Probing the structure of quantum phases of matter with holography

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Traces of holography in many settings – some better understood than others → how broad is its range of applicability?

Recently, applications to a number of condensed matter systems [see talks by Erdmenger, Gauntlett, Liu, Takayanagi,…]

• materials with unconventional scalings (e.g. ‘strange metals’)
• new poorly understood phases of matter
• entangled systems
• …

Scaling of resistivity $\rho \sim T^\alpha$
- Common feature: systems whose d.o.f. are not weakly coupled
  → no notion of quasi-particles + Boltzmann/Landau theory does not apply

- Natural setting to use holography
  • a set of analytic tools to probe mechanisms behind such systems

- On the GR side, from the dialogue between the two communities:
  • new classes of (black hole) solutions
  • new types of instabilities
  • ground states with reduced symmetries (broken translations and/or rotations, anisotropic, non-relativistic geometries …)
  • new emergent scaling IR behavior
  • …

layered structure in cuprate superconductor
My focus today:

- the **vacuum structure** of some of these novel scaling geometries (Lifshitz scaling and hyperscaling violation)
- features and questions associated with the rich landscape of IR phases

smectic order in a napoleon

charge density waves on sheets of high T superconductor (CaC$_6$)
Lifshitz scaling and hyperscaling violation

- **Non-relativistic Lifshitz scaling**
  Dynamical critical exponent $z \rightarrow$ anisotropy between space and time

  \[ \omega \sim k^z \quad x \rightarrow \lambda x \quad t \rightarrow \lambda^z t \]

  Characterizes scaling of thermo quantities

  \[ s(T) \sim T^{\frac{d}{z}} \]

- **Hyperscaling violation** $\theta \rightarrow$ anomalous scaling of free energy
  \[ s(T) \sim T^{\frac{d-\theta}{z}} \]

  shifts effective dimensionality of the system $d_{\text{eff}} = d - \theta$

\[ d_{\text{eff}} = 1 \text{ of interest for compressible states and systems w/ Fermi surface } (S_{\text{ent}} \sim A \log A) \]

[Huijse/Sachdev/Swingle, Takayanagi et al] But FS not easily captured by holography.
How do we geometrize these scalings?

‘Minimal’ model:
Exact solutions to simple EMD theory (either electric or magnetic field)

\[ \mathcal{L}_{d+2} = R - 2(\partial \phi)^2 - e^{2\alpha \phi} F^2 - V_0 e^{-\eta \phi} \]
How do we geometrize these scalings?

`Minimal` model:
Exact solutions to simple EMD theory (either electric or magnetic field)

\[
\mathcal{L}_{d+2} = R - 2(\partial \phi)^2 - e^{2\alpha \phi} F^2 - V_0
\]

\[
d s_{d+2}^2 = \left( -r^{-2z} \, dt^2 + \frac{d r^2 + d \vec{x}^2}{r^2} \right)
\]

\[
t \to \lambda^z t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \lambda r
\]

\[
\phi(r) \sim \log(r)
\]
How do we geometrize these scalings?

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\[ d s_{d+2}^2 = \left(-r^{-2z} \, d t^2 + \frac{d r^2 + d \vec{x}^2}{r^2} \right) r^{2 \theta / d} \]

\( t \rightarrow \lambda^z t, \quad \vec{x} \rightarrow \lambda \vec{x}, \quad r \rightarrow \lambda r \quad d s \rightarrow \lambda^{\theta / d} d s \)

\( \phi(r) \sim \log(r) \)

no longer scale invariant

In general, anomalous scaling of gauge field important to understand conductive properties [Gouteraux, Gouteraux/Kiritsis, Karch]
Natural question: IR endpoint of these scaling solutions?

