Hidden Geometry in Dual Heterotic/F-theory Compactifications

Lara B. Anderson

Virginia Tech

Work done in collaboration with:

W. Taylor (MIT)
J. Heckman (Harvard) and S. Katz (UIUC)

Strings 2014, Princeton
June 26th, 2014
Motivation

Much recent work: Classifying which effective theories arise from string compactifications, scanning for models/patterns

- **Goal**: Combine two approaches.

Consider $4D, N = 1$, Dual Heterotic-F-theory Vacua

- Systematically construct and study a large class of vacua
- Try to understand/classify how topology/geometry constrains effective theories
- Develop new tools for string pheno?

What possible EFTs?

Which geometries?
Heterotic on $\pi_h : X_n \xrightarrow{E} B_{n-1} \iff$ F-theory on $\pi_f : Y_{n+1} \xrightarrow{K3} B_{n-1}$

(with $\pi : Y_{n+1} \xrightarrow{E} B_n$ and $\rho : B_n \xrightarrow{\mathbb{P}^1} B_{n-1}$)

Descends from 8-dim: Het on $T^2 \iff$ F-theory on $\pi : K3 \xrightarrow{E} \mathbb{P}^1$ (Vafa)

(Rich history: Vafa, Morrison, Friedman, Morgan, Witten, Donagi, Curio, Aspinwall, Katz, Plesser, Andreas, Watari, Hayashi, Toda, Yamazaki, Schafer-Nameki, Saulina, Marsano, Cvetic...)

Where these two theories are dual, there is a finite set of geometries to study
The number of elliptically fibered CY 3-folds, $X_3$, is finite (M. Gross)

- $\mathbb{E}$-fibered 3-folds “extremal” in known examples?? (Taylor, Candelas, Ooguri-Keller, etc.)

What about the no. of vector bundles $(V_1, V_2)$ over $X_3$?

- For fixed topology $M(c(V))$ has only finitely many components
  - $rk(V): H \subset E_8$
  - Spinors: $c_1(V) = 0$
  - Anomaly cancellation:
    $$0 \leq c_2(V_i) \leq c_2(TX)$$
  - For fixed $c_2$ ⇒
    only finitely many values of $c_3$
    compatible w/ $\mathcal{N} = 1$ SUSY (e.g. Maruyama, Langer (for $H = SU(n)$))

- Bounds on $(X_3, V_i)$ non-constructive
(With W. Taylor)

- Systematically study the general properties/constraints of EFT for this class of string compactifications

- Develop a general formalism: For smooth $X_3$, possible $B_2$ classified (generalized del Pezzo). Build an algorithm to construct all $B_3$ that are non-degenerate $\mathbb{P}^1$ fibrations over any $B_2$.

- To explore/test general structure: Build dual $(X_3, Y_4)$ pairs using dataset of $61,539$ toric surfaces, $B_2$ (Morrison + Taylor)
  
  - Caveats: All fibrations w/ section. $B_3$ constructed as a $\mathbb{P}^1$-bundle over $B_2$.
  - Only $16$ of these $B_2$ lead to smooth $X_3 \Rightarrow$ Start with these $\Rightarrow 4962$ 4-folds

  (Note: Toric manifolds used as examples but constructions/constraints general)
Complex structure of $Y_4 \Leftrightarrow$ bundle moduli space of $V$

Weierstrass Model for an elliptic fibration:

$$y^2 = x^3 + f(u)x + g(u)$$

w/ $f \in H^0(B_3, K_{B_3}^{-4})$, $g \in H^0(B_3, K_{B_3}^{-6})$

E.g. $H = SU(2)$, $G = E_7$:

- **F-theory w/ $E_7$ singularity:**

  $$y^2 = x^3 + (f_3 z^3 + f_4 z^4)x + (g_5 z^5 + g_6 z^6) + \ldots$$

- In the neighborhood of the 7-brane ($z=0$):

  $$y^2 = x^3 + z^3(g_5 z^2 + f_3 x) + \ldots$$

**Heterotic**: $SU(2)$ Spectral Cover, $C$ (w/ $c_2(V) = \eta \wedge \omega_0 + \pi^*(\zeta)$):

$$a_0 \hat{Z}^2 + a_2 \hat{X} = 0$$

with $a_0 \in H^0(B_2, \mathcal{O}(\eta))$ and $a_2 \in H^0(B_2, \mathcal{O}(\eta) \otimes K_{B_2}^{\otimes 2})$

$$\{f_3 = a_2, g_5 = a_0\}, \{f_4, g_6\} \leftrightarrow X_3 \text{ Weierstrass}$$
**η: Building bundles and \( B_3 \)**

