Circuit Quantum Electrodynamics with Silicon Charge and Spin Qubits

Xiao Mi

A Dissertation
Presented to the Faculty of Princeton University in Candidacy for the Degree of Doctor of Philosophy

Recommended for Acceptance by the Department of Physics
Adviser: Jason R. Petta

June 2018
Abstract

Coherent coupling of solid-state qubits to microwave photons, in the form of circuit quantum electrodynamics (cQED), provides an elegant approach to non-demolition qubit readout and a scalable pathway toward long-range entanglement. Since its inception and monumental success in the context of superconducting qubits, enormous research efforts have been placed on developing a cQED architecture for electron spin qubits in silicon. Compared to their solid-state peers, silicon-based electron spin qubits have the virtue of exceptionally long lifetimes ($T_1$) which are on the order of seconds, fault-tolerant control fidelity and compatibility with traditional fabrication techniques. However, implementing cQED with spin qubits is inherently difficult due to the small magnetic dipole moment of a single electron spin, limiting single spin-photon coupling rates to about 10 Hz.

In this thesis, we resolve this decade-long challenge through a series of experiments: First, we demonstrate strong-coupling between a single electron charge qubit in a gate-defined silicon double quantum dot and a microwave photon in a superconducting cavity, an achievement made possible by a new device architecture capable of supporting charge coherence times two orders of magnitude longer than previously attained in a semiconductor environment. Combining electric-dipole interaction with spin-charge hybridization in the presence of a magnetic field gradient, we then achieve the strong-coupling regime between a single electron spin and a single microwave photon. Spin-photon coupling rates up to 11 MHz are supported by the device architecture, exceeding direct magnetic-dipole coupling rates by a stunning five orders of magnitude. As an immediate application of strong spin-photon coupling, we demonstrate all-electric control and dispersive readout of the single-spin qubit, laying the foundation for quantum non-demolition readout of semiconductor spin qubits. These results form a critical step toward building a quantum processor with both long lifetimes and high connectivity.
Utilizing the exquisite energy resolution offered by cQED, we conduct two further experiments exploring valley states in silicon quantum dots. Valley states are charge states with a small energy splitting (valley splitting) that is notoriously difficult to resolve using traditional transport measurement techniques. We first use the dispersive interaction between the valley states and cavity photons to perform high-resolution measurements of valley splitting. In a second experiment, we study hybridized valley-orbit states in silicon using Landau-Zener-Stückelberg-Majorana interferometry. Here we find that hybridization with valley states significantly reduces the susceptibility of silicon charge qubits to charge noise due to a flat energy dispersion, analogous to transmon qubits in superconducting systems. These findings not only hold the potential of accelerating the progress toward controlling valley splitting in Si quantum devices, but also raise the interesting prospect of turning the valley degree of freedom, often viewed as a hindrance for silicon qubits, into a powerful asset for reducing the detrimental impact of charge noise.
Acknowledgements

“A little learning is a dangerous thing; Drink deep, or taste not the Pierian spring.”
The foremost thanks goes to my advisor Prof. Jason Petta. Beyond providing me with a meticulously constructed lab without which the experiments in this thesis would not have been possible, Jason has guided me through graduate school with a myriad of inspiring personal qualities. As a physicist, he approaches any experimental undertaking with an almost self-abnegating degree of thoroughness and ensures that I have exhausted every means of empirical verification before drawing a conclusion. The high standard of “drink deep” he has set is a valuable lesson both in physics and in life. As a mentor, Jason has always made it his mission to regularly meet with me and provide insightful suggestions on the next steps. I am particularly appreciative of the fact that he has the best interests of his students at heart and goes an extra mile to help me with career choices after graduation. As a group leader, Jason is also remarkably forward-looking: every experiment we have chosen to do is not an end in itself, but always a stepping stone toward a more momentous future. I am deeply grateful to have been a member of Jason’s team these past five years.