Solutions are supported by a **running dilatonic scalar** $\phi \sim \log r$

$\rightarrow$ not expected to be a good description of the geometry in the deep IR

$$\mathcal{L} = R - 2(\partial\phi)^2 - e^{2\alpha\phi} F^2 - V_0 e^{-\eta\phi}$$

Effective gauge coupling of the theory $g \equiv e^{-\alpha\phi}$ drives system to

**strong coupling**

*(magnetic case)*

Expect modifications to $g(\phi)$, e.g.

$$\frac{1}{g^2} \rightarrow \frac{1}{g^2} + \xi_1 + \xi_2 g^2 + \ldots$$

(toy model for QM corrections)

**weak coupling**

*(electric case)*

Expect higher derivative terms no longer negligible

(tree level terms comparable to $F^4,\ldots$)

Also curvature + tidal singularities [Copsey/Mann, Horowitz/Way, Bao/Dong/Harrison/Silverstein]
In the Lifshitz case, a toy model for QM corrections generates AdS$_2 \times \mathbb{R}^2$ in deep IR [Harrison/Kachru/Wang 1202.6635]

Our starting point:

\[ \mathcal{L} = R - 2(\partial \phi)^2 - f(\phi)F^2 - V(\phi) \]

\[ f(\phi) = e^{2\alpha \phi}, \quad V(\phi) = -V_0 e^{-\eta \phi} \]

Explored **conditions for emergence of AdS$_2 \times \mathbb{R}^2$** in deep IR:

- **generic IR modifications to $f(\phi)$ and $V(\phi)$** → whether of classical or `quantum' origin (toy model of QM corrections as baby example)

AdS$_2 \times \mathbb{R}^2$ (z,θ) geometry AdS$_4$

- deep IR
  - e.g. $\xi_2 g^4 + \xi_1 g^2 \simeq 1$

- UV
  - e.g. $V = -V_0(e^{-\eta \phi} + c_1 e^{n_1 \phi})$
Main Message:

- These scaling geometries should be thought of as intermediate solutions.
- In many cases their `naïve` IR completion is \( \text{AdS}_2 \times \mathbb{R}^2 \).
Main Message:

- These scaling geometries should be thought of as intermediate solutions.

- In many cases their `naïve` IR completion is AdS$_2 \times$ R$_2$.

This picture has emerged in a number of setups:

- Dyonic charges yield stabilizing potential for scalar $\rightarrow$ AdS$_2 \times$ R$_2$ [Trivedi et al, 1208.2008].

- Higher derivative and QM corrections provide stabilization mechanism $\rightarrow$ AdS$_2 \times$ R$_2$ [Knodel/Liu, Peet et al, Cardoso/Haack et al, ...].

- Various SUGRA truncations (sometimes with `η-geometries` in IR or mid-IR) [Donos/Gauntlett/Pantelidou, Kulaxizi/Parnachev/Schalm, ...].
Well-known extensive ground state entropy of $\text{AdS}_2 \times \mathbb{R}^2$ in violation of 3rd law (highly degenerate ground state – pathology or feature?)

New phases expected to emerge $\Rightarrow \text{AdS}_2 \times \mathbb{R}^2$ should not be typical ground state
Spatially Modulated Instabilities of \((z, \theta)\) geometries

- Well-known extensive ground state entropy of \(\text{AdS}_2 \times \mathbb{R}^2\) in violation of 3rd law
  (highly degenerate ground state – pathology or feature?)

- New phases expected to emerge \(\Rightarrow \text{AdS}_2 \times \mathbb{R}^2\) should not be typical ground state

- \(\text{AdS}_2 \times \mathbb{R}^2\) suffers from spatially modulated instabilities in a variety of setups
  [Nakamura/Ooguri/Park, Donos/Gauntlett/Pantelidou,…]
  - note also non-linear instability to inhomogeneous horizons Hartnoll/Santos 1403.4612

Our logic:
- use knowledge of instabilities of \(\text{AdS}_2\) region to identify \((z, \theta)\) geometries
  which are unstable to spatially modulated phases
  \(\Rightarrow\) ubiquitous in CM systems (smectics, spin/charge density waves…)
Spatially modulated instabilities

Simple EMD setup: \[ \mathcal{L} = R - V(\phi) - 2(\partial\phi)^2 - f(\phi)F_{\mu\nu}F^{\mu\nu} \]

**Strategy:**

1) require \( f(\phi) \) and \( V(\phi) \) to give:
   - \( \text{AdS}_2 \times \mathbb{R}^2 \) in the deep IR
   - an intermediate regime of \((z, \theta)\) scaling:
     \[ f(\phi) = e^{2\alpha \phi} + \ldots \]
     \[ V(\phi) = V_0 e^{-\eta \phi} + \ldots \]
     corrections negligible in intermediate region

