Resolving Black Holes
via
Microstate Geometries

Nick Warner, Strings `14, June 24, 2014
Based on Collaborations with:
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⇒ Two distinct core ideas from *microstate geometries*

1) A string theory *mechanism* to support structure at the horizon scale

⇒ Two (at least) new scales for black-hole physics

What does horizon-scale microstate structure look like?

Fuzz/Fire/hybrid… other?

2) A semi-classical description of black-hole microstates?
   Arising from fluctuations/moduli of microstate geometries
   (in the same regime of parameters in which there is an *actual black hole*)

The *superstratum* (BPS): \[ S_{\text{semi class.}} \sim \sqrt{N_1 N_5 N_P} \]
Fixing the information problem: An old conceit

Recover information (and pure quantum state) from small corrections to GR over the evaporation time scale…

e.g. via stringy or quantum gravity \((Riemann)^n\) corrections to radiation?

Mathur (2009): **No!** Corrections cannot be small for information recovery
There must be \(O(1)\) changes to the physics at the horizon scale

Microstate geometries

A *mechanism* for resolving the problem in string theory

Firewalls

Unsupported superstructure
Microstate Geometry Program

**Microstate Geometry** ≡ Smooth, horizonless solutions to the bosonic sector of **supergravity** with the same asymptotic structure as a given black hole/ring

*Singularity resolved; Horizon removed*

**Supergravity** because we seek stringy resolutions on horizon scale
- Very long-range effects ⇒ Massless limit of strings …
- Framework within which we can *actually do calculations*

What is the form of generic, (non-)BPS, time-independent horizonless, smooth solutions in supergravity?

**Microstate Geometries/solitons** long believed **impossible** because only the presence of a horizon can restrict massless fields to a classical lump …

**Microstate Geometries** exist (how?) … and lead to new physical issues
- New physics/scales will emerge from the resolution
- What can supergravity tells us about details of microstate structure?
The Komar Mass/Smarr Formula

If there is time-translation invariance then energy is conserved:
There is a vector field (Killing vector) $K$ generating time translations.

$$K^\mu \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial t}$$

D-dimensional space-time, sectioned by hypersurfaces, $\Sigma$, with Gaussian (D-2)-spheres, $S^{D-2}$, at infinity

There is then an associated conserved ADM mass:

$$K^\mu K^\nu g_{\mu \nu} = g_{00} \approx -1 + \frac{16\pi G_D}{(D-2) A_{D-2}} \frac{M}{\rho^{D-3}} + \ldots \text{ at infinity}$$

$$\Rightarrow M = -\frac{1}{16\pi G_D} \frac{(D-2)}{(D-3)} \int_{S^{D-2}} *dK$$

If $\Sigma$ is smooth with no interior boundaries:

$$d*dK = -2 * (K^\mu R_{\mu \nu} dx^\nu)$$

$$M = \frac{1}{8\pi G_D} \frac{(D-2)}{(D-3)} \int_{\Sigma} *_D (K^\mu R_{\mu \nu} dx^\nu)$$
Bosonic sector of a generic *massless* (ungauged) supergravity

- Graviton, $g_{\mu\nu}$
- Scalars, $\Phi^A$
- Tensor gauge fields, $F^{(p)K}$

Bianchi: $d(F^{(p)K}) = 0$

Define: $G_{j,(D-p)} \equiv \star (Q_{JK}(\Phi) F^{(p)K} + \text{Chern Simons terms})$

$Q_{JK}(\Phi) = \text{Scalar matrix in kinetic terms}$

Equations of motion: $d(G_{j,(D-p)}) = 0$

Assume time-independent matter:

$$\mathcal{L}_K F^I = 0, \quad \mathcal{L}_K \Phi^A = 0 \quad \Rightarrow \quad \mathcal{L}_K G_I = 0$$

Cartan formula for forms:

$$\mathcal{L}_K \omega = d(i_K(\omega)) + i_K(d\omega)$$

$$d(i_K(F^{(p)I})) = 0, \quad d(i_K(G_{j,(D-p)})) = 0$$

Define *harmonic* forms, $H$:

$$i_K(F^{(p)I}) = H^{(p-1)}_{(p-1)} + \text{exact} \quad i_K(G_{j,(D-p)}) = H_{(D-p-1)} + \text{exact}$$
No Solitons without Topology

Smooth spatial sections with *no interior boundaries*

\[
M = \frac{1}{8\pi G_D} \frac{(D - 2)}{(D - 3)} \int_{\Sigma} \star_D (K^\mu R_{\mu\nu} dx^\nu)
\]

Equations of motion imply

\[
M = \text{const.} \int_{\Sigma} \left[ H_{I(D-p-1)} \wedge F_{(p)}^I + H_{(p-1)}^J \wedge G_J (D-p) \right]
\]

Gibbons + NPW 1305.0957; Haas 1405.3708

- Mass can be topologically supported by the cohomology \(H^*(\Sigma, \mathbb{R})\)
  
  Stationary end-state of star held up by topological flux ...
  