It has also been my true pleasure to have worked alongside other perennial dwellers of Jadwin B33. Dave Zajac, one of the most gifted experimentalists I have met, never ceases to amaze me with his analytical mind, dexterity in the cleanroom and a preternatural ability to remain calm no matter how beleaguered with difficulties. George Stehlik possesses an enviable air of omniscience, capable of answering questions about microwave circuitry, microwave radars or microwave ovens with equally high levels of expertise. I am much indebted to Dave and George for initiating me into the details of device fabrication and measurements. Sorawis Sangtawesin (Peace) has impressively ventured into unchartered waters by building a complete NV setup from ground up, all the while maintaining a hospitable apartment with George to where we occasionally flock. Jeff Cady is ever the cheerful undergraduate who helped rein in my dejection.
when resonator after resonator failed to live up to our expectations. Yinyu Liu, my compatriot and fellow frequenter of local Chinese restaurants, still holds the unbroken record on the number of papers that may be pumped out of a single sample. Stefan Putz has been an industrious comrade-at-arms with whom I shared the thrill of seeing the first sign of spin-photon vacuum Rabi splitting appear on the network analyzer screen. Anthony Sigillito is a highly experienced experimentalist and an invaluable recourse for sensible advices. I am delighted to pass the baton of the hybrid cQED project to Felix Borjans, whose punctilious and methodical approach to experiments is certain to ensure future prosperity of this burgeoning field. I also wish other past members of the Petta lab well in their present pursuits: Loren Alegria, Ke Wang, Christopher Eichler, Tom Hazard and Sonia Zhang.

I owe much of the theoretical understanding involved in my graduate research to our collaborators, Guido Burkard, Jake Taylor and Sigmund Kohler. I am also very happy to have become friends with members of the Burkard team, Monica Benito and Max Russ, with whom we have worked through a wealth of puzzles and foraged many a street for grub during various conferences. Csaba Péterfalvi, whom I have regretfully never met in person, is instrumental in threading together a neat story about valleys. Michael Gullans is always a wonderful colleague to discuss problems with and we are all excited about his recent relocation to Princeton.

A special thanks goes to my undergraduate advisor Prof. John Reppy (and his better half, Prof. Judith Reppy). A legendary figure in low temperature physics, John was a major influence during my formative years as a physicist. Not only has John personally trained me on the intricacies of dilution fridges and machining, his injunction, “extraordinary results require extraordinary proof”, is forever entrenched in my mind and strikes a chord with Jason’s principle. Since coming to Princeton, we have maintained close contact. John and Judith have also, through their wisdom, provided me with many valuable advices in matters both professional and personal.
Our crusade of finding supersolids has now been replaced with occasional strolls through the woods in the Gunks, late-night education about wine tasting, a constant urging for me to take up driving and casual discourse on Randian novels.

Research would never have run smoothly without the supportive staff in Princeton’s physics department. I am particularly thankful toward Bert Harrop, who has provided me with much assistance in terms of sample packaging through both his experience and his creativity. Similarly, Geoff Gettelfinger, Jim Kukon, Darryl Johnson, Steve Lowe, Bill Dix, Kate Brosowsky, Dale O’Brien, Ted Lewis, Lauren Callahan and Julio Lopez have all been exemplary in their alacrity to help.

Last but not least, my deepest gratitude to my parents who have endured my years of absence from home with stoic forbearance and always taken the keenest interest in every aspect of my life.

Finally, this thesis was supported by US Department of Defense under contract H98230-15-C0453, the Army Research Office through Grant No. W911NF-15-1-0149, the Gordon and Betty Moore Foundations EPiQS Initiative through grant GBMF4535, the National Science Foundation through Grants No. DMR-1409556 and No. DMR-1420541, and the Spanish Ministry of Economy and Competiveness through Grant MAT2017-86717-P. Devices were fabricated in the Princeton University Quantum Device Nanofabrication Laboratory.
To my family and friends.
The work described in this dissertation has been published in the following articles and presented at the following conferences:

Science 355, 156 (2017).

