Neutrino Interactions

So far all our considerations of the weak interaction have been restricted to particle decays which are forbidden by conservation laws of the strong and electromagnetic interactions. Greater experimental control of our studies would be possible if we could perform weak interaction scattering experiments. The only scattering processes in which weak effects would not be swamped by the strong and E&M interactions involve neutrinos. Although in the future, reactions like $e^- p \to n \nu_e$ will be studied, all weak interaction scattering experiments to date have utilized neutrino beams.

In this lecture we discuss neutrino scattering beginning with the first direct evidence for the existence of the neutrino obtained in 1956. In 1962 the distinction between ν_μ and ν_e was demonstrated. In a number of subsequent neutrino scattering experiments the structure of matter was been explored in the manner of inelastic electron-proton scattering. An extremely important result of the neutrino experiments is the demonstration of the weak neutral current in 1973. We defer until later lectures the confrontation of the Weinberg-Salam model with neutrino scattering data. But we will consider a topic of continuing interest — are neutrinos massless?

Throughout most of this lecture we will assume $M_\nu = 0$.

1. First Experimental Detection of Neutrinos

Although the neutrino was postulated by Pauli in 1930, direct evidence of its existence was first given in 1956 by Reines & Cowan [Science 124, 103 (1956)]. In a nuclear reactor the β decay of heavy nuclei, $Z \to (Z+1)\, e^- \bar\nu_e$ produces antineutrinos in great quantities. In the experiment of Reines and Cowan they put a neutrino detector near a reactor where the antineutrino flux was $10^{13}/\text{cm}^2/\text{sec}$! They looked for positrons produced by inverse β decay

$$\bar\nu p \to n e^+$$

Was it a health hazard to stand near their detector?

The typical anti-neutrino energy was 3 MeV, which would lead to positrons of energy $E_\nu - 1.3$ MeV, which were observed in a large tank of liquid scintillator.

We estimate the scattering cross section, noting that the lab frame and the c.m. frame are nearly identical in this case. So from p 80

$$\sigma \sim \frac{|\mathcal{M}|^2}{E_{cm}^2} \sim \frac{|\mathcal{M}|^2}{M_p^2}$$

$$\mathcal{M} = \text{V-A matrix element} = \frac{G}{\sqrt{2}}\underbrace{\left(\bar{v}_{e^+}\middle|\gamma_\alpha(1-\gamma_5)\middle|v_{\bar{\nu}}\right)}_{\approx E_\nu \text{ if } E_\nu \gg m_e}\underbrace{\left(\bar{u}_n\middle|\gamma_\alpha(1-\lambda\gamma_5)\middle|u_p\right)}_{\sim M_p \text{ in the non-relativistic limit (p. 291)}}$$

So $\sigma \sim G^2 E_\nu^2 \sim \dfrac{10^{-10}(3\,\text{MeV})^2}{M_p^4} \sim \dfrac{10^{-21}}{(\text{MeV})^2} \sim 10^{-43}\ \text{cm}^2$

Suppose the liquid scintillator tank was a 1 meter cube, and that liquid scintillator has density 1.

Neutrino flux $= \dfrac{10^{13}}{\text{cm}^2} \cdot 10^4\ \text{cm}^2 = 10^{17}/\text{sec}$

# of protons / $\text{cm}^2 \sim 6\times10^{23}\cdot 100 \sim 10^{26}$

So rate $=$ flux $\cdot$ # of protons/$\text{cm}^2 \cdot \sigma \sim 10^{17}\cdot10^{26}\cdot10^{-43} \sim 1$ per second

(not a health hazard). This is sufficient rate to do the experiment. Reines and Cowan measured the scattering cross section as

$$\sigma_{\bar{\nu} p \to n e^+} = 1.1 \pm 0.4 \times 10^{-43}\ \text{cm}^2 \qquad \text{at } \langle E_{\bar{\nu}}\rangle \sim 3\ \text{MeV}$$

2. Production of High Energy Neutrino Beams

Higher energy neutrino beams are obtained at particle accelerators rather than nuclear reactors. The principle of operation of these beams involves a good bit of weak interaction physics which we sketch briefly. There are 2 general techniques.

a. 'Broad Band' beams

Charged π's and K's are produced in high energy pp interactions. As many of the mesons as possible are focussed along the beam direction by a 'neutrino horn'

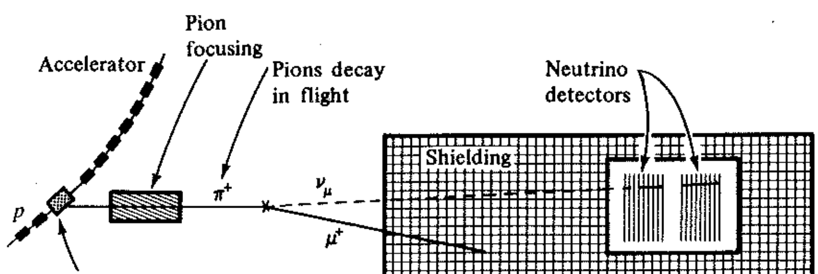
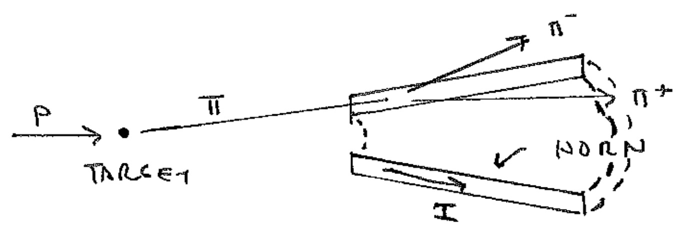


Fig. 11.9. Principle of high-energy muon neutrino experiments.

The horn is a cone of copper to which a large current pulse is applied when the protons strike the target. The azimuthal magnetic field then focuses one charge and



defocuses the other. If the horn can provide a 'kick' of $\Delta p \sim 300\ \text{MeV}/c$ then most particles (of the right sign) emerge parallel to the beam axis, as $\langle p_\perp\rangle \approx 300\ \text{MeV}/c$ in pp collisions (Lecture 12).

THE CHARGED PARTICLES, MOSTLY π AND K'S, ARE ALLOWED TO DECAY IN A LONG PIPE, BEFORE STRIKING A VERY THICK ABSORBER TO REMOVE ALL HADRONS AND MUONS. FOR A POSITIVE BEAM, NEUTRINOS ARE PRODUCED VIA

$$\pi^+ \to \mu^+ \nu_\mu \qquad K^+ \to \mu^+ \nu_\mu$$

ELECTRON NEUTRINOS COME FROM

$K^+ \to \pi^0 e^+ \nu_e$ OR $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$

THE LATTER DECAY ALSO CONTRIBUTES MUON ANTINEUTRINOS. OF COURSE SOME NEGATIVE PIONS AND KAONS CONTRIBUTE TO THE BEAM, YIELDING STILL MORE $\bar{\nu}_\mu$ ETC. THE BROAD MOMENTUM SPECTRUM OF π'S AND K'S CAPTURED BY THE HORN GIVES A BROAD ν SPECTRUM, PEAKED AT RATHER LOW ENERGY COMPARED TO THE PRIMARY PROTON ENERGY.

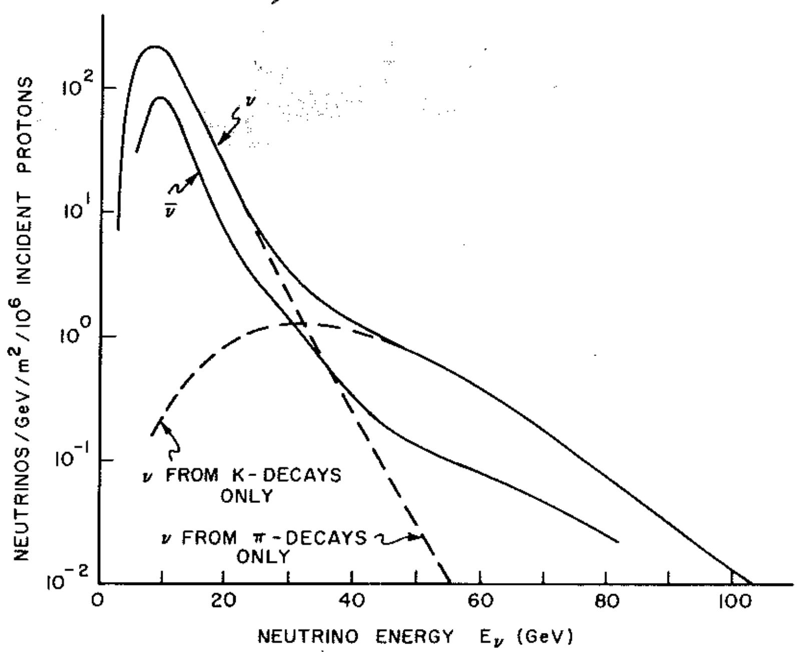


Figure 10-2 Neutrino/antineutrino spectra for a beam operating at 200-GeV primary proton energy (from Ref. 2).

