F-term axion monodromy inflation

fernando marchesano

Instituto de Física Teórica
UAM-CSIC
F-term axion monodromy inflation

Based on:
F.M., Shiu, Uranga
[1404.3040]
BICEP2 and Inflation

✦ If BICEP2 results are confirmed, most would agree that

✦ Inflation took place

✦ The energy scale of inflation is the GUT scale

\[ E_{\text{inf}} \simeq 0.75 \times \left( \frac{r}{0.1} \right)^{1/4} \times 10^{-2} M_{\text{Pl}} \]

✦ The inflaton field excursion was super-Planckian

\[ \Delta \phi \gtrsim \left( \frac{r}{0.01} \right)^{1/2} M_{\text{Pl}} \] \text{Lyth '96}

✦ Inflation is extremely sensitive to UV dynamics
Moreover, a favoured inflation model would be $V = m^2 \phi^2$:

- Loop corrections involving inflatons and gravitons small due to approximate shift symmetry
  $$\phi \mapsto \phi + \text{const.}$$

- Coupling to UV degrees of freedom in quantum gravity a priori break this shift symmetry and lead to corrections that spoil inflation, because of the large field excursions

$$\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 + \sum_{i=1}^{\infty} c_i \phi^{2i} \Lambda^{4-2i}$$
Chaotic Inflation

\[ \mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial \phi)^2 - \frac{1}{2}m^2\phi^2 + \sum_{i=1}^{\infty} c_i \phi^{2i} \Lambda^{4-2i} \]

taken from Baumann & McAllister '14
Natural Inflation

- String models where the **inflaton is an axion** in principle can avoid this problem
  - Shift symmetry broken by non-perturbative effects + UV completion, but **periodicity is exact**
  - In string theory axions generically come from p-forms, so **above the KK scale** the shift symmetry becomes a gauge symmetry

\[
\phi = \int_{\pi_p} C_p \\
F_{p+1} = dC_p \\
C_p \rightarrow C_p + d\Lambda_{p-1}
\]

Freese, Frieman, Olinto '90

Dimopoulos et al. '05
Natural Inflation

String models where the **inflaton is an axion** in principle can avoid this problem

- Shift symmetry broken by non-perturbative effects+UV completion, but **periodicity is exact**
- In string theory axions generically come from p-forms, so **above the KK scale** the shift symmetry becomes a gauge symmetry
- However, these axions have **sub-Planckian** decay constants

\[
\phi = \int_{\pi_p}^{\pi} C_p + \Delta_{p+1} = dC_p
\]

Banks et al. ’03, Svrcek & Witten ’06
Axion Monodromy Inflation

Idea: Combine chaotic inflation and natural inflation

The axion periodicity is lifted, allowing for super-Planckian displacements. The UV corrections to the potential should still be constrained by the underlying symmetry.
Axion Monodromy Inflation

Siverstein & Westphal '08

Idea:
Combine chaotic inflation and natural inflation

The axion periodicity is lifted, allowing for super-Planckian displacements. The UV corrections to the potential should still be constrained by the underlying symmetry.
Axion Monodromy Inflation

Idea: Combine chaotic inflation and natural inflation

Early developments:

- McAllister, Silverstein, Westphal → String scenarios
- Kaloper, Lawrence, Sorbo → 4d framework

see Siverstein’s talk

\[ \int C^{(2)} = c \]
Done in string theory within the moduli stabilisation program: adding ingredients like background fluxes generate superpotentials in the effective 4d theory.

F-term Axion Monodromy Inflation

Obs: Axion Monodromy ~ Giving a mass to an axion

taken from Ibañez & Uranga '12
Done in string theory within the **moduli stabilisation** program: adding ingredients like background fluxes generate **superpotentials** in the effective 4d theory.

**Obs:**

- Axion Monodromy
- Giving a mass to an axion

**Idea:**

- Use same techniques to generate an inflation potential

*taken from Ibañez & Uranga '12*
F-term Axion Monodromy Inflation

**Obs:**

- Axion Monodromy \sim \text{Giving a mass to an axion}

- Done in string theory within the moduli stabilisation program: adding ingredients like background fluxes generate superpotentials in the effective 4d theory

**Idea:**

- Use same techniques to generate an inflation potential

- Simpler models, all sectors understood at weak coupling
- Spontaneous SUSY breaking, no need for brane-anti-brane
- Clear endpoint of inflation, allows to address reheating
Toy Example: Massive Wilson line

