PHYSICS 103 LAST IN-CLASS EXAM

Instructions: When you are told to begin, check that this examination booklet contains all the numbered pages from 3 through 9.

Princeton University Undergraduate Honor Committee

This examination is administered under the Princeton University Honor Code. Students should sit one seat apart from each other, if possible, and refrain from talking to other students during the exam. All suspected violations of the Honor Code must be reported to the Honor Committee Chair at honor@princeton.edu.

The checked items below are permitted for use on this examination. Any item that is not checked may not be used and should not be in your working space. Assume items not on this list are not allowed for use on this examination. Please place items you will not need out of view in your bag or under your working space at this time. University policy does not allow the use of electronic devices such as cell phones, PDAs, laptops, MP3 players, iPods, etc. during examinations. Students may not wear headphones during an examination.

☐ Course textbooks  ☐ Course Notes  ☐ Other printed materials

NOTHING IS ALLOWED EXCEPT THIS EXAMINATION, A PEN OR PENCIL, AND YOUR CALCULATOR. THE CALCULATOR POLICY (SEE NEXT PAGE) IS IN PLACE AS ALWAYS.

Students may only leave the examination room for a very brief period without the explicit permission of the instructor. The exam may not be taken outside of the examination room.

This is a timed examination. You will have 50 minutes to complete this exam. During the examination, the Professor or a preceptor will be outside the exam room.

Rewrite and sign the pledge: I pledge my honor that I have not violated the Honor Code during this examination.

Signature
Problem 1. Traveling to the Future (30 pts)

Your goal is to visit the Earth 10 years in the future. You decide to do this by setting out in a very fast rocket (speed $v$) and traveling for 6 months, then turning around and traveling straight back at the same speed. (Neglect the difficulties associated with starting, stopping, and turning, and assume that you can do those maneuvers in negligible time.)

a) (10 pts) How fast do you need to go? Give values for both the Lorentz factor $\gamma$ and for $v/c$. Write one or two sentences explaining your reasoning. Also show your analytical and numerical work.

\[
\begin{align*}
S &= \text{observer on Earth} \\
S' &= \text{observer on rocket} \\
\Delta t_S &= 10 \text{ years} \\
\Delta t_{S'} &= 1 \text{ year} \\
S' &= \text{measures the proper time} \\
\Delta t_S &= \gamma \Delta t_{S'} \\
\gamma &= 10, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \\
\beta &= \frac{v}{c} \\
1 - \beta^2 &= 0.01 \\
\beta &= \sqrt{1 - 0.01} \approx 1 - \frac{1}{2} 0.01 = 0.995 \\
v &= 2.985 \times 10^8 \text{ m/s}
\end{align*}
\]

b) (10 pts) How far will you go in your outbound trip, as measured in the rest frame of the Earth? If you did not find an answer to (a), you may leave your answer here in terms of $v$.

\[
\begin{align*}
\Delta S &= \text{proper length} \\
\Delta S &= \frac{\Delta t_{S'}}{2} \\
\Delta S &= \frac{v \Delta t_{S'}}{2} = 4.975 \text{ ly} \\
\Delta S &= \gamma \Delta S' = \gamma v \frac{\Delta t_{S'}}{2} = 10 \times 0.995 c \times 0.5 \text{ year} = 4.975 \text{ ly}
\end{align*}
\]
c) (10 pts) One last question about special relativity: the muon has a mean lifetime after creation of about 2 $\mu$s. Light can travel a distance $(3 \times 10^8 \text{ m/s}) \times (2 \times 10^{-6} \text{ s}) = 600 \text{ m}$ in 2 $\mu$s. Muons with velocities near the speed of light are created in the upper atmosphere, at height $h = 15 \text{ km}$. Nearly all of the muons make it to the ground where they can be detected in simple spark chambers, such as the one set up during the last lecture. Noting that $h > 600 \text{ m}$, why does this work? Circle the best answer among the choices below.

i) When moving with $v$ near the speed of light, the distance an object covers in time $t$ is no longer given by $d = vt$.

ii) In the muon rest frame, the distance between the top and bottom of the atmosphere is smaller than 15 km, so the shorter time for the trip is not a problem if the muon is fast enough.

iii) In the Earth rest frame, the muon clock runs more slowly and so it has longer to traverse the atmosphere before it decays.

iv) All of the above.

v) None of the above.

