses about the direction of  $\mathbf{H}$  with angular velocity

$$\boldsymbol{\omega}_H = \Gamma\mathbf{H} = \begin{cases} \frac{eH}{2mc} \hat{\mathbf{H}} & \text{(orbital),} \\ \frac{eH}{mc} \hat{\mathbf{H}} & \text{(spin).} \end{cases} \quad (24)$$

The sense of precession is the same for any direction of the moment  $\boldsymbol{\mu}$ . A small leap in the classical argument is that the precession occurs even when the moment is exactly parallel or antiparallel to the external field  $\mathbf{H}$ .

Numerically,  $eH/mc = 1.8 \times 10^{11}$  for  $H = 10,000$  gauss, so the precession frequency in laboratory experiments is typically small compared to optical frequencies. Hence, we ignore the effect of the magnetic field  $\mathbf{H}_w$  of the incident wave on the magnetic moments, and take  $\mathbf{H}$  in eq. (24) to be only the external static field  $H_0 \hat{\mathbf{z}}$ .

We now consider the interaction of the magnetic medium with an optical wave that propagates along the  $z$ -axis. As in eq. (17), we can decompose this wave into left- and right-handed circularly polarized components,

$$\mathbf{E}_w = E_{w-} e^{i(kz - \omega t)} \hat{\mathbf{e}}_+ + E_{w+} e^{i(kz + \omega t)} \hat{\mathbf{e}}_-, \quad (16)$$

The electric field vector of the left-handed component,  $E_{w-} e^{i(kz - \omega t)} \hat{\mathbf{e}}_+$ , rotates with angular velocity  $\omega \hat{\mathbf{z}}$  at a fixed value of  $z$ , while that of the right-handed component rotates with angular velocity  $-\omega \hat{\mathbf{z}}$ . We argue that because of the precession (24), the component fields rotate at angular velocities

$$\omega_{\pm} = \omega \pm \omega_H \quad (25)$$

relative to the electronic structure of the medium, so that the index of refraction for this component is

$$n_{\pm} = n(\omega_{\pm}) = n(\omega) \pm \omega_H \frac{dn(\omega)}{d\omega}. \quad (26)$$

As before, we consider the sum and difference of the indices of refraction,

$$\Delta n = n_+ - n_- = 2\omega_H \frac{dn(\omega)}{d\omega}, \quad n_{\text{ave}} = \frac{n_+ + n_-}{2} = n(\omega), \quad (27)$$

such that the electric field at distance  $z = L$  within the magnetic medium is

$$\mathbf{E}_w(z = L) = E_w \left( \cos \frac{\Delta n \omega L}{2c} \hat{\mathbf{x}} + \sin \frac{\Delta n \omega L}{2c} \hat{\mathbf{y}} \right) e^{i[kL - \omega t + (n_{\text{ave}} - 1)\omega L/c]}, \quad (28)$$

when the wave entered the medium at  $z = 0$  with linear polarization in the  $x$ -direction. The wave at  $z = L$  is linearly polarized at angle

$$\Delta\phi = \frac{\Delta n \omega L}{2c} = \frac{\omega \omega_H}{c} \frac{dn}{d\omega} L = \Gamma \frac{\omega}{c} \frac{dn}{d\omega} H_0 L = -\Gamma \frac{\lambda}{c} \frac{dn}{d\lambda} H_0 L \equiv V H_0 L \quad (29)$$

with respect to the  $x$ -axis, where

$$V = -\Gamma \frac{\lambda}{c} \frac{dn}{d\lambda} = \begin{cases} -\frac{e}{2mc^2} \lambda \frac{dn}{d\lambda} & \text{(orbital),} \\ -\frac{e}{mc^2} \lambda \frac{dn}{d\lambda} & \text{(spin).} \end{cases} \quad (30)$$

is the Verdet constant of the medium. Note that  $dn/d\lambda$  is negative for typical optical materials.

The Verdet constant for some diamagnetic materials is reasonably close to the form of eq. (30) for orbital magnetization. See, for example, [8]. However, the Verdet constant for many diamagnetic materials is closer to 1/2 of the orbital prediction [9]. The largest Verdet constants are obtained with glassy materials doped with paramagnetic ions [10], for which, however, eq. (30) is not a particularly good description.

The wave takes time  $\Delta t = Ln_{\text{ave}}/c$  to traverse the medium of length  $L$ , so the direction of polarization of the wave inside the medium precesses at rate

$$\Omega = \frac{\Delta\phi}{\Delta t} = \frac{\Delta n \omega}{2n_{\text{ave}}} = -\frac{\lambda}{n} \frac{dn}{d\lambda} \omega_H. \quad (31)$$

*Although our discussion of the Faraday rotation in a magnetic medium appeared to emphasize the magnetic properties of that medium, the presence of the index of refraction  $n = \sqrt{\epsilon\mu}$  in eq. (29) implies that the dielectric properties of the medium are relevant also. Indeed, if  $\epsilon - 1 \gg \mu - 1$  we reconsider the microscopic model of the index from sec. 2.1, then taking  $n = n_{\text{ave}}$  according to eq. (18) leads to*

$$\frac{dn}{d\omega} \approx \frac{\omega \omega_p^2 [(\omega_0^2 - \omega^2)^2 + \omega_0^2 \omega_H^2]}{[(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2]^2} \approx \frac{\omega \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2}, \quad (32)$$

and the Faraday rotation associated with orbital magnetization is

$$\Delta\phi = \frac{e}{2mc^2} \omega \frac{dn}{d\omega} H_0 L \approx \frac{e}{2mc^2} \frac{\omega^2 \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega^2 \omega_H^2} H_0 L, \quad (33)$$

which is the same as that found in eq. (19).

Thus our two derivations of the Faraday rotation are essentially equivalent if the index of refraction is largely due to the dielectric properties of the medium, as is the case in most magneto-optic materials. It is perhaps counterintuitive that a derivation that begins with consideration of magnetic moments ends up with a form in which aspects of electric dipole moments are prominent. The latter form is, however, more similar to that found in derivations based on quantum theory. See, for example, [11].

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