6. DICHROMATIC BEAMS

WHEN A π OR A K OF A FIXED MOMENTUM DECAYS TO μν, THERE IS A MAXIMUM AND MINIMUM LABORATORY ENERGY GIVEN TO THE ν. AMAZINGLY THE MINIMUM ENERGY ν FROM K DECAY IS ALMOST EXACTLY EQUAL TO THE MAXIMUM ENERGY ν FROM π DECAY. SO IF ONE SELECTS ONLY π'S AND K'S OF A NARROW RANGE OF MOMENTUM A DICHROMATIC ν SPECTRUM CAN BE OBTAINED. OF COURSE THE ν FLUX IS MUCH REDUCED COMPARED TO THAT OBTAINED WITH THE BROAD BAND TECHNIQUE.

A DICHROMATIC BEAM HAS THE FURTHER ADVANTAGE THAT THE NEUTRINO ENERGY IS WELL CORRELATED WITH ITS PRODUCTION ANGLE.

Fig. 13.12. Schematic layout of narrow band neutrino beam of the CDHS group at CERN. (From Dydak, 1978.)

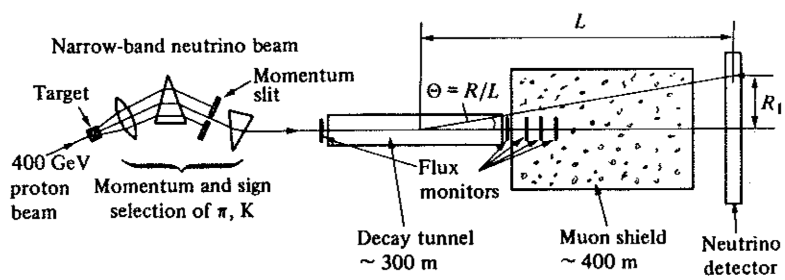
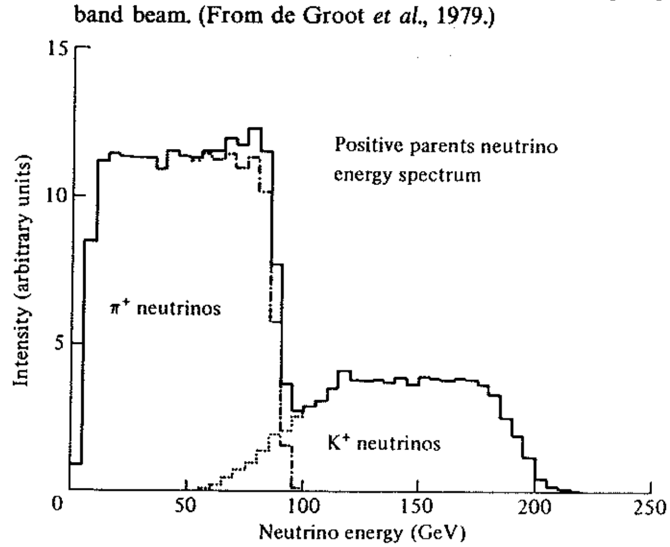
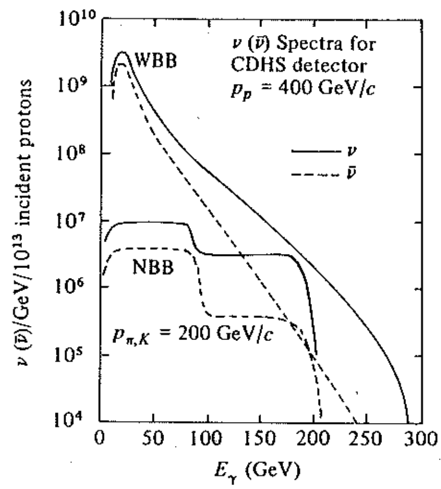


Fig. 13.13. Neutrino energy spectrum of the CDHS group's narrow band beam. (From de Groot *et al.*, 1979.)





LORENTZ TELLS US THAT

$$E^* = \gamma E_{LAB} (1 - \beta \cos \theta_{LAB})$$

WHERE $E^* = \nu$ ENERGY IN π OR K REST FRAME

$$= \frac{M_\pi^2 - M_\mu^2}{2 m_\pi} \quad \text{or} \quad \frac{M_K^2 - M_\mu^2}{2 M_K}$$

$$\gamma = \frac{E_{\pi,LAB}}{M_\pi} \quad \text{or} \quad \frac{E_{K,LAB}}{M_K}$$

THUS E_ν IS CORRELATED WITH THE POSITION OF THE NEUTRINO AS IT STRIKES THE DETECTOR!

3. DEMONSTRATION OF THE TWO-NEUTRINO HYPOTHESIS

IN EITHER OF THE NEUTRINO BEAMS DISCUSSED MANY MORE MUON NEUTRINOS ARE PRODUCED THAN ELECTRON NEUTRINOS. THEN WE EXPECT NEUTRINO SCATTERING TO PROCEED MAINLY VIA

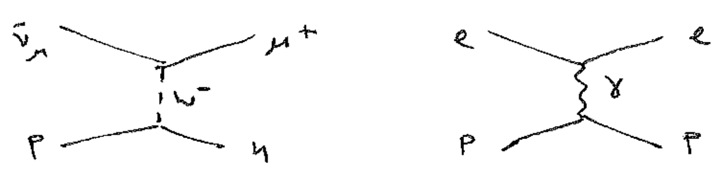
$\nu_\mu n \to p \mu^-$ or $\bar{\nu} p \to n \mu^+$ RATHER THAN $\nu_e n \to p e^-$ or $\bar{\nu} p \to n e^+$

IF THIS IS ACTUALLY OBSERVED WE HAVE GOOD EVIDENCE FOR THE HYPOTHESIS THAT THE MUON NEUTRINO AND ELECTRON NEUTRINO ARE DISTINCT, HAVING LEPTON NUMBER L_μ AND L_e RESPECTIVELY.

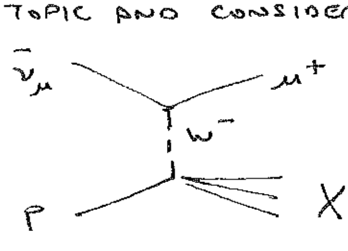
THE EXPERIMENT WAS PERFORMED BY DANBY ET AL [P.R.L. 9, 36 (1962)] WHICH YIELDED $N_e/N_\mu \sim 2\%$, CONSISTENT WITH THE EXPECTED FRACTION OF ELECTRON NEUTRINOS IN THE BEAM. THIS EXPERIMENT WAS THE FIRST NEUTRINO BEAM EXPERIMENT AT ANY PARTICLE ACCELERATOR, AND GOT THE FIELD OF NEUTRINO SCATTERING OFF TO AN AUSPICIOUS START.

4. INELASTIC NEUTRINO-PROTON SCATTERING

THE REACTION $\bar{\nu}_\mu p \to n \mu^+$ IS THE WEAK ANALOG OF e-p ELASTIC SCATTERING



AS SUCH IT MIGHT BE USED TO EXPLORE THE ELASTIC FORM FACTOR OF THE NUCLEON. HOWEVER, AS DISCUSSED IN LECTURE 7, OTHER STUDIES OF THE ELASTIC FORM FACTOR HAVE NOT LED TO GREAT INSIGHT. SO WE SKIP OVER THIS TOPIC AND CONSIDER INELASTIC NEUTRINO-PROTON SCATTERING



AS IN LECTURE 8 WE CAN PROCEED IN A FORMAL MANNER GUIDED BY DIRACOLOGY TO DESCRIBE THE POSSIBLE BEHAVIOR OF THE SCATTERING CROSS SECTION. BUT AS BEFORE, THE GREATEST UNDERSTANDING COMES FROM A QUARK MODEL INTERPRETATION. THE LATTER IS RELATIVELY STRAIGHTFORWARD, BUT TO INTRODUCE CERTAIN STANDARD NOTATIONS WE BEGIN WITH THE DIRACOLOGY.

