Magnetars

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1 Problem

The x-ray pulsar SGR1806-20 has recently been observed to have a period T of 7.5 s and a relatively large "spindown" rate $\left|\dot{T}\right| = 8 \times 10^{-11}$ [1].

Calculate the maximum magnetic field at the surface of this pulsar, assuming it to be a standard neutron star of mass $1.4 M_\odot = 2.8 \times 10^{30}$ kg and radius 10 km, that the mass density is uniform, that the spindown is due to electromagnetic radiation, and that the angular velocity vector is perpendicular to the magnetic dipole moment of the pulsar.

Compare the surface magnetic field strength to the so-called QED critical field strength $m^2c^3/e\hbar = 4.4 \times 10^{13}$ gauss, at which electron-positron pair creation processes become highly probable.

2 Solution

In Gaussian units, the rate of magnetic dipole radiation is

$$\frac{dU}{dt} = \frac{2}{3}\frac{\ddot{\mathbf{m}}^2}{c^3} = \frac{2}{3}\frac{m^2\omega^4}{c^3}, \tag{1}$$

where $\omega = 2\pi/T$ is the angular velocity, taken to be perpendicular to the magnetic dipole moment $\mathbf{m}$.

The radiated power (1) is derived from a decrease in the rotational kinetic energy, $U = I\omega^2/2$, of the pulsar:

$$\frac{dU}{dt} = -I\omega\dot{\omega} = \frac{2}{5}MR^2\omega\,|\dot{\omega}|\,, \tag{2}$$

where the moment of inertia I is taken to be that of a sphere of uniform mass density. Combining eqs. (1) and (2), we have

$$m^2 = \frac{3}{5}\frac{MR^2\,|\dot{\omega}|\,c^3}{\omega^3}. \tag{3}$$

Substituting $\omega = 2\pi/T$, and $|\dot{\omega}| = 2\pi\left|\dot{T}\right|/T^2$, we find

$$m^2 = \frac{3}{20\pi^2}MR^2T\left|\dot{T}\right|c^3. \tag{4}$$

The static magnetic field $\mathbf{B}$ due to dipole $\mathbf{m}$ is

$$\mathbf{B} = \frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{r^3}, \tag{5}$$

so the peak field at radius R is

$$\mathbf{B} = \frac{2\mathbf{m}}{R^3}. \tag{6}$$

Inserting this in eq. (4), the peak surface magnetic field is related by

$$B^2 = \frac{3}{5\pi^2} \frac{MT \left| \dot{T} \right| c^3}{R^4} = \frac{3}{5\pi^2} \frac{(2.8 \times 10^{33})(7.5)(8 \times 10^{-11})(3 \times 10^{10})^3}{(10^6)^4} = 2.8 \times 10^{30} \text{ gauss}^2. \tag{7}$$

Thus, $B_{\rm peak} = 1.7 \times 10^{15}$ G $= 38 B_{\rm crit}$, where $B_{\rm crit} = 4.4 \times 10^{13}$ G.

When electrons and photons of kinetic energies greater than 1 MeV exist in a magnetic field with $B > B_{\rm crit}$, they rapidly lose this energy via electron-positron pair creation.

Kouveliotou *et al.* report that $B_{\rm peak} = 8 \times 10^{14}$ G without discussing details of their calculation.

References

[1] C. Kouveliotou *et al.*, *An X-ray pulsar with a superstrong magnetic field in the soft γ-ray repeater SGR1806-20*, Nature **393**, 235-237 (1998),
http://puhep1.princeton.edu/~mcdonald/examples/EM/kouveliotou_nature_393_235_98.pdf