- **Idea:** Choose topology of bundles \((V_1, V_2) \Leftrightarrow \text{Build } \rho : B_3 \to \mathbb{P}^1 \to B_2\)**

**Heterotic:**

- Can expand:
  \[ c_2(V_i) = \eta_i \wedge \omega_0 + \zeta_i, \]
  where \( \eta_i \) (resp. \( \zeta_i \)) \( \{1, 1\} \) (resp. \( \{2, 2\} \)) forms on \( B_2 \) and \( \omega_0 \) dual to the zero section.

- Anomaly Cancellation \( \Rightarrow \)
  \[ \eta_{1,2} = 6c_1(B_2) \pm t \]

In Het/F-dual pairs, two \( t \)'s are the same (FMW), (Grimm + Taylor)

- Can build \( B_3 \) over \( B_2 \) by
  "twisting" the \( \mathbb{P}^1 \) fibration
  (analog of \( \mathbb{F}_n \) surfaces in \( 6D \))
  \[ B_3 = \mathbb{P}(\mathcal{O} \oplus \mathcal{L}) \]

- \( c_1(B_3) = c_1(B_2) + 2\Sigma + t \)
  where \( \Sigma \) is dual to the zero-section of the \( \mathbb{P}^1 \)-fibration

Next: Bounds on twists \( \Rightarrow \) finite \# \( B_3 \) sol'ns/enumeration
\[ N = 1 \text{ SUSY} \]

- **Heterotic**: \( X_3 \text{ CY. Bundles, } V_i \) satisfy the Hermitian-YM Eq.s:
  \[ F_{ab} = F_{\bar{a} \bar{b}} = 0 \quad g^{\bar{a} \bar{b}} F_{a \bar{b}} = 0 \]

- **F-theory**: \( Y_4 \) can be resolved into a smooth Calabi-Yau 4-fold

Need vanishing degrees of
\[ (f, g, \Delta) \leq (4, 6, 12) \] on every divisor in \( B_3 \)

- \( f, g \) cannot vanish to orders 4, 6 on any curve.

\[ \Rightarrow t \Rightarrow \eta \text{ an effective curve class in } B_2. \]

**4D Symmetries**

Only certain divisors can carry singular fibers

- \( \eta_i \text{ base point free } \Rightarrow \text{ implies } \not\exists \text{ any eff. curve of negative self-intersection, } D \) such that \( \eta_i \cdot D < 0 \)
  \[ (-6K_2 \pm t) \cdot D \geq 0 \]

- \( H = SU(n) \Rightarrow \eta \text{ bpf.} \)

- \( D^2 = -2 \Rightarrow \text{ non-bpf egs:} \)
  \[ H = SO(8), G_2, F_4, E_6, E_7, E_8, \] over gen. del Pezzos
Sample Question: How does topology constrain 4D Gauge Symmetry?

I.e. given a CY 3-fold, $X$, does $\exists$ a stable bundle with \textit{given} rank ($rk(V)$), structure group ($H \subset E_8$) and total Chern class ($c(V)$)?

\textbf{Step 1:}
- Study all possible $Y_4$’s with perturbative heterotic duals.
- Constrain $M(c(V))$