THE SITUATION IS DIRECTLY ANALAGOUS TO OUR STUDY OF e-p INELASTIC SCATTERING ON p 127 ff.

$$\frac{d\sigma}{dE_\mu \, d\Omega_\mu} = \frac{E_\mu \, |\mathcal{M}|^2}{32\pi^2 E_\nu M_p} \quad \text{IN THE LAB FRAME}$$

$$\mathcal{M} = \frac{G}{\sqrt{2}} \, J_\alpha^{\text{LEPTON}} \, J_\alpha^{\text{HADRON}}$$

WHERE WE KNOW THAT $J_\alpha^{\text{LEPTON}} = (\bar{\upsilon}_{\mu^+} | \gamma_\alpha (1-\gamma_5) | \upsilon_{\bar{\nu}})$

$$\text{THEN } |\mathcal{M}|^2 = \frac{G^2}{2} \cdot \frac{1}{2} \sum_{\text{SPIN}} J_\alpha J_\beta^+ \, J_\alpha J_\beta^+$$

THE $\bar{\nu}$ HAS ONLY ONE POSSIBLE SPIN ; THE PROTON HAS TWO

$$\equiv \frac{G^2}{4} \, \ell_{\alpha\beta} \, L_{\alpha\beta}$$

$\ell_{\alpha\beta}$ CAN BE EVALUATED EXACTLY BY TRACE METHODS

$$\ell_{\alpha\beta} = (P_\nu)_\alpha (P_\mu)_\beta + (P_\mu)_\alpha (P_\nu)_\beta - \delta_{\alpha\beta} \, (P_\nu P_\mu) + i \, \epsilon_{\alpha\beta\gamma\delta} (P_\nu)_\gamma (P_\mu)_\delta$$

FOR WHAT IT'S WORTH

AGAIN, $L_{\alpha\beta}$ IS NOT KNOWN PRECISELY BUT CAN BE EXPRESSED IN TERMS OF LORENTZ SCALAR STRUCTURE FUNCTIONS $W_i(q^2, \nu)$, MULTIPLIED BY VARIOUS POSSIBLE TENSORS FORMED OUT OF THE KINEMATIC 4 VECTORS OF THE PARTICLES.

$$\text{AS BEFORE} \quad \nu \equiv E_\nu - E_\mu \quad ; \; q_\alpha = (P_\nu)_\alpha - (P_\mu)_\alpha \quad ; \; P_\alpha \equiv (P_\nu)_\alpha + (P_p)_\alpha$$

BECAUSE THE WEAK INTERACTION VIOLATES PARITY THERE CAN BE ONE MORE STRUCTURE FUNCTION IN NEUTRINO SCATTERING THAN IN ELECTRON SCATTERING. THIS NEW FUNCTION WILL COUPLE TO THE ANTISYMMETRIC PIECE OF $\ell_{\alpha\beta}$ LISTED ABOVE.

$$L_{\alpha\beta} = 2M_p \left[-\delta_{\alpha\beta} W_1 + \frac{P_\alpha P_\beta}{M_p^2} W_2 - \frac{i \, \epsilon_{\alpha\beta\gamma\delta} P_\gamma P_\delta}{2 M_p^2} W_3 \right]$$

AND

$$\frac{d\sigma_{\bar{\nu}p}}{d\nu \, dq^2} = \frac{G^2}{2\pi} E_\mu^2 \left[2W_1 \sin^2 \theta/2 + W_2 \cos^2 \theta/2 - W_3 \left(\frac{E_\nu + E_\mu}{M_p} \right) \sin^2 \theta/2 \right]$$

A NOTABLE FEATURE IS THE ABSENCE OF THE $\frac{1}{\sin^4 \theta/2}$ TERM DUE TO THE PHOTON PROPAGATOR

Neutrino scattering results are usually not described by variables ν and q^2, perhaps because $\nu = E_\nu - E_\mu$ is confusing. Instead we use the variable $x = \frac{q^2}{2M_p\nu}$ which was introduced

in Lecture 8 in association with the parton model. We gave the interpretation that x = fraction of proton's momentum carried by a quark.

The 2nd standard variable is $y \equiv \frac{\nu}{E_\nu} = \frac{E_\nu - E_\mu}{E_\nu} = 1 - \frac{E_\mu}{E_\nu}$

Finally people redefine the structure functions slightly, to give

$$\frac{d\sigma_{\bar\nu p}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x y^2 F_1(x) + (1-y)F_2(x) - xy\left(1-\frac{y}{2}\right)F_3(x)\right]$$

If a neutrino beam is used instead of an antineutrino beam, the sign of the term $\epsilon_{\alpha\beta\gamma\delta}(P_\nu)(P_\mu)_\delta$ is reversed, which leads to

$$\frac{d\sigma_{\nu p}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x y^2 F_1(x) + (1-y)F_2(x) + xy\left(1-\frac{y}{2}\right)F_3(x)\right]$$

The $F_i(x)$ need not be the same for ν and $\bar\nu$ beams.

We now show how the quark parton model leads quite simply to the above forms, allowing a more detailed interpretation of the experimental results.

The reaction $\nu p \to \mu^- X$ is thought to actually be

$\nu q_1 \to \mu^- q_2$ with the other quarks

acting as spectators. In the following we will only illustrate the Cabibbo favored couplings, setting $\theta_c \to 0$

If we also restrict our considerations to u, d & s quarks and antiquarks only 2 diagrams are possible with a neutrino beam

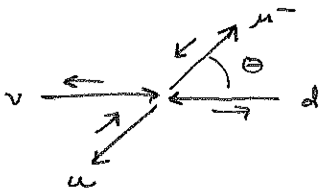
This already gives an indication how inelastic neutrino scattering will probe nucleon structure quite differently from electron scattering.

We can readily estimate the cross section $\frac{d\sigma}{d\Omega}\,\nu q \to \mu q'$ in the high energy limit. On dimensional grounds, $\sigma \sim G^2 E^2$

AT VERY HIGH ENERGIES THE RELEVANT ENERGY E WILL NOT INVOLVE ANY PARTICLE MASSES. SO WE EXPECT

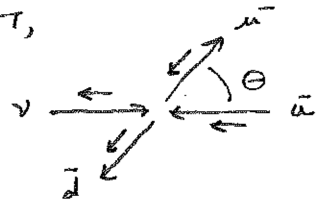
$$\frac{d\sigma}{d\Omega}\nu q \sim G^2 S_{\nu q} \quad \text{WHERE} \quad S_{\nu q} = (E_{cm} \text{ OF THE } \nu q \text{ SYSTEM})^2$$

BECAUSE OF THE LEFT HANDED COUPLING IN THE V-A THEORY OF THE WEAK INTERACTION, THERE IS AN ANGULAR FACTOR. THIS DEPENDS ON THE QUARK FLAVOR AS IS READILY SEEN IN PICTURES



FOR $\nu d \to \mu^- u$ THE INITIAL AND FINAL STATES HAVE SPIN ZERO. SO WE EXPECT AN ISOTROPIC ANGULAR DISTRIBUTION.

BUT,



FOR $\nu \bar{u} \to \mu^- \bar{d}$ BOTH INITIAL AND FINAL STATES HAVE $J_z = -1$ ALONG THE RESPECTIVE DIRECTIONS OF MOTION. CLEARLY SCATTERING AT 180° IS FORBIDDEN. THE ANGULAR DISTRIBUTION IS OBTAINED BY RECALLING THE SPIN 1 ROTATION MATRIX (p 118)

$$R^1_{-1-1} = \frac{1+\cos\theta}{2} \quad \text{PROJECTS } J_z' = -1 \text{ ALONG ANGLE } \theta \text{ ONTO } J_z = -1$$

WE ALSO NOTE THAT A DETAILED V-A CALCULATION GIVES A FACTOR $1/4\pi^2$ TIMES OUR SIMPLE ESTIMATES

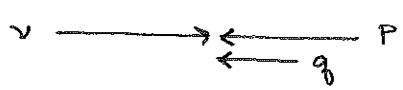
THEN $$\frac{d\sigma}{d\Omega}\nu d \to \mu^- u = \frac{G^2 S_{\nu q}}{4\pi^2}$$

$$\frac{d\sigma}{d\Omega}\nu \bar{u} \to \mu^- \bar{d} = \frac{G^2 S_{\nu q}}{4\pi^2}\left(\frac{1+\cos\theta}{2}\right)^2$$

IF WE USE AN ANTI-NEUTRINO BEAM THE ABOVE EXPRESSIONS HOLD IF WE CHANGE ALL PARTICLE AND ANTIPARTICLE LABELS (CP INVARIANCE).

AS NOTED ON p 347, THE νq OR $\bar{\nu} q$ SCATTERING TAKES PLACE INSIDE THE APPARENT νp OR $\bar{\nu} p$ REACTION. A VIEW IN THE νp C.M. FRAME IS

$$P_\nu = P_p \sim \frac{E_{cm}}{2}$$



AND $P_q = x P_p = x\frac{E_{cm}}{2}$

WE USE THE INTERPRETATION THAT x = FRACTION OF PROTON MOMENTUM CARRIED BY THE QUARK.