International Workshop on Silicon Quantum Electronics, August 2017, Hillsboro, Oregon.
International School and Symposium on Nanoscale Transport and Photonics, November 2017, Atsugi, Japan.
American Physical Society Meeting, March 2018, Los Angeles, California.
Contents

Abstract  .......................................................... iii
Acknowledgements  ................................................. v
List of Tables  ...................................................... xiii
List of Figures  ...................................................... xiv

1  Introduction  ...................................................... 1
   1.1  Cavity Quantum Electrodynamics  ........................... 1
   1.2  Circuit-QED with Superconducting Qubits  .................. 4
   1.3  Semiconductor Spin Qubits  ................................ 7
   1.4  Coupling Spins to Microwave Photons  ...................... 11
   1.5  Outline of Contents  ........................................ 13

2  Magneto-Transport Study of Undoped Si/SiGe Heterostructures  14
   2.1  Silicon Germanium Heterostructures  ......................... 15
   2.2  Characterization at $T = 4.2$ K  .......................... 17
   2.3  High Mobility Sample  ..................................... 20
   2.4  Low Mobility Sample  ..................................... 25
   2.5  Estimate of Defect Densities  .............................. 28
   2.6  Metal-to-Insulator Transition  ............................. 29
   2.7  Valley Splitting  ......................................... 31
   2.8  Conclusion  ............................................... 33
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4</td>
<td>Data with Unequal Valley Splittings</td>
<td>102</td>
</tr>
<tr>
<td>6.5</td>
<td>Conclusion</td>
<td>104</td>
</tr>
<tr>
<td>7</td>
<td>Electrically Protected Valley-Orbit States in Silicon</td>
<td>105</td>
</tr>
<tr>
<td>7.1</td>
<td>Strongly Coupled DQD-cQED Device with Low-Lying Valley States</td>
<td>106</td>
</tr>
<tr>
<td>7.2</td>
<td>Microwave Spectroscopy of Valley States</td>
<td>109</td>
</tr>
<tr>
<td>7.3</td>
<td>Landau-Zener-Stückelberg-Majorana Interferometry of Valley-Orbit States</td>
<td>111</td>
</tr>
<tr>
<td>7.4</td>
<td>Charge Noise Spectroscopy</td>
<td>115</td>
</tr>
<tr>
<td>7.5</td>
<td>A Charge-Noise-Insensitive Valley-Orbit Qubit</td>
<td>116</td>
</tr>
<tr>
<td>7.6</td>
<td>Conclusion</td>
<td>117</td>
</tr>
<tr>
<td>8</td>
<td>Outlook</td>
<td>119</td>
</tr>
<tr>
<td>A</td>
<td>Theory of Spin-Photon Coupling in DQDs</td>
<td>121</td>
</tr>
<tr>
<td>A.1</td>
<td>Input-Output Theory for Cavity Transmission</td>
<td>121</td>
</tr>
<tr>
<td>A.2</td>
<td>Theoretical Models for Spin-Photon Coupling and Spin Decoherence</td>
<td>123</td>
</tr>
<tr>
<td>A.3</td>
<td>Asymmetric Line-Shapes of Spin-Photon Vacuum Rabi Splitting</td>
<td>126</td>
</tr>
<tr>
<td>B</td>
<td>Theory for Cavity Response under Finite Bias</td>
<td>128</td>
</tr>
<tr>
<td>C</td>
<td>Theory for Landau-Zener-Stückelberg-Majorana Interferometry of Valley-Orbit States</td>
<td>138</td>
</tr>
<tr>
<td>C.1</td>
<td>Theoretical Model for Decoherence of the Valley-Orbit States</td>
<td>138</td>
</tr>
<tr>
<td>C.2</td>
<td>Numerical Calculation of Landau-Zener-Stückelberg Interference</td>
<td>140</td>
</tr>
<tr>
<td>C.3</td>
<td>Analytical Approximations</td>
<td>143</td>
</tr>
<tr>
<td>Bibliography</td>
<td></td>
<td>147</td>
</tr>
</tbody>
</table>
List of Tables

B.1 DQD Capacitances . . . . . . . . . . . . . . . . . . . . . . . . . . . . 130

C.1 Stationary Phase Points . . . . . . . . . . . . . . . . . . . . . . . . . 144
List of Figures