\textbf{Step 2:}
- Add in G-flux on $Y_4$ to fully determine $M(c(V))$

<table>
<thead>
<tr>
<th>base $B_2$</th>
<th>$h_{1,1}$</th>
<th>$# B_3$’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 1, 1)$</td>
<td>(2)</td>
<td>14</td>
</tr>
<tr>
<td>$(0, 0, 0, 0)$</td>
<td>(F$_0$)</td>
<td>2</td>
</tr>
<tr>
<td>$(1, 0, -1, 0)$</td>
<td>(F$_1$)</td>
<td>2</td>
</tr>
<tr>
<td>$(2, 0, -2, 0)$</td>
<td>(F$_2$)</td>
<td>2</td>
</tr>
<tr>
<td>$(0, 0, -1, -1, -1)$</td>
<td>(dP$_2$)</td>
<td>3</td>
</tr>
<tr>
<td>$(1, -1, -1, -2, 0)$</td>
<td>3</td>
<td>173</td>
</tr>
<tr>
<td>$(-1, -1, -1, -1, -1)$</td>
<td>(dP$_3$)</td>
<td>4</td>
</tr>
<tr>
<td>$(0, -1, -1, -2, -1, -1)$</td>
<td>4</td>
<td>729</td>
</tr>
<tr>
<td>$(0, 0, -2, -1, -2, -1)$</td>
<td>4</td>
<td>312</td>
</tr>
<tr>
<td>$(1, 0, -2, -2, -1, -2)$</td>
<td>4</td>
<td>62</td>
</tr>
<tr>
<td>$(-1, -1, -2, -1, -2, -1, -1)$</td>
<td>5</td>
<td>1119</td>
</tr>
<tr>
<td>$(0, -1, -1, -2, -2, -1, -2)$</td>
<td>5</td>
<td>406</td>
</tr>
<tr>
<td>$(-1, -1, -2, -1, -2, -2, -1, -2)$</td>
<td>6</td>
<td>351</td>
</tr>
<tr>
<td>$(-1, -2, -1, -2, -1, -2, -1, -2)$</td>
<td>6</td>
<td>214</td>
</tr>
<tr>
<td>$(0, -2, -1, -2, -2, -2, -1, -2)$</td>
<td>6</td>
<td>83</td>
</tr>
<tr>
<td>$(-1, -2, -2, -1, -2, -2, -1, -2, -2)$</td>
<td>7</td>
<td>36</td>
</tr>
</tbody>
</table>
Questions in Deformation theory

Begin with 4d symmetry \( G \subset E_8 \):

**Heterotic:** Begin with \( H \)-bundle \( V \), \( \text{rank}(V) = n \)

- “Higgs” \( G \Rightarrow \text{Deform} \ V \oplus \mathcal{O}_{X_3}^\oplus m \) to \( V' \) with \( \text{rank}(V') = n + m \)
- “Enhance” to \( G' \): “Break” \( V \rightarrow V_1 \oplus V_2 \oplus \mathcal{O}_{X_3} \oplus \ldots \) with \( \text{rank}(V_i) < \text{rank}(V) \)

**F-theory:** Singular \( Y_4 \)

- “Higgs” \( G \Rightarrow \text{Deform} \) complex structure of \( Y_4 \) to smooth singularities
- “Enhance” to \( G' \): \( G \subset G' \subset E_8 \Rightarrow “Tune” \) complex structure of \( Y_4 \) to produce more singular space
## Generic Symmetries

<table>
<thead>
<tr>
<th>×</th>
<th>.</th>
<th>su2</th>
<th>su3</th>
<th>g2</th>
<th>so8</th>
<th>f4</th>
<th>e6</th>
<th>e7</th>
<th>e8</th>
</tr>
</thead>
<tbody>
<tr>
<td>.</td>
<td>712</td>
<td>499</td>
<td>121</td>
<td>589</td>
<td>276</td>
<td>1245</td>
<td>184</td>
<td>890</td>
<td>15</td>
</tr>
<tr>
<td>su2</td>
<td></td>
<td>47</td>
<td>11</td>
<td>62</td>
<td>14</td>
<td>74</td>
<td>2</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>su3</td>
<td></td>
<td></td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>g2</td>
<td></td>
<td></td>
<td></td>
<td>34</td>
<td>12</td>
<td>54</td>
<td>2</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>so8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>f4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>32</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
“Generic” symmetries on $Y_4$ provide rank($V$)-dependent vanishing criteria for $\mathcal{M}(c(V))$. (First studied by Rajesh and Berglund & Myer)

Also constraints on which symmetries can be enhanced

non-Higgsable $SU(2), SU(3) \not\rightarrow SU(5)$

Can be pinned at exactly one symmetry (or a sparse set)

Intriguing for string pheno...

<table>
<thead>
<tr>
<th>$H$</th>
<th>$\eta \geq Nc_1(B_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(n)$</td>
<td>$n \ (n \geq 2)$</td>
</tr>
<tr>
<td>$SO(7)$</td>
<td>$4$</td>
</tr>
<tr>
<td>$SO(m)$</td>
<td>$\frac{m}{2} \ (m \geq 8)$</td>
</tr>
<tr>
<td>$Sp(k)$</td>
<td>$2k \ (k \geq 2)$</td>
</tr>
<tr>
<td>$F_4$</td>
<td>$\frac{13}{3}$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$\frac{7}{2}$</td>
</tr>
<tr>
<td>$E_6$</td>
<td>$\frac{9}{2}$</td>
</tr>
<tr>
<td>$E_7$</td>
<td>$\frac{14}{3}$</td>
</tr>
<tr>
<td>$E_8$</td>
<td>$5$</td>
</tr>
</tbody>
</table>
Issues with G-flux