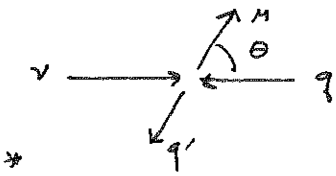
$$S_{\nu q} = \left\{\left(\frac{E_{cm}}{2}, 0, 0, \frac{E_{cm}}{2}\right) + \left(x\frac{E_{cm}}{2}, 0, 0, -x\frac{E_{cm}}{2}\right)\right\}^2$$
$$= x E_{cm}^2 = x S_{\nu p}$$

$S_{\nu p}$ CAN BE RELATED TO LABORATORY QUANTITIES

$$S_{\nu p} = \{ (E_\nu, 0, 0, E_\nu) + (M_p, 0, 0, 0) \}^2 = 2 M_p E_\nu$$

$$\text{So} \quad S_{\nu q} = 2 x M_p E_\nu$$

WE CAN ALSO CALCULATE E_μ IN THE LAB FRAME FROM QUANTITIES IN THE νq C.M. FRAME



$$E_{\mu, LAB} = \gamma E_\mu^* (1 + \cos\theta)$$

$$E_{\nu, LAB} = \gamma E_\nu^* (1 + \cos 0^\circ) = 2\gamma E_\nu^* = 2\gamma E_\mu^*$$

$$\text{so} \quad E_{\mu, LAB} = E_{\nu, LAB} \left(\frac{1 + \cos\theta}{2} \right)$$

$$\text{or} \quad \frac{1 + \cos\theta}{2} = \frac{E_\mu}{E_\nu} = 1 - y \quad \text{WHERE } y = \frac{E_\nu - E_\mu}{E_\nu} \text{ AS ON p 347}$$

$$d\cos\theta = 2 dy \Rightarrow d\Omega = 4\pi \, dy$$

$$\text{AND} \quad \frac{d\sigma}{d\Omega}\nu q = \frac{1}{4\pi} \frac{d\sigma}{dy}\nu q = \frac{G^2}{4\pi^2} 2 x M_p E_\nu \begin{Bmatrix} 1 \\ (1-y)^2 \end{Bmatrix}$$

FINALLY, WE REINTRODUCE THE IDEA OF THE QUARK DISTRIBUTION FUNCTIONS $f_q(x)$ = PROBABILITY OF FINDING A QUARK OF MOMENTUM FRACTION x

$$\text{THEN} \quad \frac{d\sigma}{dy}\nu p = \sum_{\text{QUARKS}} f_q(x) \, dx \, \frac{d\sigma_{\nu q}}{dy}$$

$$\text{OR} \quad \frac{d\sigma_{\nu p}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi} \left[2x\, d(x) + 2x (1-y)^2 \bar{u}(x) \right]$$

$$\text{AND} \quad \frac{d\sigma_{\bar{\nu} p}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi} \left[2x\, \bar{d}(x) + 2x (1-y)^2 u(x) \right]$$

THESE RESULTS ARE TO BE COMPARED WITH THE GENERAL FORMS GIVEN ON p 347. THEY ARE INDEED THE SAME IF WE IDENTIFY

$$2x F_1(x) = F_2(x) \quad \text{AND} \quad F_2^{\nu p}(x) = 2x \left(d(x) + \bar{u}(x) \right)$$

$$F_3^{\nu p}(x) = 2 \left(d(x) - \bar{u}(x) \right)$$

THE EQUATION $2x F_1(x) = F_2(x)$ IS THE CALLAN - GROSS RELATION ENCOUNTERED ON p. 132. IT IS A CONSEQUENCE OF THE SPIN 1/2 NATURE OF THE QUARKS.

5. EXPERIMENTAL RESULTS ON INELASTIC NEUTRINO - NUCLEON SCATTERING

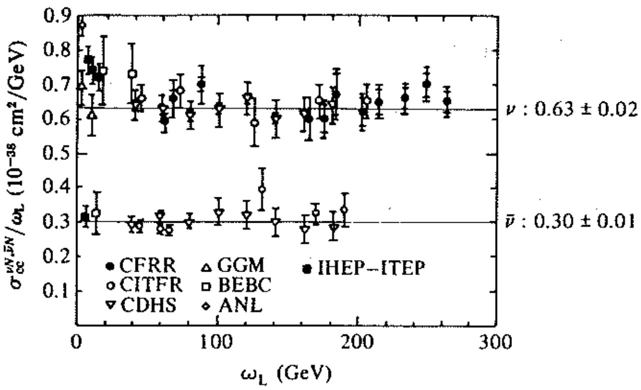
THE SIMPLEST PREDICTION OF THE ANALYSIS OF SEC. 4 IS THAT

$$\sigma_{\nu p} \sim E_\nu$$

THIS DEPENDS MAINLY ON BEING IN THE HIGH ENERGY LIMIT: $E_\nu \gg M_p$.
THE DATA ARE IN GOOD AGREEMENT WITH THIS FOR $5 < E_\nu < 260$ GeV

ALSO $\dfrac{\sigma_{\bar\nu N}}{\sigma_{\nu N}} \sim \dfrac{1}{2}$

Figure 8.9. Neutrino–nucleon and antineutrino–nucleon total cross section for isoscalar targets. [Source: Kelly et al. (80) and ●, Barish et al. (79*a*); ○, Barish et al. (77); ▽, de Groot et al. (79); △, Ciampolillo et al. (79); □, Colley et al. (79); ◇, Barish et al. (79*b*); and ■, Asratyan et al. (78).]



WE NOTE THAT MOST NEUTRINO SCATTERING EXPERIMENTS ARE DONE WITH HEAVY TARGETS SUCH AS IRON TO INCREASE THE INTERACTION RATE. IN THIS CASE WE SHOULD ALSO INDICATE THE QUARK MODEL PREDICTION FOR νn SCATTERING. BY ISOSPIN SYMMETRY WE EXPECT THAT THE DISTRIBUTION OF d QUARKS IN THE NEUTRON IS THE SAME AS THAT FOR u QUARKS IN THE PROTON. $\quad d^n(x) = u^p(x) \;;\; u^n(x) = d^p(x)$

THEN $$\frac{d\sigma_{\nu n}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[2x\,u(x) + 2x(1-y)^2\,\bar d(x)\right]$$

WHERE u AND $\bar d$ REFER TO THE QUARK DISTRIBUTIONS FOR THE PROTON

WE ALSO DEFINE $N = \dfrac{n+p}{2}$, AND WRITE

$$\frac{d\sigma_{\nu N}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x\left(u(x)+d(x)\right) + x(1-y)^2\left(\bar u(x)+\bar d(x)\right)\right]$$

$$\frac{d\sigma_{\bar\nu N}}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x\left(\bar u(x)+\bar d(x)\right) + x(1-y)^2\left(u(x)+d(x)\right)\right]$$

THEN WE PREDICT $$\frac{\sigma_{\bar\nu N}}{\sigma_{\nu N}} = \frac{\int x\,dx\left[\frac{1}{3}(u+d) + (\bar u+\bar d)\right]}{\int x\,dx\left[(u+d) + \frac{1}{3}(\bar u+\bar d)\right]}$$

IF ANTIQUARKS COULD BE IGNORED, $\dfrac{\sigma_{\bar\nu N}}{\sigma_{\nu N}} \to \dfrac{1}{3}$

INSTEAD THE DATA SHOW A VALUE $\sim \frac{1}{2}$ FOR THIS RATIO.

WE AGAIN INTRODUCE THE IDEA OF VALENCE QUARKS AND SEA QUARKS. THE VALENCE QUARKS ARE THOSE THAT MAKE UP THE uud CONTENT OF THE PROTON; THE SEA QUARKS ARE ALWAYS PRODUCED IN $q\bar q$ PAIRS FROM GLUONS.

THEN $\int u_v(x)\,dx = 2$ AND $\int d_v(x) = 1$ FOR VALENCE QUARKS IN THE PROTON. ALSO $u_s(x) = \bar{u}_s(x)$; $d_s(x) = \bar{d}_s(x)$ WHERE S = SEA.

THE TOTAL MOMENTUM CARRIED BY VALENCE QUARKS IS $P_v = \int x\,dx\,(u_v(x) + d_v(x))$

WHILE SEA QUARKS CARRY $P_s = \int x\,dx\,(u_s(x) + d_s(x) + \bar{u}_s(x) + \bar{d}_s(x)) = 2\int x\,dx\,(\bar{u} + \bar{d})$

THEN
$$\frac{\sigma_{\bar{\nu}N}}{\sigma_{\nu N}} = \frac{\frac{1}{3}P_v + \frac{2}{3}P_s}{P_v + \frac{2}{3}P_s} \sim \frac{1}{2} \Rightarrow \frac{P_s}{P_v} \sim \frac{1}{3}$$

I.E., SEA QUARKS CARRY 1/3 AS MUCH MOMENTUM AS VALENCE QUARKS!