1.1 Representation of a CQED Experiment ........................................... 2
1.2 Vacuum Rabi Splitting of an Atom in an Optical Cavity ..................... 3
1.3 Circuit-QED with Superconducting Qubits ........................................ 5
1.4 Spin Qubits in Semiconductor Gate-Defined Quantum Dots ................. 8
1.5 Silicon-Based Spin Qubits .............................................................. 9
1.6 Coupling of a Spin Ensemble to a Microwave Cavity .......................... 11

2.1 Si/SiGe Heterostructures ............................................................. 16
2.2 Mobility Variation between Wafers ............................................... 18
2.3 Integer Quantum Hall Effect of the High Mobility Wafer .................... 20
2.4 Variation of Mobility over Carrier Density ....................................... 22
2.5 Quantum Lifetime of the High Mobility Wafer .................................. 23
2.6 Variation of Quantum Lifetime over Temperature ............................. 25
2.7 Mobility and Quantum Lifetime of the Low Mobility Wafer ................. 26
2.8 Metal-to-Insulator Transition of the Si/SiGe 2DEG ............................. 30
2.9 Valley Splitting of the Si/SiGe 2DEG .............................................. 32

3.1 Schematic of a CPW Resonator ...................................................... 37
3.2 Bare Resonator Variants ............................................................. 40
3.3 Bare Resonator Transmissions ....................................................... 42
3.4 Power and Temperature Dependences of Bare Resonators .................... 43
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 Magnetic Field Dependences of Resonators with Al Air-Bridges</td>
<td>45</td>
</tr>
<tr>
<td>3.6 Early Design of Hybrid cQED Devices with Si/SiGe QDs</td>
<td>48</td>
</tr>
<tr>
<td>3.7 Test Resonator Design for Understanding Microwave Leakage</td>
<td>49</td>
</tr>
<tr>
<td>3.8 Circuit model of a Resonator with One Coupling Port</td>
<td>51</td>
</tr>
<tr>
<td>3.9 Circuit model of a Resonator with Two Coupling Ports</td>
<td>51</td>
</tr>
<tr>
<td>3.10 Circuit model of a Resonator with One LC-filtered Coupling Port</td>
<td>53</td>
</tr>
<tr>
<td>3.11 Circuit model of a Resonator with Two LC-filtered Coupling Ports</td>
<td>54</td>
</tr>
<tr>
<td>3.12 Hybrid Si DQD-cQED Device</td>
<td>56</td>
</tr>
<tr>
<td>3.13 Compact LC-Filter Design and Improved Cavity Quality Factor</td>
<td>58</td>
</tr>
<tr>
<td>4.1 Microwave Measurement Circuit Diagram</td>
<td>62</td>
</tr>
<tr>
<td>4.2 Charge-Photon Interaction</td>
<td>64</td>
</tr>
<tr>
<td>4.3 Single Electron Charge Coherence</td>
<td>66</td>
</tr>
<tr>
<td>4.4 Charge-Photon Vacuum Rabi Splitting</td>
<td>68</td>
</tr>
<tr>
<td>5.1 Spin-Photon Interface</td>
<td>74</td>
</tr>
<tr>
<td>5.2 Single Spin-Photon Strong Coupling</td>
<td>76</td>
</tr>
<tr>
<td>5.3 Electrical Control of Spin-Photon Coupling</td>
<td>79</td>
</tr>
<tr>
<td>5.4 Quantum Control and Dispersive Readout of a Single Spin</td>
<td>82</td>
</tr>
<tr>
<td>5.5 Spin Relaxation at $\epsilon = 0$</td>
<td>84</td>
</tr>
<tr>
<td>5.6 Photon Number Calibration</td>
<td>86</td>
</tr>
<tr>
<td>5.7 Prospect for Long-Range Spin-Spin Coupling</td>
<td>87</td>
</tr>
<tr>
<td>6.1 Cavity-Coupled Si DQD with Valley States</td>
<td>92</td>
</tr>
<tr>
<td>6.2 Valley States Detection in Thermal Equilibrium</td>
<td>95</td>
</tr>
<tr>
<td>6.3 High-temperature Enhancement of Valley States Visibility</td>
<td>97</td>
</tr>
<tr>
<td>6.4 Valley States Re-population under Positive Source-Drain Bias</td>
<td>99</td>
</tr>
<tr>
<td>6.5 Valley States Re-population under Negative Source-Drain Bias</td>
<td>101</td>
</tr>
<tr>
<td>6.6 Spectroscopy of Devices with Unequal Valley Splittings</td>
<td>103</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Cavity Quantum Electrodynamics

