1. Ultrasounds are used to monitor the velocity of the beating heart of a fetus. To that end, ultrasonic waves of frequency $f_s = 2,252,343$ Hz are sent to the heart. (Notice that to solve this problem you will need to calculate frequencies to six significant digits.) The waves reach the heart when it is pulsing toward the source with velocity $v_{\text{heart}} = 0.2263$ m/s. The speed of sound in human tissue is $v = 1542$ m/s.

1a. [2pt] What is the frequency $f_{o1}$ that would be detected by an observer on the moving surface of the fetus’ heart?

$$f_{o1} = f_s (1 + \frac{v_{\text{heart}}}{v})$$

$f_{o1} = 2252674$ Hz

1b. [2pt] What is the frequency $f_{o2}$ of the wave that reflects off the surface of the fetus’ heart as detected by an observer at the location of the ultrasound wave generator?

$$f_{o2} = \frac{f_{o1}}{1 - \frac{v_{\text{heart}}}{v}}$$

$f_{o2} = 2253004$ Hz

1c. [1pt] What is the beat frequency of the interference between the initial wave and the reflected wave? (Notice that medical instruments invert this calculation to infer the velocity of the fetus’ heart from a measurement of the beat frequency.)

$$f_{\text{beat}} = f_{o2} - f_s$$

$f_{\text{beat}} = 661$ Hz
2a. [3 pt] A middle-age man typically has poorer hearing than a middle-age woman. A man begins to hear a sound only when its intensity level is increased by 7.8 dB relative to the minimum intensity level that a woman can detect. What is the ratio of the sound intensity just detected by the man to that just detected by the woman?

\[ \beta_{\text{man}} - \beta_{\text{woman}} = 10\text{dB} \quad [\log \frac{I_{\text{man}}}{I_0} - \log \frac{I_{\text{woman}}}{I_0}] = 10\text{dB} \quad \log \frac{I_{\text{man}}}{I_{\text{woman}}} = 7.8\text{dB}. \]

Hence \( \log \frac{I_{\text{man}}}{I_{\text{woman}}} = 0.78 \), which implies that the ratio between the two intensities is

\[ \frac{I_{\text{man}}}{I_{\text{woman}}} = 6.03 \]

2b. [2 pt] Suppose that the man in question 2a is located at a distance \( r_{\text{man}} = 8.50 \text{ m} \) from the source, and he can barely hear the sound. At what distance from the source can the woman be located and still (barely) be able to hear?

In part 2a have computed the ratio between the two intensities. We know that the intensities are inversely proportional to the distances. It follows that

\[ \frac{I_{\text{man}}}{I_{\text{woman}}} = \frac{r^2_{\text{woman}}}{r^2_{\text{man}}} = 6.03 \]

Hence,

\[ r_{\text{woman}} = 22.2 \text{ m} \]