- Simple example of axion: (4+d)-dimensional gauge field integrated over a circle in a compact space $\Pi_d$

\[ \phi = \int_{S^1} A_1 \quad \text{or} \quad A_1 = \phi(x) \eta_1(y) \]

- $\phi$ massless if $\Delta \eta_1 = 0 \Rightarrow S^1$ is a non-trivial circle in $\Pi_d$
  exact periodicity and (pert.) shift symmetry

- $\phi$ massive if $\Delta \eta_1 = -\mu^2 \eta_1 \Rightarrow kS^1$ homologically trivial in $\Pi_d$
  (non-trivial fibration)
Simple example of axion: (4+d)-dimensional gauge field integrated over a circle in a compact space $\Pi_d$

$$\phi = \int_{S^1} A_1 \quad \text{or} \quad A_1 = \phi(x) \eta_1(y)$$

- $\phi$ massless if $\Delta \eta_1 = 0 \Rightarrow S^1$ is a non-trivial circle in $\Pi_d$
  - exact periodicity and (pert.) shift symmetry
- $\phi$ massive if $\Delta \eta_1 = -\mu^2 \eta_1 \Rightarrow kS^1$ homologically trivial in $\Pi_d$
  - (non-trivial fibration)

$$F_2 = dA_1 = \phi \, d\eta_1 \sim \mu \phi \, \omega_2 \Rightarrow \text{shifts in } \phi \text{ increase energy via the induced flux } F_2$$

$\Rightarrow$ periodicity is broken and shift symmetry approximate
MWL and twisted tori

Simple way to construct massive Wilson lines: consider compact extra dimensions $\Pi_d$ with circles fibered over a base, like the twisted tori that appear in flux compactifications.

There are circles that are not contractible but do not correspond to any harmonic 1-form. Instead, they correspond to torsional elements in homology and cohomology groups:

$$\text{Tor} \; H_1(\Pi_d, \mathbb{Z}) = \text{Tor} \; H^2(\Pi_d, \mathbb{Z}) = \mathbb{Z}_k$$
MWL and twisted tori

- Simple way to construct massive Wilson lines: consider compact extra dimensions $\Pi_d$ with circles fibered over a base, like the twisted tori that appear in flux compactifications.

- There are circles that are not contractible but do not correspond to any harmonic 1-form. Instead, they correspond to torsional elements in homology and cohomology groups.

\[
\text{Tor } H_1(\Pi_d, \mathbb{Z}) = \text{Tor } H^2(\Pi_d, \mathbb{Z}) = \mathbb{Z}_k
\]

- Simplest example: twisted 3-torus $\tilde{T}^3$

\[
H_1(\tilde{T}^3, \mathbb{Z}) = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_k
\]

\[
d\eta_1 = kdx^2 \wedge dx^3 \quad \Rightarrow \quad F = \phi k \, dx^2 \wedge dx^3
\]

\[
\mu = \frac{kR_1}{R_2R_3}
\]

under a shift $\phi \rightarrow \phi + 1$

$F_2$ increases by $k$ units
MWL and monodromy

Do the monodromy and approximate shift symmetry help preventing wild UV corrections?
Torsion and gauge invariance

Twisted tori torsional invariants are not just a fancy way of detecting non-harmonic forms, but are related to a hidden gauge invariance of these axion-monodromy models.

Let us again consider a 7d gauge theory on $M^{1,3} \times \mathbb{T}^3$

Instead of $A_1$ we consider its magnetic dual $V_4$

\[
V_4 = C_3 \wedge \eta_1 + b_2 \wedge \sigma_2 \quad \Rightarrow \quad dV_4 = dC_3 \wedge \eta_1 + (db_2 - kC_3) \wedge \sigma_2
\]
Torsion and gauge invariance

- Twisted tori torsional invariants are not just a fancy way of detecting non-harmonic forms, but are related to a hidden gauge invariance of these axion-monodromy models.

- Let us again consider a 7d gauge theory on $M^{1,3} \times \mathbb{T}^3$.
  
  - Instead of $A_1$ we consider its magnetic dual $V_4$.
  
  $$V_4 = C_3 \wedge \eta_1 + b_2 \wedge \sigma_2$$
  
  $$dV_4 = dC_3 \wedge \eta_1 + (db_2 - kC_3) \wedge \sigma_2$$

- From dimensional reduction of the kinetic term:
  
  $$\int d^7 x |dV_4|^2 \rightarrow \int d^4 x |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

  - Gauge invariance: $C_3 \rightarrow C_3 + d\Lambda_2$, $b_2 \rightarrow b_2 + k\Lambda_2$.
  