- In the previous discussion we have ignored G-flux.
- Does gauge symmetry of the theory match Kodaira/Tate singular fibers of $Y_4$?
- Up until recently the consensus would have said yes.... (in M-theory limit, Abelian flux cannot break non-Abelian symmetries).
- But in the singular limit, F-theory can be more subtle.
- Can never have more symmetry than indicated by Kodaira/Weierstrass. Could have less with G-flux in the singular limit...
- D-branes idea (Donagi, Katz, Sharpe) ⇒ much recent work in local F-theory ("T-branes" (Cecotti,Cordova, Heckman, Vafa) or "Gluing data", (Donagi,Wijnholt))
Consider the simplest possible heterotic solution. The so-called “Standard Embedding”, \( V = TK3, c_2(V) = 24 \).

Problem: F-theory dual \( y^2 = x^3 + g_5 z^5 + \ldots \)
This is an \( E_8 \) singularity not \( E_7 \)

Even worse, \( \Delta_Y = z^{10} (g_{24})(\ldots) \) with \( g_{24} = \Delta_{K3} \)

To get a smooth CY4, must blow up the base at \( g_{24} = \Delta_{K3} \) \( \Rightarrow \) This is the dual of Heterotic Small Instantons at 24 \( l_1 \) fibers over pts in \( \mathbb{P}^1 \).

Question: How can \( TK3 \) and \( I_{\Delta_{K3}} \) have the same F-theory dual?
**T-branes (Local Description)**

- **Gauge fields on the 7-brane: Hitchin’s Equations**

\[
F - \frac{i}{2} [\Phi, \Phi^\dagger] = 0 \ , \ \ \tilde{\partial}_A \Phi = 0 \quad (\Phi \in H^1(End(V) \times K))
\]

- **Spectral Equation:** \( det(\Phi - \lambda I) = 0 \) reproduces local transverse d.o.f.

  E.g. If \( y^2 = x^3 + z^5 \) (i.e. \( E_8 \) on \( z = 0 \)) can turn on \( SU(2) \) gauge flux to break to \( E_7 \)

\[
\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix} \quad \Rightarrow \quad y^2 = x^3 + \phi^2 z^3 x + z^5
\]

- **T-brane:**

\[
\Phi = \begin{bmatrix} 0 & \phi \\ 0 & 0 \end{bmatrix}
\]

- **This still breaks \( E_8 \to E_7 \), but no longer visible in the complex structure.**
Global Geometry + T-branes

(with J. Heckman and S. Katz)

- How to extend local T-brane description to concrete global geometry?
- G-flux defined in Deligne Cohomology:

\[ 0 \to J^3(X) \to D \to H^{2,2}(X, \mathbb{Z}) \to 0 \]

- Need an intrinsic notion of these d.o.f in singular limit \((X_t \to X_0)\)
- Key new ingredient: Diaconescu, Donagi, Panete demonstrated that the moduli space of the Hitchin system over a curve can be identified with the moduli (complex structure and intermediate Jacobian) of a non-compact CY 3-fold....
We found a partial compactification of the DDP results

“Emergent” Hitchin System

Limiting mixed Hodge structure analysis identifies the fibers of the parabolic Hitchin systems with part of limits of intermediate Jacobians $J(X_t)$ of 1-parameter smoothings $X_t$

A “Transition function” to patch open/closed string descriptions in limit $X_t \to X_0$

$M_{lo}c = \text{Moduli of 7-brane curve}$

$H = \text{Hitchin Moduli space}$

$\tilde{M}_{cx} = \text{Comp. Struc. of resolved CY}$
Conclusions and Future Directions

- $N = 1$ Heterotic/F-theory geometries are a fruitful arena for classifying/enumerating (a finite set) of dual geometries/string vacua.
- Developed an algorithm to systematically build all 4-folds (w/ $\mathbb{P}^1$ bundle base $B_3$, over $B_2$ (gdP)).
- Explicitly constructed all Heterotic/F-theory dual pairs over toric bases (such that $X_3$ smooth).
- Non-trivially matched topological consistency conditions ($\eta$ eff., bpf, etc) & developed vanishing conditions for $\mathcal{M}(c(V))$ on $X_3$.
- A classification requires understanding G-flux in the singular limit.
- 6D Global T-branes ⇒ limiting mixed Hodge structures and emergent Hitchin systems.
- A first step in a systematic study...
Thank you!