FURTHER ANALYSIS CAN BE DONE BY TAKING THE SUM AND DIFFERENCE OF $\sigma_{\nu N}$ AND $\sigma_{\bar{\nu}N}$

$$\frac{d(\sigma_{\nu N} + \sigma_{\bar{\nu}N})}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x\,\underbrace{(u(x) + d(x) + \bar{u}(x) + \bar{d}(x))}_{u_v(x) + d_v(x) + 2(u_s(x) + d_s(x))}(1 + (1-y)^2)\right]$$

$$\frac{d(\sigma_{\nu N} - \sigma_{\bar{\nu}N})}{dx\,dy} = \frac{G^2 M_p E_\nu}{\pi}\left[x\,\underbrace{(u(x) + d(x) - \bar{u}(x) + \bar{d}(x))}_{u_v(x) + d_v(x)}(1 - (1-y)^2)\right]$$

IN THIS WAY ONE CAN EXTRACT THE VALENCE AND SEA QUARK DISTRIBUTIONS RATHER DIRECTLY FROM THE DATA.

FITS FOR u_v, d_v AND $u_s \equiv d_s$ USING BOTH νN, AND eN INELASTIC SCATTERING RESULTS WERE SHOWN ON p 150, AND ALSO →

A CHECK ON THIS ANALYSIS IS THAT THE TOTAL NUMBER OF VALENCE QUARKS SHOULD BE 3

$$3 = \int dx\,(u(x) + d(x) - \bar{u}(x) - \bar{d}(x)) = \frac{3\pi}{2G^2 M_p E_\nu}\int dy \int \frac{dx}{x}\,\frac{d(\sigma_{\nu N} - \sigma_{\bar{\nu}N})}{dx\,dy}.$$

EXPERIMENT YIELDS 3.0 ± 0.5 FOR THIS INTEGRAL, IN GOOD AGREEMENT WITH THE QUARK MODEL.

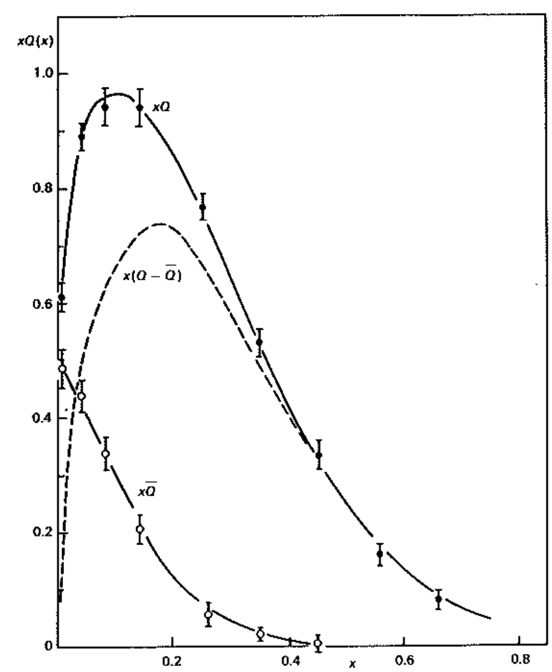


Fig. 7.22 Momentum distributions of quarks (Q) and antiquarks ($\bar{Q}$) in the nucleon, at a value of q^2 of order 10 GeV^2, obtained from results on neutrino and antineutrino scattering in experiments at CERN and Fermilab. The neutrino and antineutrino differential cross-sections measure the structure functions F_2 and F_3 in Eq. (7.50), and the difference and sum of these, through Eq. (7.56), give the quark and antiquark populations weighted by the momentum fraction x. The antiquarks ($\bar{Q}$) are concentrated at small x, the region of the so-called quark-antiquark "sea". The "valence" quarks of the static quark model ($Q - \bar{Q}$) are concentrated towards $x = 0.2$.

ANOTHER PREDICTION OF THE QUARK MODEL IS THAT THE STRUCTURE FUNCTIONS OBSERVED IN eN SCATTERING SHOULD BE BASICALLY THE SAME AS IN νN SCATTERING. THE NOTATION USED IN LECTURE 8 CAN BE TRANSLATED INTO THAT OF THE PRESENT ANALYSIS NOTING

$\nu W_2 \leftrightarrow F_2$ WE ALREADY INDICATED HOW TO CHANGE $dE'd\Omega$ INTO $dx\,dy$

THEN $$\frac{d\sigma_{eN}}{dx\,dy} = \frac{8\pi\alpha^2}{q^2}\left[F_2^{eN}(x)\left(1+(1-y)^2\right)\right]$$

ASSUMING THE CALLAN-GROSS RELATION $2xF_1 = F_2$

IN eN SCATTERING THE PHOTON COUPLING WEIGHTS EACH QUARK BY THE SQUARE OF ITS CHARGE: $$F_2^{eN} = \sum Q_q^2\, x f(x) = x\left\{\frac{4}{9}\left(u(x)+\bar{u}(x)\right) + \frac{1}{9}\left(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right)\right\}$$

WHILE $F_2^{\nu N} = x\left[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)\right]$ (IGNORING TERMS IN $\sin^2\theta_C$)

IF A COMPARISON IS MADE WITH DATA TAKEN USING A TARGET SUCH AS DEUTERIUM OR IRON, THEN $u(x) = d(x)$ AS WE HAVE EQUAL NUMBERS OF U AND d QUARKS IN ISOSPIN SYMMETRIC COMBINATIONS. (HOWEVER IN THIS SENSE $u(x) \neq u^P(x)$ AS USED ON P351)

IF WE IGNORE THE SEA QUARK CONTRIBUTION, WE PREDICT

$$\frac{F_2^{eN}}{F_2^{\nu N}} = \frac{\frac{4}{9}+\frac{1}{9}}{2} = \frac{5}{18}$$

THIS HOLDS SURPRISINGLY WELL ON COMPARING WITH DATA.

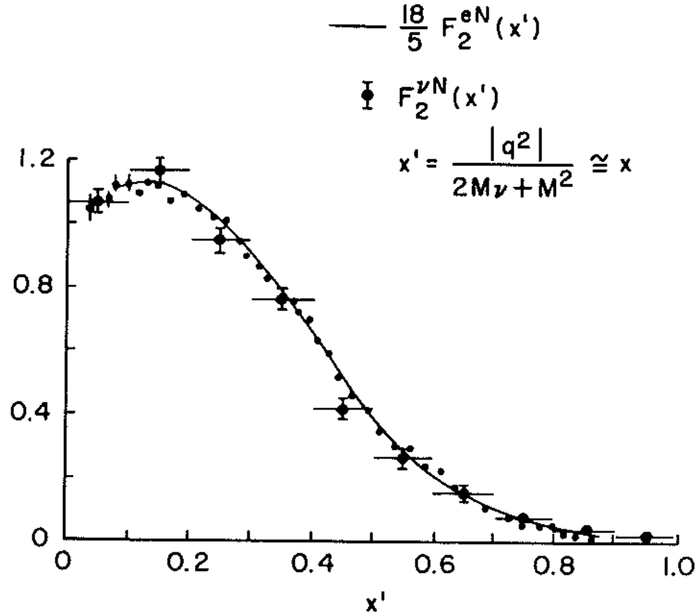


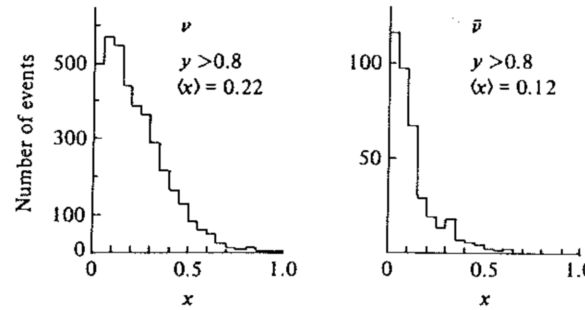
Figure 10-11 Comparison between the electron-nucleon structure function $F_2^{eN}(x)$ and the neutrino-nucleon structure function $F_2^{\nu N}(x)$ (from Ref. 15).

THUS FAR ALL OUR CONSIDERATIONS HAVE INVOLVED INTEGRATION OVER THE VARIABLE y. BUT MORE CAN BE LEARNED IF IN PARTICULAR WE CONSIDER ONLY THE REGION $y \sim 1$. REFERRING TO P350

$$\frac{d\sigma_{\nu N}}{dx} \xrightarrow{y\to 1} x\left(u(x)+d(x)\right)$$

$$\frac{d\sigma_{\bar{\nu} N}}{dx} \xrightarrow{y\to 1} x\left(\bar{u}(x)+\bar{d}(x)\right)$$

Figure 8.12. x distributions for νN and $\bar{\nu}N$ at large y. In the quark model, the νN plot is the quark distribution $Q(x)$ and the $\bar{\nu}N$ plot is the antiquark distribution $\bar{Q}(x)$ (CDHS data). (From de Groot, et al. 79. Reprinted with permission.)