The physics of cavity quantum electrodynamics (CQED) explores the coherent interaction between a two-level atom (or qubit, in the language of quantum information) and photons inside a resonant cavity, as illustrated by Fig. 1.1. The Hamiltonian of the system $H_{\text{sys}}$ consists of three parts $H_{\text{sys}} = H_a + H_c + H_{\text{int}}$. Here $H_a = \hbar \omega_a \sigma_z$ is the atom Hamiltonian where $\sigma_z$ is a Pauli operator and $\hbar$ is Planck’s constant. $H_c = \hbar \omega_c (a^\dagger a + \frac{1}{2})$ is the cavity Hamiltonian where $a^\dagger(a)$ is the photon creation (annihilation) operator.

Semi-classically, the interaction Hamiltonian $H_{\text{int}}$ may be written as:

$$H_{\text{int}} = -q (r - r_0) \dot{E}(r_0),$$  \hspace{1cm} (1.1)

where $q$ is the electric charge of the atom, $r$ refers to the coordinates of the atom with $r_0$ being the origin and $E$ is the electric field inside the cavity. In CQED, the electric field within the cavity is quantized and takes the form:

$$E(r) = i \sqrt{\frac{\hbar \omega_c}{2V\epsilon_0}} (a - a^\dagger) \hat{U}(E),$$  \hspace{1cm} (1.2)
Figure 1.1: Sketch of a CQED experiment. A two-level atom with ground state |g⟩ and excited state |e⟩ is placed within a resonant cavity having a frequency ω_c and a decay rate κ. The atom has a transition energy ℏΩ_a and a characteristic decoherence rate γ. The coherent coupling rate between the atom and the cavity is g.

where V is the “mode volume” of the cavity, ε₀ is the permittivity of free space and ˆU(E) is a unit vector representing the direction of the cavity electric field. One can re-write the interaction Hamiltonian as $H_{\text{int}} = 1 H_{\text{int}} 1$ where $1 = |g⟩⟨g| + |e⟩⟨e|$ is the identity operator of the atom. We assume the integral $q \int_{V} |ψ_{g,e}(r)|^2 E(r) \cdot dr = 0$ where $ψ_{g}(r)$ [$ψ_{e}(r)$] is wave-function of the ground [excited] state of the atom. This leads to $⟨g|H_{\text{int}}|g⟩ = ⟨e|H_{\text{int}}|e⟩ = 0$. It then follows that:

$$H_{\text{int}} = \hbar (g |e⟩⟨g| - g^* |g⟩⟨e|)(a - a^\dagger) = \hbar (g σ_+ - g^* σ_-)(a - a^\dagger),$$  \hspace{1cm} (1.3)

where $σ_+ = |e⟩⟨g|$ and $σ_- = |g⟩⟨e|$ are the raising and lowering operators of the atom, respectively. Also,

$$g = -qi \sqrt{\frac{ω_c}{2hVε₀}} ˆU(⟨e| (r - r_0) |g⟩)$$ \hspace{1cm} (1.4)

is the coherent coupling rate between the cavity and the atom. Using the rotating wave approximation (neglecting fast-rotating terms $a^\dagger σ_+$ and $aσ_-$), the total Hamil-
Figure 1.2: (a) Setup of an early CQED experiment, showing a stream of Cesium atoms which are allowed to pass through an optical cavity. (b) Plot of intra-cavity photon number, $\bar{n}$, as a function of probe frequency $\Omega$, when the average number of atoms in the cavity is $N = 1$. Figures taken from Ref. [3].