2) identify conditions for existence of IR instabilities (modes that violate \( \text{AdS}_2 \) bound)

   \rightarrow \text{instability conditions for generic } f(\phi_h) \text{ and } V(\phi_h)

3) map to conditions on \((z, \theta)\) and remaining parameters in the theory

Magnetic case 1212.4172
Purely electric in 1310.3279
Spatially modulated instabilities

Turn on spatially modulated fluctuations about IR AdS$_2 \times \mathbb{R}^2$

Purely magnetic case [1212.4172]:

\[
\begin{align*}
\delta g_{tt} &= L^2 r^2 h_{tt}(r) \cos(kx), \\
\delta g_{xx} &= L^2 b^2 h_{xx}(r) \cos(kx), \\
\delta g_{yy} &= L^2 b^2 h_{yy}(r) \cos(kx), \\
\delta A_y &= a(r) \sin(kx), \\
\delta \phi &= w(r) \cos(kx),
\end{align*}
\]

**k=0 case**

No unstable modes for AdS$_2 \times \mathbb{R}^2$ (Almuhairi/Polchinski, Donos/Gauntlett/Pantelidou)

Spectrum of scaling dimensions:

\[
\delta_{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + L^2 V'' + \frac{f''}{f}} \quad V_{eff}'' > 0
\]

Possible instabilities controlled by curvature of effective scalar potential

$\Rightarrow$ none if scalar settles to min of its effective potential (e.g. AdS$_2 \times \mathbb{R}^2$)
Spatially modulated instabilities

Finite k case

- spectrum of scaling dimensions (small k expansion):

\[
\delta_{1,2,3,4} = \frac{1}{2} \pm \sqrt{\text{mess}} \\
\text{mess} = [k = 0 \text{ terms}] \pm k^2 \left[ \frac{3}{2} \pm \frac{1}{2} \pm \frac{2}{8 - V_{eff}''} \frac{f'^2}{f^2} \right]
\]

\[
V_{eff}'' = \frac{f''}{f} + L^2 V''
\]

AdS$_2 \times$ R$^2$ unstable to spatial modulations (for some k-range) whenever

\[
8 - 2 \left( \frac{f'}{f} \right)^2 < \frac{f''}{f} + L^2 V'' < 8 + \left( \frac{f'}{f} \right)^2
\]

- so far we have a **generic scalar potential and gauge kinetic function** (here constant B field but analogous results for purely electric case)
Connecting with the intermediate regime

Require $f(\phi)$ and $V(\phi)$ to give rise to intermediate scaling regime, e.g.

$$f(\phi) = e^{2\alpha\phi} + \ldots$$

$$V(\phi) = V_0 e^{-\eta\phi} + \ldots$$

corrections negligible in intermediate region

Fully specifying $f(\phi)$ and $V(\phi) \rightarrow$ values of $(z, \theta)$ associated with instabilities
Connecting with the intermediate regime

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Fully specifying $f(\phi)$ and $V(\phi)$ $\Rightarrow$ values of $(z, \theta)$ associated with instabilities

Concrete example [from electric case 1310.3279]

\[
    f(\phi) = e^{\alpha\phi}, \quad V = V_0 e^{-\eta\phi} + \mathcal{V}(\phi)
\]

spatially modulated instabilities provided constraint is obeyed

\[
    \frac{8}{\theta - 2z + 2} = L^2 \left( \mathcal{V}''(\phi_0) - \frac{\theta^2}{(\theta - 2)(\theta - 2z + 2)} \mathcal{V}(\phi_0) \right)
\]
Take home message:

- evidence for spatially modulated phases (‘stripes’) as possible ground states of (certain) $(z, \theta)$ geometries

We took a shortcut to identify instabilities and used the naïve IR $\text{AdS}_2 \times \mathbb{R}^2$

- one should be able to see these unstable modes by analyzing the $(z, \theta)$ geometries directly \(\rightarrow\) Iizuka, Maeda 1301.5677
An AdS$_4$ IR completion of (z, $\theta$) geometries
[ Work with J. Bhattacharya and B. Gouteraux, 1407.????]