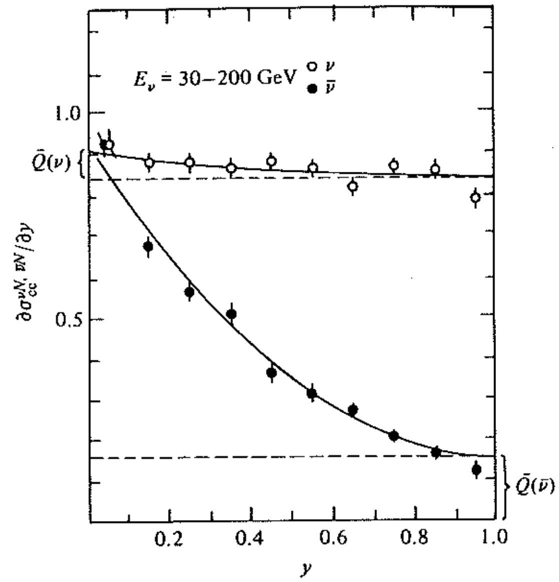


OR, SUPPOSE WE INTEGRATE OVER x TO GET $\frac{d\sigma}{dy}$ THEN

$$\frac{\frac{d\sigma_{\bar{\nu}N}}{dy}}{\frac{d\sigma_{\nu N}}{dy}} \xrightarrow{y \to 1} \frac{\frac{1}{2}P_S}{P_V + \frac{P_S}{2}} \approx .15$$

$\Rightarrow$ $\frac{P_S}{P_V} \sim .4$ USING THE NOTATION OF p 351. AGAIN THE SEA QUARKS CARRY A SIGNIFICANT AMOUNT OF THE NUCLEON'S MOMENTUM.

Figure 8.11. y dependence of cross section for νN and $\bar{\nu}N$ scattering, showing portion attributable to sea antiquarks. Note that antiquarks make up a different fraction of the total at low y (measured by νN scattering) than at high y (from $\bar{\nu}N$ scattering). (From de Groot et al. 79.) (Reprinted with permission.)



FINALLY, WE NOTE THAT

$$\int_0^1 F_2^{\nu N}(x)\, dx \sim \tfrac{1}{2} \quad \text{EXPERIMENTALLY}$$

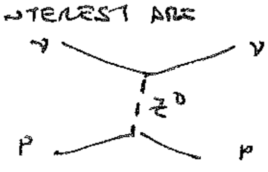
IN OUR MODEL THIS IS JUST $P_V + P_S$ = TOTAL MOMENTUM CARRIED BY THE QUARKS. AS WAS REMARKED FOR THE CASE OF eN SCATTERING, IT APPEARS THAT HALF THE NUCLEON MOMENTUM IS CARRIED BY PARTICLES WHICH INTERACT NEITHER ELECTROMAGNETICALLY NOR WEAKLY. THIS IS CONSISTENT WITH THE IDEA OF GLUONS MAKING UP NEARLY HALF THE PROTONS MOMENTUM. (HOW DOES THIS INFLUENCE OUR THINKING ABOUT THE MASS SPECTRUM OF BOUND QUARK STATES??)

6. THE DISCOVERY OF WEAK NEUTRAL CURRENTS

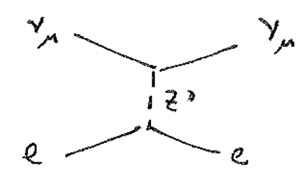
ALL OF THE NEUTRINO SCATTERING REACTIONS CONSIDERED THUS FAR INVOLVE $W^\pm$ EXCHANGE, AND ARE TERMED CHARGED CURRENT INTERACTIONS. WE NOW LOOK FOR EVIDENCE OF POSSIBLE NEUTRAL CURRENT INTERACTIONS, IN WHICH THE HEAVY BOSON Z^0 IS EXCHANGED. IN LECTURE 17 p 316 WE REMARKED HOW THE GIM MECHANISM MAKES IT ALMOST IMPOSSIBLE TO DETECT EFFECTS DUE TO THE Z^0 IN 'ORDINARY' REACTIONS SO LONG AS $q^2 \ll M_{Z^0}^2$.

NEUTRINO SCATTERING VIA Z^0 EXCHANGE LEADS TO REACTIONS WHICH ARE COMPLETELY FORBIDDEN BY OTHER MECHANISMS. ELASTIC NEUTRINO SCATTERING REACTIONS OF INTEREST ARE

$\nu p \to \nu p$

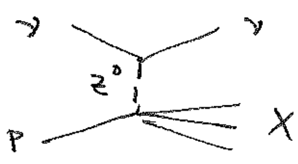


AND $\nu_\mu e \to \nu_\mu e$



INELASTIC SCATTERING IS ALSO POSSIBLE

$\nu p \to \nu X$



ALL OF THESE REACTIONS CONTAIN AN ENERGETIC NEUTRINO IN THE FINAL STATE, WHICH IS PRACTICALLY IMPOSSIBLE TO DETECT. THE OTHER REACTION PRODUCTS HAVE RATHER LOW ENERGY. IN THE 1960'S NEUTRINO BEAMS OFTEN HAD SMALL CONTAMINATION OF NEUTRONS, WHICH CAN INTERACT TO PRODUCE SIMILAR FINAL STATES AS IN NEUTRINO SCATTERING

THUS

LEADS TO A LOW ENERGY FINAL STATE NEUTRON WHICH MAY ESCAPE DETECTION, LEAVING AN APPARENTLY LONE PROTON.

ALSO

MIGHT LEAD TO OBSERVATION OF A LONE ELECTRON VIA $\pi^0 \to \gamma\gamma$ FOLLOWED BY CONVERSION OF ONE PHOTON TO e^+e^- ...

THESE DIFFICULTIES MASKED THE SIGNAL FOR NEUTRAL CURRENT EVENTS FOR SOME TIME.

IN 1973 GOOD QUALITY EVIDENCE FOR A FEW NEUTRAL CURRENT INTERACTIONS CAME FROM A BUBBLE CHAMBER EXPERIMENT AT CERN [HASERT ET AL, PHYS. LETT. B46, 138 (1973); NUC. PHYS. B73, 1 (1974)] ALL 3 TYPES OF NEUTRAL CURRENT EVENTS SKETCHED ABOVE HAVE BEEN OBSERVED. THE ELASTIC SCATTERING EVENTS ARE STILL RARE - ONLY A FEW HUNDRED HAVE BEEN DETECTED TO DATE. MOST SUBSEQUENT WORK HAS BEEN DONE WITH INELASTIC SCATTERING, INCLUDING MEASUREMENTS

$$\frac{\sigma_{\nu N \to \nu X}}{\sigma_{\nu N \to \mu^- X}} \sim \frac{1}{4} \qquad \frac{\sigma_{\bar\nu N \to \bar\nu X}}{\sigma_{\bar\nu N \to \mu^+ X}} \sim \frac{1}{2} \left[\begin{array}{l} \text{P.350: } \dfrac{\sigma_{\bar\nu N \to \mu^+ X}}{\sigma_{\nu N \to \mu^- X}} \sim \dfrac{1}{2} \\ \text{so } \dfrac{\sigma_{\nu N \to \nu X}}{\sigma_{\bar\nu N \to \bar\nu X}} \sim 1 \end{array} \right]$$

THESE SHOW THAT THE COUPLINGS OF THE Z^0 TO LEPTONS AND QUARKS CANNOT BE EXACTLY LIKE THE V-A COUPLINGS OF THE $W^\pm$ BOSONS. ON THE OTHER HAND, THE COUPLING CANNOT BE TOO DIFFERENT. THE WEINBERG-SALAM MODEL PROVIDES A PREDICTION OF THE Z^0 COUPLINGS WHICH CAN BE WELL TESTED WITH NEUTRAL CURRENT SCATTERING DATA. HOWEVER WE DEFER DISCUSSION OF THIS UNTIL LATER IN THE COURSE.

7. Neutrino Mass

IN HIS 1934 PAPER ON THE THEORY OF THE WEAK INTERACTION FERMI INDICATED HOW TO DECIDE IF THE NEUTRINO IS MASSLESS. NAMELY THAT THE SPECTRUM OF THE ELECTRON EMITTED IN NUCLEAR β DECAY IS NOTICABLY AFFECTED AT ITS MAXIMUM END POINT IF $M_\nu \neq 0$. NOTE ALSO THAT IF $M_\nu > 0$ THEN THE MAXIMUM ENERGY WHICH THE ELECTRON CAN ATTAIN IS REDUCED SLIGHTLY. FERMI CONCLUDED THAT THE DATA AVAILABLE IN 1934 WERE CONSISTENT WITH $M_\nu = 0$.