Both ii) and iii) are valid explanations.
Problem 2. Oscillating (30 pts)

A particle of mass \( m \) experiences a potential energy \( U(r) \) as a function of its distance \( r \) from the origin which has the form:

\[
U(r) = \frac{A}{r^2} - \frac{B}{r},
\]

where \( A \) and \( B \) are positive constants. Aside from the force giving rise to this potential, no other forces act on the particle.

a) (10 pts) At what position \( r = r_0 \) is the particle in stable equilibrium? (Hint/reminder: a particle can only be in equilibrium if the net force on it is zero.)

Equilibrium requires \( U' = -F' = 0 \)

\[
U' = \frac{dU}{dr} = -2 \frac{A}{r^3} + \frac{B}{r^2}
\]

\[
U' = 0 \Rightarrow \frac{B}{r_0^2} = 2 \frac{A}{r_0^3} \Rightarrow r_0 = \frac{2A}{B} \quad \text{Full credit}
\]

One should check stability:

\[
U'' = \frac{6A}{r^4} - \frac{2B}{r^3}
\]

\[
U''(r_0) = -6 \frac{A}{(2A)^4} - \frac{2B^4}{(2A)^3} = \frac{6B^4}{16A^3} - \frac{2B^4}{8A^3} = \frac{B^4}{8A^3} > 0
\]

\( \therefore U'(r_0) \) is min.

\( r_0 \) is stable equilibrium.
b) (10 pts) Find the approximate force on the particle in the vicinity of \( r_0 \) in terms of the constants \( A \) and \( B \). You may parameterize the radial coordinate with \( r = r_0 + \epsilon \), where \( \epsilon/r_0 \ll 1 \). Simplify your answer as far as you can for full credit. (Hint: it may be helpful to use the binomial expansion theorem which says \((1 + a)^n \approx 1 + na\), as long as \( a \ll 1\).)

\[ F(r) = F(r_0) + F'(r_0)(r - r_0) \]

\( = 0 + F'(r_0) \epsilon \)  
(equilibrium)

But \( F' = -U'' \), so from part (a) \( F'(r_0) = -\frac{1}{8} \frac{B^4}{A^3} \)

and \[ F = -\frac{1}{8} \frac{B^4}{A^3} \epsilon \] where \( \epsilon = r - r_0 \)

Method 2:

\[ F = \frac{2A}{r^3} - \frac{B}{r^2} = 2A \left( r_0 + \epsilon \right)^{-3} - B \left( r_0 + \epsilon \right)^{-2} \]

\[ = \frac{2A}{r_0^3} \left( 1 + \frac{\epsilon}{r_0} \right)^{-3} - \frac{B}{r_0^2} \left( 1 + \frac{\epsilon}{r_0} \right)^{-2} \approx \frac{2A}{r_0^3} \left( 1 - 3 \frac{\epsilon}{r_0} \right) - \frac{B}{r_0^2} \left( 1 - 2 \frac{\epsilon}{r_0} \right) \]

\[ \approx \text{binomial expansion} \]

\[ F = -\frac{B^4}{8A^3} \epsilon \]

\[ \text{(c) This is of form } F = -k\epsilon, \text{ where } k = \frac{B^4}{8A^3} \]

\[ \text{so } \omega = \sqrt{\frac{B^4}{8A^3 m}} \text{ and } T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{8A^3 m}{B^4}} \]

Avoid expanding \( U \); too much work. Binomial expansion and Taylor-Series.
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Problem 3. Waves (15 pts)

Two loudspeakers emit sound waves of frequency \( f = 1.0 \text{ kHz} \) and identical amplitudes, but with a fixed phase difference \( \Delta \phi \). The speed of sound is \( c_s = 343 \text{ m/s} \).

a) (5 pts) First, the two loudspeakers are placed side-by-side on the \( x \)-axis at \( x = 0 \). Both speakers face toward positive \( x \). (You may assume that no sound waves are emitted from the backs of the loudspeakers.) It is observed that no sound is heard by observers standing anywhere on the \( x \)-axis until the second loudspeaker is moved slightly along the \( x \)-axis. What is the minimum positive value for \( \Delta \phi \)?