  - Generalization of the Stückelberg Lagrangian.

Quevedo & Trugenberger '96
Effective 4d theory

- The effective 4d Lagrangian

\[ \int d^4x \, |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2 \]

describes a massive axion, has been applied to QCD axion ⇒ generalised to arbitrary \( V(\phi) \)

Kalosh et al. '95
Dvali, Jackiw, Pi '05
Dvali, Folkerts, Franca '13

- Reproduces the axion-four-form Lagrangian proposed by Kaloper and Sorbo as 4d model of axion-monodromy inflation with mild UV corrections

\[ \int d^4x \, |F_4|^2 + |d\phi|^2 + \phi F_4 \]

\[ F_4 = dC_3 \]
\[ d\phi = *_4 db_2 \]

Kaloper & Sorbo '08

- It is related to an F-term generated mass term

Groh, Louis, Sommerfeld '12
Effective 4d theory

- Effective 4d Lagrangian

\[
\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \mu^2 \phi^2 + \Lambda^4 \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}}
\]

\[
\sum_n c_n \frac{F^{2n}}{\Lambda^{4n}} \rightarrow \mu^2 \phi^2 \sum_n c_n \left( \frac{\mu^2 \phi^2}{\Lambda^4} \right)^n
\]

\[\Rightarrow \text{suppressed corrections up to the scale where } V(\phi) \sim \Lambda^4\]

\[\Rightarrow \text{effective scale for corrections } \Lambda \rightarrow \Lambda_{\text{eff}} = \Lambda^2/\mu\]
Effective 4d theory

- Effective 4d Lagrangian

\[ \int d^4 x \left| dC_3 \right|^2 + \frac{\mu^2}{k^2} \left| dB_2 - kC_3 \right|^2 \]

\[ F_4 = dC_3 \]
\[ d\phi = *_4 dB_2 \]

- Gauge symmetry \( \Rightarrow \) UV corrections only depend on \( F_4 \)

\[ \Lambda \rightarrow \Lambda_{\text{eff}} = \Lambda \left( \frac{\Lambda}{\mu} \right) \]
Discrete symmetries and domain walls

The integer $k$ in the Lagrangian

$$\int d^4x |F_4|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

corresponds to a discrete symmetry of the theory broken spontaneously once a choice of four-form flux is made. This amounts to choose a branch of the scalar potential

![Graph showing scalar potential with different branches for $n=0, 1, 2, 3$ and $k=4$.](taken from Kaloper & Lawrence '14)
Discrete symmetries and domain walls

- The integer \( k \) in the Lagrangian

\[
\int d^4x |F_4|^2 + \frac{\mu^2}{k^2}|db_2 - kC_3|^2
\]

corresponds to a discrete symmetry of the theory broken spontaneously once a choice of four-form flux is made. This amounts to choose a branch of the scalar potential.

- Branch jumps are made via nucleation of domain walls that couple to \( C_3 \), and this puts a maximum to the inflaton range.

- Domain walls analysed in string constructions:
  
  Berasaluce-Gonzalez, Camara, F.M., Uranga '12

  - They correspond to discrete symmetries of the superpotential/landscape of vacua, and appear whenever axions are stabilised.
  - \( k \) domain walls decay in a cosmic string implementing \( \phi \rightarrow \phi+1 \).
Massive Wilson lines in string theory

- Simple example of MWL in string theory: D6-brane on $M^{1,3} \times \mathbb{TP}^3$

- An inflaton vev induces a non-trivial flux $F_2$ proportional to $\phi$ but now this flux enters the DBI action

$$\sqrt{\det (G + 2\pi \alpha' F_2)} = dvol_{M^{1,3}} (|F_2|^2 + \text{corrections})$$
Massive Wilson lines in string theory

- Simple example of MWL in string theory: D6-brane on $M^{1,3} \times \tilde{T}^3$

- An inflaton vev induces a non-trivial flux $F_2$ proportional to $\phi$ but now this flux enters the DBI action

$$\sqrt{\det (G + 2\pi \alpha' F_2)} = d\text{vol}_{M^{1,3}} (|F_2|^2 + \text{corrections})$$

- For small values of $\phi$ we recover chaotic inflation, but for large values the corrections are important and we have a potential of the form

$$V = \sqrt{L^4 + \langle \phi \rangle^2} - L^2$$

Similar to the D4-brane model of Silverstein and Westphal except for the inflation endpoint
Massive Wilson lines and flattening