Hamiltonian becomes:

$$H_{\text{sys}} = \hbar \omega_c \left( a^\dagger a + \frac{1}{2} \right) + \hbar \frac{\omega_a}{2} \sigma_z + \hbar g \left( a^\dagger |\psi^-\rangle + a |\psi^+\rangle \right).$$ (1.5)

This is the well-known Jaynes-Cummings Hamiltonian that characterizes most CQED systems [1,2]. An interesting aspect of this Hamiltonian is that in the resonant regime where the atom and the cavity have the same frequency ($\omega_a = \omega_c$), the eigenstates of the system become the dressed states $|\psi_\pm\rangle = (|g\rangle|n + 1\rangle \pm |e\rangle|n\rangle)/\sqrt{2}$, where $n = 0, 1, 2...$ refers to the number of photons inside the cavity and the states are split by an energy difference $2g\sqrt{n}$. The atom and the cavity are hybridized in this case.

A physical manifestation of this hybridization is that if an excited atom is placed within an empty cavity, it will oscillate between the ground and excited states at a rate $2g$, emitting and re-absorbing a single photon in each cycle. Such a phenomenon is known as a vacuum Rabi oscillation. In the frequency domain, this hybridization leads to vacuum Rabi splitting, which is a pair of resonant peaks in the transmission spectrum of the cavity separated by the vacuum Rabi frequency $2g$. 


All real systems have loss, of course, which often make vacuum Rabi splitting challenging to observe in experiments. For CQED systems, these losses are characterized by two rates: $\gamma$, which refers to the decoherence rate of the atom, and $\kappa$, which refers to the rate at which photons leak out of the cavity. The consequence of these loss processes is a broadening of each vacuum-Rabi-split peak into a finite width of $\gamma + \kappa/2$. It is only when the vacuum Rabi frequency exceeds such a width, $2g > \gamma + \kappa/2$, that vacuum Rabi splitting may be well-resolved. This is the so-called strong-coupling regime of CQED which was first demonstrated in atomic systems. The experimental setup and result of an early experiment are shown in Fig. 1.2: A cloud of Cesium atoms are allowed to pass an optical cavity. When the average number of atoms in the cavity is reduced to 1, vacuum Rabi splitting is observed in the average intra-cavity photon number as a function of the probe frequency.

After early demonstrations in atomic systems, experimental efforts were placed to realize CQED with solid-state systems. An example setup is optically addressed self-assembled quantum dots, where a cavity may be realized either with a distributed Bragg reflector made from alternating layers of dielectric materials or with a defect in a photonic band-gap material. Another setup for solid-state CQED, particularly relevant to this thesis, is with superconducting qubits and superconducting resonators. We will discuss this setup in detail in the following section.

### 1.2 Circuit-QED with Superconducting Qubits

A particularly successful arena for CQED is its implementation with superconducting qubits, which is often referred to as circuit-QED or cQED in this particularly context. Superconducting qubits are non-linear oscillators composed of Josephson junctions which have a non-linear inductance. As a result, the ground to 1st excited state transition has a frequency that is different from the transition frequency between
other states in the system, allowing these non-linear circuits to behave as effective solid-state artificial atoms, or qubits. By fabricating microwave-frequency co-planar waveguide (CPW) cavities (see Chapter 3 for details on such cavities) next to these qubits, strong-coupling may be achieved, as illustrated in Fig. 1.3.

Following the early demonstration of strong-coupling, cQED has been utilized to achieve two tremendously useful practical applications. The first such application is dispersive readout. Here the system is operated in the dispersive regime, where the qubit-cavity detuning $\Delta = |\omega_a - \omega_c| \gg g$. In this case, the Jaynes-Cummings Hamiltonian may be expanded in powers of $g/\Delta$ to yield [8]:

$$H_{\text{sys}} = \hbar \left( \omega_c + \frac{g^2}{\Delta} \sigma_z \right) \left( a^\dagger a + \frac{1}{2} \right) + \hbar \omega_a \sigma_z \frac{\sigma_z}{2}. \quad (1.6)$$
The Hamiltonian now resembles that of an uncoupled qubit-cavity system, except that the cavity resonance frequency is shifted by an amount \( \frac{g^2}{\Delta} \sigma_z \), the sign of which depends on the state of the qubit. In practice, this dispersive shift may be detected through a change in the amplitude or phase of the transmission through the cavity, allowing readout of the qubit state with near-unity visibility \([9]\). Besides high readout fidelity, another significant advantage of dispersive readout is that it is highly quantum non-demolition (QND). The large detuning \( \Delta \) ensures that the qubit is (almost) projected to the state that is being measured and leaves its state approximately unperturbed at the end of the measurement.