Natural question: are there other possible ground states?
- not all (z, $\theta$) solutions are unstable to spatial modulations or even approach AdS$_2 \times$ R$^2$

Emergent conformal symmetry in the IR?
An AdS$_4$ IR completion of $(z,\theta)$ geometries

[Work with J. Bhattacharya and B. Gouteraux, 1407.????]

Natural question: are there other possible ground states?

- not all $(z,\theta)$ solutions are unstable to spatial modulations or even approach $\text{AdS}_2 \times \mathbb{R}^2$

Emergent conformal symmetry in the IR?

Picture we are exploring:

*Analog of ground state of holographic superconductor but with intermediate hyperscaling violating regime*

[Gubser/Rocha 0807.1737, Gubser/Nellore 0908.1972 ... , Horowitz/Roberts 0908.3677]
Our toy model \[ \mathcal{L} = R - \frac{1}{2} (\partial \phi)^2 - f(\phi) F^2 - W(\phi) A^2 - V(\phi) \]

\( \rightarrow \) broken-symmetry phase of theory w/ U(1) symmetry and charged complex scalar

Note:
- scalar potential must be carefully engineered to get intermediate scaling
- massive gauge field needed to source IR AdS\(_4\)
- intermediate scaling regime sensitive to where charge density is concentrated
Features

Emergent conformal symmetry

\[ V(\phi_{UV}) = -\frac{6}{L_{UV}^2} \]
\[ V(\phi_{IR}) = -\frac{6}{L_{IR}^2} \]

- new stable ground state for scaling solutions w/out extensive entropy issues
- \((z,\theta)\) scaling regime in mid-infrared region \(\Rightarrow\) tunable knobs
- expect interplay between different scalings at different energy scales
  \(\Rightarrow\) Applications to transport?
Rich structure of IR phases

AdS$_4$

intermediate

Lifshitz and hyperscaling

violating geometry

spatially modulated geometries

AdS$_4$

AdS$_2 \times$ R$_2$

$\#(\text{AdS}_2 \times \text{R}_2)$

Lifshitz
This story falls into the recent efforts to classify IR geometries (Gouteraux + Kiritsis, Iizuka et al, Kachru et al, …)

- scaling IR asymptotics at finite density
- homogeneous Bianchi geometries (e.g. helical structure)
- broken translations (‘smectic’ order) and/or rotations (‘nematic’ order)

Breaking of translational invariance crucial for transport
→ lots of work on resulting phenomenology [e.g. talks by Erdmenger and Gauntlett]

IR phases breaking only rotations can also be realized and mirror CM systems

\[ ds^2 = -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^{2p} dx^2 + r^{2q} dy^2 \]
\[ t \rightarrow \lambda^z t, \quad x \rightarrow \lambda^p x, \quad y \rightarrow \lambda^q y, \quad r \rightarrow \lambda^{-1} r \]

underlying theme → ground states with reduced symmetries
A number of interesting questions once symmetries are relaxed, e.g.

- Can we geometrize the interplay between different phases and scalings?
  - e.g. nematic phases with no smectic instabilities at weak coupling. At strong coupling? or Isotropic – nematic – smectic transitions?

- Competing orders?
  - competition between possible sources of instabilities and phases  [new $\omega$-deformed SO(8) gauged SUGRA theories, work in progress with Y. Pang, C. Pope, J. Rong]

- Holographic RG flows w/out Lorentz invariance: any monotonicity?
  [SC + Xi Dong, arXiv:1311.3307]
  - generically breakdown of monotonicity w/out Lorentz invariance
  - in certain cases, one can still identify criteria on UV geometry that ensure monotonicity (c-function from entanglement entropy of a strip)
  - more fundamental understanding? [e.g. talk by Casini]
To wrap up…

- The structure of phases from gravity is much richer than anticipated, with interesting emergent IR behaviors
- Novel ground states with reduced symmetries
- As the dialogue between gravity and quantum field theories continues, we gain more insight into the mechanisms driving strongly coupled phases of matter

Thank You