FROM FERMI (1934).
DETECTOR RESOLUTION WILL CAUSE SIGNIFICANT MODIFICATION OF THESE IDEALIZED SHAPES!

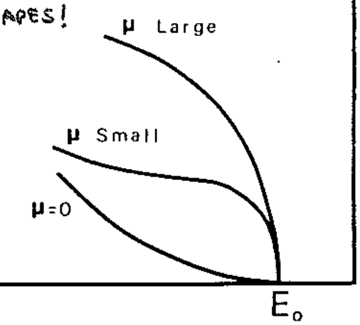


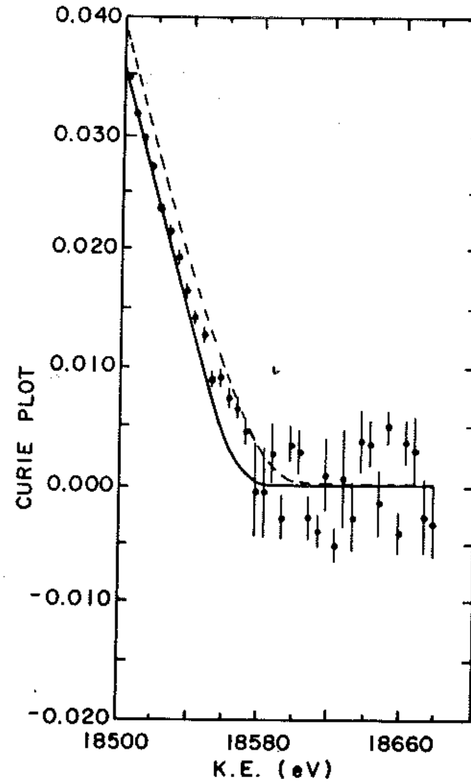
FIG. 1. The end of the distribution curve for $\mu=0$ and for large and small values of μ.

SOME OF THE CONTINUING INTEREST IN THE QUESTION OF NEUTRINO MASS IS GENERATED BY PERSISTENT REPORTS FROM A RUSSIAN EXPERIMENT OF A NON-ZERO MASS FOR ν_e. THEY EXAMINE TRITIUM β DECAY:

$$t \to He^3 + e^- + \bar{\nu}_e$$

THE MAXIMUM ENERGY OF THE ELECTRON IS 18.6 KeV. ON THE BASIS OF THE SPECTRUM SHOWN THEY CLAIM $M_{\bar{\nu}_e} = 33 \pm 1$ eV!

THIS ENERGY IS TYPICAL OF MOLECULAR BINDING! WE ARE DEFINITELY NOT IN THE HIGH ENERGY LIMIT.



LUBIMOV ET AL. PHYS. LETT 94B, 266 (1980)

SEE BENNETT ET AL. PR C31, 197 (1985) FOR CAUTIONS ABOUT ATOMIC PHYSICS EFFECTS. ALSO, BERGKVIST, PHYS. LETT. 154B, 224 (1985).

SEE BORIS ET AL, PHYS. LETT. 159B, 217 (1985) FOR THE RUSSIAN REBUTTAL.

Fig. 4. The edge of the spectrum (Kurie plot). The solid line: $M_\nu = 33$ eV; $E_o = 18583$ eV; $\alpha = -1.84\times10^{-9}$ eV^{-2}. The broken line: $M_\nu=0$; E_o=18583 eV; α=-1.84x10^{-9} eV.

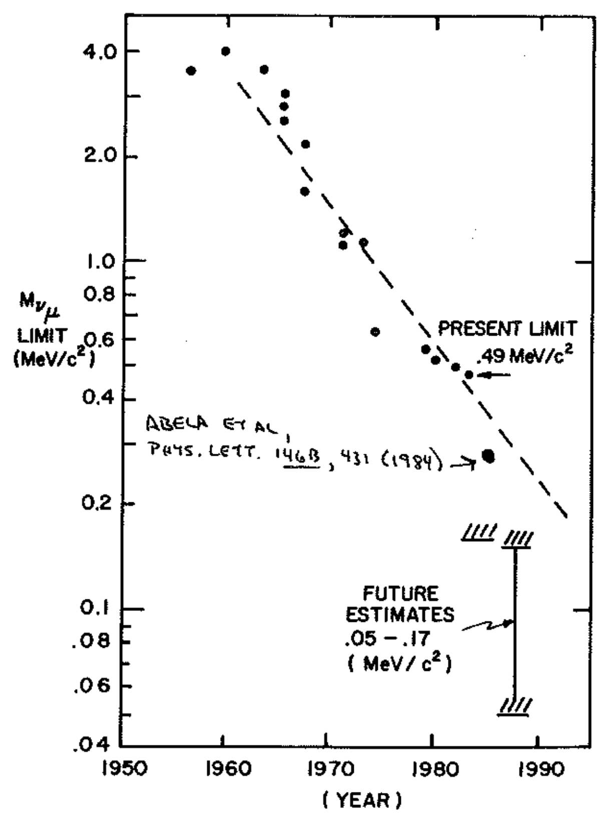
NEEDLESS TO SAY, THIS RESULT IS CHALLENGING. ABOUT 10 EXPERIMENTS ARE UNDERWAY TO CHECK THE TRITIUM β DECAY SPECTRUM. IN ADDITION EFFORTS ARE BEING MOUNTED TO UTILIZE ELECTRON CAPTURE: $Z + e^- \to (Z-1) + \nu_e$ IN WHICH $Q = E_Z - E_{Z-1}$ IS VERY SMALL. PEOPLE AT PRINCETON HAVE BEEN CONCERNED WITH ^{60}Ho WHICH HAS $Q \sim 2.4$ KeV. ANOTHER POSSIBILITY IS ^{158}Tb WITH $Q \sim 150$ eV!

MEASUREMENTS OF THE MASS OF ν_μ ARE MUCH CRUDER. THEY COME FROM DETAILED ANALYSIS OF THE μ SPECTRUM IN $K^0 \to \pi^- \mu^+ \nu_\mu$ AND $\pi^+ \to \mu^+ \nu_\mu$.

THE PRESENT LIMIT IS $M_{\nu_\mu} \lesssim M_e/2$

FOR THE τ NEUTRINO, LIMITS ARE SET VIA DECAYS $\tau \to \nu_\tau e \bar{\nu}_e$ AND $\tau \to \nu_\tau \pi$.

PRESENTLY, $M_{\nu_\tau} < 50$ MeV.
SET BY THE CLEO COLLABORATION (1986).



Muon neutrino mass limits vs. time.

IT IS VERY POSSIBLE THAT NEUTRINOS HAVE SMALL BUT NON-ZERO MASS. CERTAINLY THIS WOULD MAKE MANY ASTROPHYSICISTS HAPPY. IF $M_\nu \sim 10\,eV$ IT IS ESTIMATED THAT MOST OF THE MASS OF THE UNIVERSE WOULD BE IN NEUTRINOS. THEN 'ORDINARY' MATTER BECOMES A RELATIVELY UNIMPORTANT SUBJECT ON THE COSMOLOGICAL SCALE.

8. Neutrino Oscillations

IF IN ADDITION TO SUPPOSING THAT NEUTRINOS HAVE MASS, WE ALSO GIVE UP THE IDEA OF LEPTON NUMBER CONSERVATION, THEN THE PHENOMENON OF NEUTRINO OSCILLATIONS IS POSSIBLE.

OUR EVIDENCE IN FAVOR OF LEPTON NUMBER CONSERVATION ACTUALLY COMES FROM ONLY A FEW SOURCES. WE ARE PARTICULARLY INTERESTED IN THE IDEA THAT ELECTRONS AND MUONS HAVE A DIFFERENT LEPTON NUMBER.

$$\frac{\Gamma_{\mu \to e\gamma}}{\Gamma_{\mu \to \mathrm{ALL}}} \lesssim 10^{-10}$$ WHICH SEEMS RATHER CONVINCING

$$\frac{\sigma_{\nu_\mu p \to n e^+}}{\sigma_{\nu_\mu p \to n \mu^+}} \lesssim 1\%$$ AS IN SEC. 3 ABOVE.