- The DBI modification
  \[ \langle \phi \rangle^2 \rightarrow \sqrt{L^4 + \langle \phi \rangle^2} - L^2 \]
  can be interpreted as corrections due to UV completion

- E.g., integrating out moduli such that \( H < m_{\text{mod}} < M_{\text{GUT}} \)
  will correct the potential, although not destabilise it
  \[ \text{Kaloper, Lawrence, Sorbo '11} \]

- In the DBI case the potential is flattened: argued general effect due to couplings to heavy fields
  \[ \text{Dong, Horn, Silverstein, Westphal '10} \]

- Large vev flattening also observed in examples of confining gauge theories whose gravity dual is known [Witten’98]
  \[ \text{Dubovsky, Lawrence, Roberts '11} \]
Other string examples

- We can integrate a bulk p-form potential $C_p$ over a p-cycle to get an axion:

$$F_{p+1} = dC_p, \quad C_p \rightarrow C_p + d\Lambda_{p-1}, \quad c = \int_{\pi_p} C_p$$

- If the p-cycle is torsional we will get the same effective action:

$$\int d^{10}x |F_{9-p}|^2 \quad \rightarrow \quad \int d^4x |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$
Other string examples

- We can integrate a bulk $p$-form potential $C_p$ over a $p$-cycle to get an axion

$$F_{p+1} = dC_p, \quad C_p \rightarrow C_p + d\Lambda_{p-1} \quad c = \int_{\pi_p} C_p$$

- If the $p$-cycle is torsional we will get the same effective action

$$\int d^{10}x |F_{9-p}|^2 \quad \rightarrow \quad \int d^4x |dC_3|^2 + \frac{\mu^2}{k^2} |db_2 - kC_3|^2$$

- The topological groups that detect this possibility are

$$\text{Tor } H_p(X_6, \mathbb{Z}) = \text{Tor } H^{p+1}(X_6, \mathbb{Z}) = \text{Tor } H^{6-p}(X_6, \mathbb{Z}) = \text{Tor } H_{5-p}(X_6, \mathbb{Z})$$

one should make sure that the corresponding axion mass is well below the compactification scale (e.g., using warping)

Franco, Galloni, Retolaza, Uranga ’14
Other string examples

- Axions also obtain a mass with background fluxes

- Simplest example: \( \phi = C_0 \) in the presence of NSNS flux \( H_3 \)

\[
W = \int_{x_6} (F_3 - \tau H_3) \wedge \Omega \quad \tau = C_0 + i/g_s
\]

- We also recover the axion-four-form potential

\[
\int_{M^{1,3} \times x_6} C_0 H_3 \wedge F_7 = \int_{M^{1,3}} C_0 F_4 \quad F_4 = \int_{PD[H_3]} F_7
\]
Other string examples

- Axions also obtain a mass with background fluxes

- **Simplest example:** $\phi = C_0$ in the presence of NSNS flux $H_3$

  $$W = \int_{x_6} (F_3 - \tau H_3) \wedge \Omega \quad \tau = C_0 + i/g_s$$

- We also recover the **axion-four-form potential**

  $$\int_{M^{1,3} \times x_6} C_0 H_3 \wedge F_7 = \int_{M^{1,3}} C_0 F_4 \quad F_4 = \int_{\text{PD}[H_3]} F_7$$

- **M-theory version:** Beasley, Witten '02

- A rich set of superpotentials obtained with **type IIA fluxes**

  $$\int_{x_6} e^{J_c} \wedge (F_0 + F_2 + F_4) \quad J_c = J + iB$$

  potentials higher than quadratic

- Massive axions detected by **torsion groups in K-theory**
Conclusions

* Axion monodromy is an elegant idea that combines chaotic and natural inflation, aiming to prevent disastrous UV corrections to the inflaton potential

* We have discussed its implementation in a new framework, dubbed F-term axion monodromy inflation compatible with spontaneous supersymmetry breaking

* In a simple set of models the inflaton is a massive Wilson line. They show the mild UV corrections for large inflaton vev.

* Effective action reproduces the axion-four-form action proposed by Kaloper and Sorbo. Discrete symmetries classified by K-theory torsion groups.

* Inflaton mass should be hierarchically smaller than the Kaluza-Klein modes and the compactification moduli. (e.g. via warping)
Thank you!