Hyperdeuterons

Kirk T. McDonald
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
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1 Problem

Estimate the relative binding energies of the 64 possible pairs of baryons in the basic octet: n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^-, Ξ^0.

For this use a simplified 'one-pion-exchange' model that the nuclear force is entirely due to exchanges of a single pi meson, and that the operator $g\vec{\tau}_1 \cdot \vec{\tau}_2$ characterizes the 'charge independence of this interaction. Here g is a coupling constant, and $\vec{\tau}$ is the isospin-1 operator. That is, ignore electromagnetic effects and spin-dependent effects.

(A harder version of the problem would be to deduce that $g\vec{\tau}_1 \cdot \vec{\tau}_2$ is the appropriate operator.)

2 Solution

The hint is that the Hamiltonian relevant to binding of the dibaryons is $H \propto g\vec{\tau}_1 \cdot \vec{\tau}_2$. Hence we should consider the matrix elements $\langle N_1 N_2 | \vec{\tau}_1 \cdot \vec{\tau}_2 | N_1 N_2 \rangle$, where N is any member of the baryon octet. As for electricity, we infer that a negative matrix element implies an attractive force, while a positive matrix element implies repulsion.

We must recall that

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \frac{1}{2}(\tau_{+,1}\tau_{-,2} + \tau_{-,1}\tau_{+,2}) + \tau_{z,1}\tau_{z,2},$$

where

$$\tau_+ \equiv \tau_x + i\tau_y \qquad \tau_- \equiv \tau_x - i\tau_y,$$

and

$$\tau_+|t, t_z\rangle = \sqrt{(t + t_z + 1)(t - t_z)}|t, t_z + 1\rangle,$$

$$\tau_-|t, t_z\rangle = \sqrt{(t + t_z)(t - t_z + 1)}|t, t_z - 1\rangle,$$

Thus $\tau_+|n\rangle = |p\rangle$, and $\tau_-|p\rangle = |n\rangle$. Of course, $\tau_z|p\rangle = \frac{1}{2}|p\rangle$ and $\tau_z|n\rangle = -\frac{1}{2}|n\rangle$. (That is, the τ operators can be represented by Pauli spin matrices times 1/2.)

One can proceed in gory detail: For example, start with the two nucleon states: pp, nn and np. The pp diagram gets a factor $\frac{1}{2}$ at each vertex for an overall strength of $\frac{1}{4}$ (times g^2 which we ignore when comparing strengths). For the nn diagram we get $(-\frac{1}{2})(-\frac{1}{2}) = \frac{1}{4}$ also. Now for the np case we get $(-\frac{1}{2})(\frac{1}{2}) = -\frac{1}{4}$ for the diagram with π^0 exchange, and $(\frac{1}{2})(1)(1) = \frac{1}{2}$ for the diagram with π^+ exchange. But these two diagrams interfere, so we add the amplitudes, and the result is again $\frac{1}{4}$. Charge independence!

We have glossed over an important point in the above. There are really two kinds of np states: $\sqrt{\frac{1}{2}}(|np\rangle + |pn\rangle)$ and $\sqrt{\frac{1}{2}}(|np\rangle - |pn\rangle)$. The charge-independent result above is for the first state, which we recognize as the partner of $|pp\rangle$ and $|nn\rangle$ in the symmetric isospin-1 triplet. The second np state is the antisymmetric isospin-0 singlet. Show that this state is an eigenstate of $\vec{\tau}_1 \cdot \vec{\tau}_2$ with eigenvalue $-\frac{3}{4}$.

Thus we infer that the single np state is bound with a relatvie strength of $-3/4$ units, while the triplet pp, np, nn states are unbound (but a continuum level exists at $+1/4$ relative units).

Having examined 4 of the 64 dibaryon states one could now turn to the other 60. Note that

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \frac{1}{2}\left(\tau^2 - \tau_1^2 - \tau_2^2\right).$$

Also, charge independence means you don't have to look at each of the 64 pairs separately, but you can more simply consider pairs of isospin multiplets, each of which leads to one or more multiplets of total isospin exactly as for combinations of ordinary spin. For this note that the nucleons, N, and the cascade particles, Ξ, each form an isodoublet, the Λ is an isosinglet, and the Σ's form an isotriplet.

I found that 11 of the 64 pairs should have bounds states, and that none of these would be more weakly bound than the deuteron. Some of the proposed bound states have an interesting feature in their isospin wave function...

No dibaryon bound state other than the deuteron has ever been observed, but I'm not sure anyone has ever looked for the other nine predicted states....