No sound is heard.

Hence, the two speakers are emitting sound waves with phase difference \( \Delta \phi = \pi \), (\( \because \Delta \phi \) is positive)

180° phase delay for maximum destructive interference.

b) (10 pts). Now the first loudspeaker is left at \( x = 0 \) and the second is moved along the positive \( x \)-axis. When it is at position \( x_1 \), the sound intensity heard by all observers on the positive \( x \)-axis is maximum. Find the minimum value for \( x_1 \).

\[
\frac{1}{x=0} \quad \quad \frac{1}{x=x_1}
\]

We want \( A \sin(kx - \omega t) + A \sin(k(x-x_1) - \omega t + \Delta \phi) \) to be maximum constructive interference.

Therefore, \( -kx_1 + \Delta \phi = 0, 2\pi, 4\pi, \ldots \)

For minimum positive \( x_1 \), we find \( kx_1 = \Delta \phi \)

\[
x_1 = \frac{\Delta \phi}{k} = \frac{\pi}{2\pi/2} = \frac{2}{2} = \frac{c_s}{2f} = \frac{343}{2 \times 1000} = 0.1715 \text{ m}
\]
Problem 4. Grab Bag (25 pts)

a) (5 pts) The figure above shows the time evolution of the air displacement at a spot in space due to the superposition of two sound waves of frequencies $f_1$ and $f_2$. What are those two frequencies?

\[
\sin \omega_1 t + \sin \omega_2 t = 2 \sin \left( \frac{\omega_1 + \omega_2}{2} t \right) \cos \left( \frac{\omega_1 - \omega_2}{2} t \right)
\]

Slowly varying envelope $\rightarrow \frac{f_2 - f_1}{2} \times 2$ second $= 1$ period

Fast oscillating $\rightarrow \frac{f_2 + f_1}{2} \times 2$ second $= 21$ periods

Therefore $f_1 = 10 \text{ Hz}$, $f_2 = 11 \text{ Hz}$

b) (5 pts) In the 1970's, a commercial for loudspeakers aired in which Ella Fitzgerald shattered a crystal wineglass by singing one loud clear note, amplified through the loudspeakers. Circle the physics phenomenon below which played the dominant role in this feat.

i) critical damping

\textbf{ii) resonance}

iii) overdamping

iv) interference

v) beats
c) (15 pts) A traveling wave moves in the x direction along an extremely long string. The figure above shows a snapshot of the string at time $t = 250$ s on the top, and plots the $y$ position of the piece of string at $x = 0$ m as a function of time in the lower panel. Use these two graphs to write down the function $y(x, t)$. Define all constants which appear in your function.

From the two plots, we find

$$\lambda = \frac{400}{4} = 100 \text{ m} \quad \text{and} \quad f = \frac{2}{1000} = 2 \times 10^{-3} \text{ Hz}$$

$$y(0, t) = 2 \cos (\pm 2\pi ft)$$

$$y(x, 250) = 2 \cos \left( \frac{2\pi}{100} x \pm 2\pi \times 2 \times 10^{-3} x \times 250 \right)$$

$$= 2 \cos \left( \frac{\pi}{50} x \pm \pi \right) \times \text{OK}.$$  

Therefore, if the wave moves in positive x direction

$$y(x, t) = A \cos \left( \frac{\pi}{\lambda} x - 2\pi ft \right)$$

$$= 2 \cos \left( \frac{\pi}{50} x - 4\pi \times 10^{-3} t \right).$$