In the dispersive regime, the Jaynes-Cummings Hamiltonian may also be rewritten to emphasize the effect of dispersive interaction on the qubit \([8]\):

\[
H_{\text{sys}} = \hbar \omega_c \left( a^\dagger a + \frac{1}{2} \right) + \hbar \left( \omega_a + \frac{2g^2}{\Delta} a^\dagger a + \frac{g^2}{\Delta} \right) \sigma_z.
\] (1.7)

It can be seen that the qubit now acquires a photon-number dependent frequency shift (the ac-Stark shift) equal to \( 2ng^2/\Delta \) and a vacuum-fluctuation induced “Lamb” shift \( g^2/\Delta \). The ac-Stark shift effect allows convenient \textit{in-situ} calibration of the intra-cavity photon number.

A second application of cQED is to engineer non-local qubit interactions. Long-range coupling between qubits can, for example, be accomplished by mapping the qubit state onto a Fock state of the cavity through vacuum Rabi oscillation. The cavity Fock state is then mapped onto the state of another qubit in a reverse manner \([10]\). This method, however, suffers from cavity-induced loss. Another method for cavity-mediated two-qubit interaction is using virtual photons \([11, 12]\). Here both qubits are dispersively coupled to the cavity, such that the qubit-cavity detunings \( \Delta_1 = \omega_{a,1} - \omega_c \) and \( \Delta_2 = \omega_{a,2} - \omega_c \) satisfy \( \Delta_1 \gg g_1 \) and \( \Delta_2 \gg g_2 \) respectively. Using
second order perturbation theory, the effective Hamiltonian for the system is:

\[ H_{\text{sys}} = \frac{\hbar \omega_{a,1}}{2} \sigma_1^z + \frac{\hbar \omega_{a,2}}{2} \sigma_2^z + \hbar (\omega_c + \chi_1 \sigma_1^z + \chi_2 \sigma_2^z) + \hbar J (\sigma_1^- \sigma_2^+ + \sigma_2^- \sigma_1^+) \]. (1.8)

Here \( \chi_1 \) and \( \chi_2 \) are the magnitudes of the dispersive shifts of both qubits. The transverse exchange interaction \( J \) depends on both \( \Delta_{1,2} \) and the qubit-cavity coupling rates \( g_{1,2} \) of each qubit: \( J = \frac{g_1 g_2}{2} (1/\Delta_1 + 1/\Delta_2) \). An important advantage of this coupling scheme is that since both qubits are far detuned from the cavity, no real photons are emitted or absorbed in the process, thereby circumventing cavity-induced loss. However, such a coupling also requires large qubit-cavity coupling rates \( g_{1,2} \) to be effective.

1.3 Semiconductor Spin Qubits

Another popular class of solid-state qubits are based on the spin degrees of freedom of electrons confined in gate-defined semiconductor quantum dots (QDs). Such “spin qubits” were originally proposed by Loss and DiVincenzo [Fig. 1.4(a)] in the form of a double quantum dot (DQD) [13]. Each dot contains a single electron, the spin state of which is Zeeman-split by an externally applied magnetic field. The spin-up and spin-down state of each electron therefore forms a single qubit. To perform two-qubit gates, an exchange interaction of the form \( J \mathbf{S}_1 \cdot \mathbf{S}_2 \) exists between the two spins, where \( J = 4t_C^2/U \) depends the interdot tunnel coupling \( t_C \) and \( U \) is the charging energy of a single dot. Since \( t_C \) may be quickly controlled by the voltages on the gate electrodes, this exchange interaction may be voltage-controlled as well.