  - A new object: A Topological Star
  - Black-Hole Microstate?

- No spatial topology \(\Rightarrow M = 0 \Rightarrow\) Space-time is flat/empty

Only assumed time independence: *Not simply for BPS objects*

Applies to all time-independent smooth remnants in massless ungauged supergravity
A Decade of BPS Microstate Geometries

Bubbled geometries in five or six dimensions ⇒ 2 or 3 cycles

★ There are vast families of smooth, horizonless microstate geometries

★ New physics at the horizon scale

⇒ The cap-off and the non-trivial topology, “bubbles,” arise at the original horizon scale

★ Families of solutions: Large moduli spaces of cycles; fluctuations around cycles

★ Special class: KK reduction yields multi-centered solutions of Denef

★ There are scaling microstate geometries with AdS throats that can be made arbitrarily long but cap off smoothly

★ Holography in the long AdS throat: All these solutions represent black-hole microstates

⇒ Semi-classical sampling of black-hole microstate structure

“Topological stars” = coherent microstates of black holes

★ New physical scales …
This is an example of a phase/geometric transition in string theory ... Analogous to holography of confinement and chiral symmetry breaking.

★ Magnitude of fluxes, $\sigma = \text{Order parameter of new phases}$
★ Size of the bubbles, $\lambda_T = \text{Transition Scale}$ is a new scale in the topological phase

Supergravity equations $\Rightarrow \lambda_T \sim \text{Magnitude of fluxes, \(\sigma\)}$

Balance: Gravity $\leftrightarrow$ Flux expansion force

Classically: Freely choosable parameter. Can have $\lambda_T > \ell_p$

Quantum mechanics: Could $\lambda_T$ be dynamically generated?

Black holes: Could large $\lambda_T$ be entropically favored?
Scale 2: The Energy Gap

\[ \lambda_{\text{gap}} = \text{maximally redshifted wavelength, at infinity of lowest collective mode of bubbles at the bottom of the throat.} \]

\[ E_{\text{gap}} \sim (\lambda_{\text{gap}})^{-1} \]

★ The gap is determined by “maximum redshift,” \( z_{\text{max}} \), and size of black hole

Traditional black holes: \( E_{\text{gap}} = 0 \)

★ \( E_{\text{gap}} \) determines where microstate geometries begin to differ from black holes

BPS black holes

Semi-classical quantization of the moduli of the geometry:

★ The throat depth, or \( z_{\text{max}} \), is not a free parameter

★ \( E_{\text{gap}} \) is determined by the flux structure of the geometry

Exactly matches \( E_{\text{gap}} \) for the stringy excitations underlying the original state counting of Strominger and Vafa ..... 


Semi-classical Microstate Structure: **Superstrata on** $R^{5,1} \times T^4$

IIB: D1-D5-P system compactified on $T^4$ (or K3)

**Six Dimensions:**
- Profile in $R^{5,1}$
- Add KKM dipole and Angular Momentum

**Smooth BPS** configurations that depend upon **functions of two variables**: $F(z, \psi)$

Back-reacted geometry: 3 homology

$\Rightarrow$ BPS shape modes on 3 cycles
- **functions of two variables**
D1-D5-P System: Microstate Counting

D1-D5 SCFT Fields

\[
\begin{align*}
X_{(r)}^{\dot{A} A}(z, \bar{z}) & \quad \psi^{\alpha \dot{A}}_{(r)}(z) & \quad \bar{\psi}^{\dot{\alpha} \dot{A}}_{(r)}(\bar{z}) \\
\end{align*}
\]

\[r = 1, \ldots, N = N_1 N_5\]

\[c = 6 N = 6 N_1 N_5\] (4,4) supersymmetry

Define:

\[
J^{\alpha \beta}_{(r)}(z) \equiv \frac{1}{2} \psi^{\alpha \dot{A}}_{(r)}(z) \epsilon_{\dot{A} \dot{B}} \psi^{\beta \dot{B}}_{(r)}(z), \quad \bar{J}^{\dot{\alpha} \dot{\beta}}_{(r)}(\bar{z}) \equiv \frac{1}{2} \bar{\psi}^{\dot{\alpha} \dot{A}}_{(r)}(\bar{z}) \epsilon_{\dot{A} \dot{B}} \bar{\psi}^{\dot{\beta} \dot{B}}_{(r)}(\bar{z})
\]

\[= \text{CFT Degrees of freedom “visible” in } \mathbb{R}^4\]

\[
= \frac{(SU(2)_{(1)}^L \times SU(2)_{(1)}^R)^N}{S_N} \quad c = N = N_1 N_5
\]

\[\frac{1}{4} \text{ BPS states} = (R,R)-\text{ground states}\]

\[\frac{1}{8} \text{ BPS states} = (\text{any left-moving state, R ground state}) \quad \text{Momentum, } P = L_{0,\text{left}}\]

The left-handed currents, \(J^{(r)\alpha \beta}(z), (c = N_1 N_5)\) create left-moving momentum states visible in \(\mathbb{R}^4 \iff \text{BPS Shape modes of the superstratum}/S^3\)
The Holographic Dual: \( \text{AdS}_3 \times S^3 \times T^4 \)

\[
\text{AdS}_3 \times S^3 \times T^4
\]

Modes: \( SU(2)_L \times SU(2)_R \) quantum numbers \((j, m; \hat{j}, \hat{m})\)

\( |j - \hat{j}| \) = space-time spin of underlying field

\( \frac{1}{4} \) BPS states = \((R, R)\)-ground states \(\leftrightarrow\) quantum numbers \((j, j; \hat{j}, \hat{j})\)

\( \rightarrow \) D1-D5 supertube shape modes on \( S^3 \)

\( \leftrightarrow \) one arbitrary function, Fourier modes, \( j \)

Lunin and Mathur; Lunin, Maldacena and Maoz; Mathur; Skenderis and Taylor …

\( \Rightarrow \) Semi-classical entropy of a supertube:

\[
S \sim \sqrt{Q_1 Q_2} \sim Q
\]

\( \frac{1}{8} \) BPS states: Act with modes of \( J_{(r)}^{\alpha \beta}(z) \) \(\leftrightarrow\) quantum numbers \((j, m; \hat{j}, \hat{j})\)

\( \rightarrow \) D1-D5 superstratum shape modes on \( S^3 \)

\( \leftrightarrow \) two arbitrary functions, Fourier modes, \((j, m)\)

Bena, Shigemori and NPW 1406.4506
Semi-classical black-hole microstate structure

★ Linearized supergravity modes can at least capture, semi-classically, the momentum excitations corresponding to the CFT currents, \( J_{(r)}^{\alpha\beta}(z) \).

★ Deep, scaling microstate geometries have \( E_{\text{gap}} \sim (N_1 N_5)^{-1} \)

Add momentum charge \( N_P \) using \( J_{(r)}^{\alpha\beta}(z) \)

\[
S_{\text{semi class.}} = 2\pi \sqrt{\frac{1}{6} N_1 N_5 N_P} \sim \sqrt{N_1 N_5 N_P}
\]

★ Semi-classical quantization gives a dense enough sampling of microstate structure to recover entropy corresponding to a macroscopic horizon scale

\[
\Rightarrow \text{Typical microstates must have the scale of the original black-hole horizon?}
\]
Summary

• String theory has new phases dominated by topological fluxes that can prevent the formation of black holes → Topological Stars/Black-hole microstates

• Transition to new phase ↔ Formation of bubbles supported by flux → Order parameter and new scale in Nature: $\lambda_T$ = Transition Scale

• The new phase smoothly caps-off the space-time before a horizon forms: → Limits the red-shift and the lowest-energy states: $E_{\text{gap}} > 0$

• The new phases represent new “infra-red” vacua of string theory
  This viewpoint is a natural and direct outgrowth of holographic field theories …

• Discussion of near-horizon physics, like the infall problem and even firewalls, will be enriched/clarified by separating $\lambda_T$ and $E_{\text{gap}}$ from the Planck scale.
  Ignoring this possibility is probably a serious mistake …

• Vast families of BPS examples explicitly constructed

• Superstrata/BPS fluctuations as functions of two variables give semi-classical entropy with correct growth as a function of the charges

• These ideas can be extended to non-BPS, extremal and near-extremal …