[EVIDENCE THAT $\nu_e \neq \bar{\nu}_e$ COMES FROM THE ABSENCE OF NEUTRINOLESS DOUBLE BETA DECAY {SEE CALDWELL ET AL. P.R.L. 54, 281 (1985). AND THE SUPPRESSION OF THE REACTION $\bar{\nu}_e + {}^{37}Cl \to {}^{37}Ar + e^-$]

THE POSSIBILITY OF NEUTRINO OSCILLATIONS WAS RAISED ALREADY IN 1957 BY PONTECORVO, ABOUT THE TIME THAT THE 2 COMPONENT THEORY OF THE NEUTRINO WAS ESTABLISHED. AT THAT TIME THERE WERE ONLY 2 TYPES OF NEUTRINOS TO CONSIDER, ν_e AND ν_μ. PERHAPS THERE ARE TRANSITIONS POSSIBLE BETWEEN ν_e AND ν_μ. THEN ν_e AND ν_μ AS PRODUCED IN WEAK INTERACTIONS CANNOT BE THE EIGENSTATES OF THE HAMILTONIAN OF THE NEUTRINOS. RATHER THERE MIGHT BE 2 OTHER STATES ν_1 AND ν_2, WHICH HAVE MASSES M_1 AND M_2. THESE ARE RELATED TO ν_μ AND ν_e BE A CABIBBO-LIKE ROTATION

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

WITH 3 KINDS OF NEUTRINOS ONE HAS THE POSSIBILITY OF 'MIXING' ACCORDING TO A KOBAYASHI-MASKAWA MATRIX

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} & & \\ & K-M & \\ & & \end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

THEN CP VIOLATION MIGHT BE POSSIBLE AS WELL....

We sketch how the mixing matrix leads to neutrino oscillations for the case of two types of neutrinos ν_e and ν_μ. This is very similar to the strangeness oscillations of K^0 and $\bar{K}^0$, except that we ignore the possibility that the neutrinos might decay.

The mixing is only possible if the neutrinos have mass. Then there is a rest frame for the neutrinos, and we begin our analysis there.

In this frame $\quad \nu_1(t^*) = \nu_1(0)\, e^{-iM_1 t^*} \qquad \nu_2(t^*) = \nu_2(0)\, e^{-iM_2 t^*}$

where t^* = proper time. Suppose at $t = t^* = 0$ we create a ν_μ in a weak interaction. i.e. $\nu_\mu(0) = 1 \quad \nu_e(0) = 0$. Then using the inverse of the mixing matrix

$$\nu_1(0) = -\sin\theta \qquad \nu_2(0) = \cos\theta$$

In general $\quad \nu_\mu(t^*) = -\sin\theta\, \nu_1(t^*) + \cos\theta\, \nu_2(t^*)$

$$\text{so} \qquad \nu_\mu(t^*) = \sin^2\theta\, e^{-iM_1 t^*} + \cos^2\theta\, e^{-iM_2 t^*}$$

$$P_\mu(t^*) = |\nu_\mu(t^*)|^2 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta\,\cos(\Delta M t^*)$$

$$= 1 - \frac{1}{2}\sin^2 2\theta\,(1 - \cos\Delta M t^*)$$

$$= 1 - \sin^2 2\theta\, \sin^2\left(\frac{\Delta M t^*}{2}\right)$$

We now transform to the lab frame : $\qquad t = \gamma t^*$

where $\gamma = \dfrac{E_{LAB}}{M_{eff}} \sim \dfrac{2E_{LAB}}{M_1 + M_2}$ so $\dfrac{\Delta M t^*}{2} = \dfrac{(M_1^2 - M_2^2)t}{4E_{LAB}} \equiv \dfrac{\delta m^2 t}{4E}$

$$P_\mu(t) = 1 - \sin^2 2\theta\, \sin^2\left(\frac{\delta m^2 t}{2E}\right)$$

$P_e(t) = 1 - P_\mu(t) \qquad$ as there is no decay.

Note that our approximations only hold if $\dfrac{\Delta M}{M} \ll 1$

A characteristic oscillation length is then $\dfrac{4\pi E}{\delta m^2} = \dfrac{5\,E(\mathrm{MeV})}{\delta m^2(\mathrm{eV})^2}$ meters

Experimentally one might look for the apparent vanishing of ν_μ or ν_e as a function of distance (depending on which is produced originally). Some people hoped that this might be a resolution to the 'solar neutrino problem', that they can't find the neutrino from nuclear processes in the sun. Another experimental technique is to look for periodic appearance of ν_e in a ν_μ beam – a variation on the original two neutrino experiment.

AFTER ONE INITIAL POSITIVE REPORT OF A NEUTRINO OSCILLATION EFFECT [REINES ET AL. P.R.L. 45, 1307 (1980)], A LARGE NUMBER OF NEGATIVE RESULTS HAVE BEEN OBTAINED.

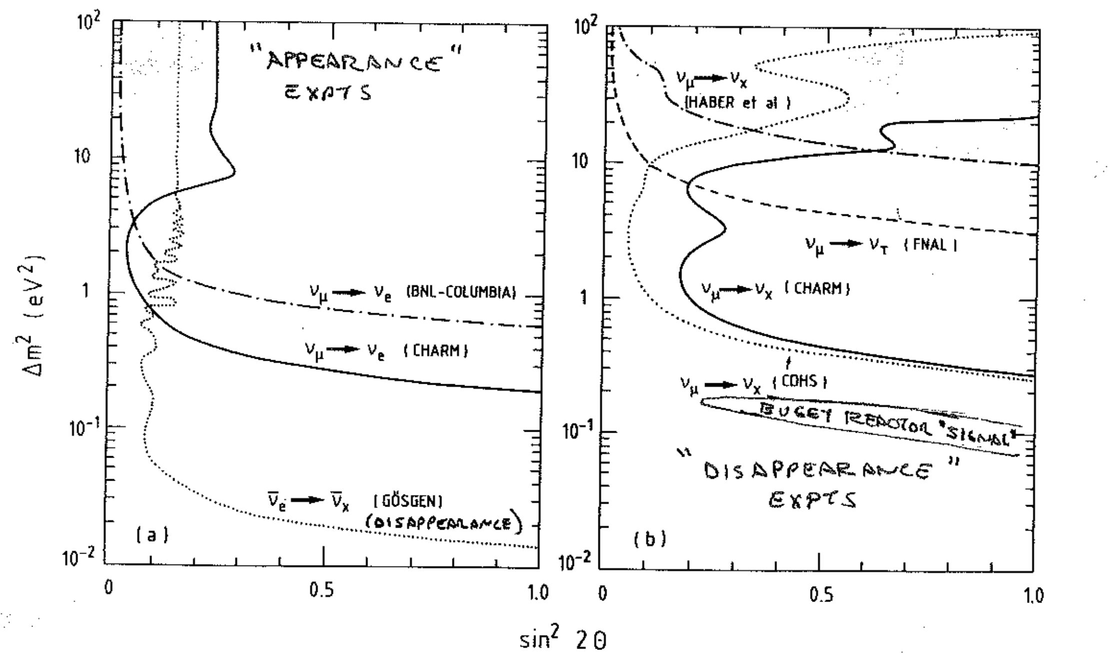


Fig. 3. (a) The 90% confidence limit obtained in the appearance experiment $\nu_\mu \to \nu_e$ is compared with the best previously obtained limits for oscillations of the types: $\nu_\mu \to \nu_e$ (Baker et al. [3]) and $\bar{\nu}_e \to \bar{\nu}_x$ (Vuillemier et al. [6]). (b) The 90% confidence limit obtained in the disappearance experiment is compared with previous limits on ν_μ oscillations to ν_τ (Ushida et al. [4]) and $\nu_\mu \to \nu_x$ (CDHS Collab. [8] and Haber et al. [4]).

RECENT RESULTS INCLUDE:

GABATHULER ET AL	PHYS. LETT 138B, 449 (1984)
STOCKDALE ET AL	P.R.L. 52, 1384 (1984)
BERGSMA ET AL	PHYS. LETT. 142B, 103 (1984)
DYDAK ET AL.	PHYS. LETT 134B, 281 (1984)

AN EXPERIMENT AT THE BUGEY REACTOR (FRANCE) CLAIMS A SIGNAL OF DISAPPEARING $\bar{\nu}_e$'S. CAVAIGNAC ET AL. PHYS. LETT 148B, 387 (1984)
THIS DISAGREES WITH THE GÖSGEN REACTOR RESULT (GABATHULER ET AL, ABOVE)
SEE ALSO ZACEK ET AL, PHYS. LETT 164B, 193 (1985) FOR AN UPDATE.

IF $\sin^2 2\theta > 0.1$ THEN $M^2_{\nu_\mu} - M^2_{\nu_e} \lesssim 1\,eV^2$

FROM THE TRITIUM β DECAY EXPT WE CAN SURELY SAY $M_{\nu_e} < 50\,eV$

WHICH IMPLIES $M_{\nu_\mu} < 50\,eV$ ALSO (SO LONG AS $\sin^2 2\theta > 0.1$)

THIS LIMIT IS MUCH TIGHTER THAN THE DIRECT MEASUREMENT $M_{\nu_\mu} < 250\,keV$ (p 355)