Early realizations of spin qubits were done with doped GaAs/AlGaAs heterostructures as substrates. As an example, the SEM image of a GaAs DQD is shown in Fig. 1.4(b). Selective biasing of each gate electrode allows the confinement of a single electron in each quantum dot. This particular experiment, in fact, formed a qubit
Figure 1.4: (a) The Loss-DiVincenzo proposal: A pair of electrons are confined within a gate-defined double quantum dot (DQD) system. The Zeeman-split spin state of each spin forms a single qubit. Two-qubit gates are enabled by the exchange interaction, which may be voltage-controlled [13]. (b) A depletion mode GaAs DQD device from an early experiment [14], which uses a nearby QPC for charge state readout. Spin-to-charge conversion using Pauli blockade allows the spin states of the DQD to be read out as well. (c) Coherent oscillation between the singlet $(|↑,↓⟩ - |↓,↑⟩)/\sqrt{2}$ and the triplet $(|↑,↓⟩ + |↓,↑⟩)/\sqrt{2}$ states [14]. (d) Coherent oscillation between spin-up $|↑⟩$ and spin-down $|↓⟩$ states of a single electron spin in a GaAs DQD device [15].
using the two-electron spin states: a singlet state $|S\rangle = (|↑, ↓⟩ - |↓, ↑⟩)/\sqrt{2}$ and a triplet state $|T_0\rangle = (|↑, ↓⟩ + |↓, ↑⟩)/\sqrt{2}$. To distinguish between these two states, a nearby quantum point contact (QPC) first reads out the charge state of the DQD $(2,0)$ and $(1,1)$, where $(2,0)$ is a charge state where both electrons are in one dot and $(1,1)$ is a charge state where the electrons are in separate dots. Due to Pauli spin blockade, the triplet state cannot tunnel into the $(2,0)$ state, hence allowing the two spin states to be distinguished using this process of “spin-to-charge conversion”. Coherent oscillation between the singlet and the triplet states is driven by the difference in the nuclear magnetic field of each dot, as shown in Fig. 1.4(c).

Soon after the demonstration of this “exchange qubit”, single spin qubits were demonstrated in GaAs as well. In Fig. 1.4(d), coherent oscillation between the spin-up and spin-down states of a single electron in the DQD is measured in DC current flowing through the DQD. Here a single spin rotation is driven by an applied ac magnetic field, and spin state readout is again enabled by Pauli blockade [15].
Spin qubits in GaAs-based devices have a crucial shortcoming: The spin-carrying nuclei of the substrate cause rapid dephasing of the electronic spin states due to fluctuation of the nuclear spin bath. To address this limitation, the spin qubit community began exploring QD spin qubits on silicon-based heterostructures such as Si/SiO$_2$ or Si/SiGe. The naturally abundant nuclear isotope of Si, $^{28}$Si, carries zero nuclear spin and therefore significantly increases the dephasing time of Si-based spin qubits. Moreover, isotopic purification may increase the dephasing time even further. Initial efforts to make QDs on Si/SiGe heterostructures employed modulation-doped substrates which faced tuning challenges. More recently, undoped Si/SiGe heterostructures were used to make Si-based QDs, which were greeted with immense success. Figure 1.5(a) shows an array of nine QDs defined on an undoped Si/SiGe heterostructure, and it was shown that the charging energy and orbital splitting of each QD are highly uniform throughout the array. The coherence times of isotopically purified Si spin qubits are also very long. Figure 1.5(b) shows the high-quality Rabi oscillation of a single spin qubit [17]. The $T_2$ of such a sample is as long as 3 ms using dynamic decoupling, and a single-qubit control fidelity of 99.9% is achieved.

Due to the environmental isolation of electronic spin states, the relaxation times ($T_1$) of QD spin qubits are typically very long, ranging from 100 ms to as long as a few seconds [18]. As such, continuous materials and fabrication improvements are expected to lead to further increase in the coherence times of such qubits, making them highly promising candidates for a future quantum processor. However, also due to this degree of environmental isolation, spin qubits are typically difficult to couple. Exchange interaction, which has enabled demonstrations of two-qubit gates [19, 21], is limited to an effective distance of about 100 nm. Scaling up spin qubits under this constraint is a very difficult task, motivating the search for alternative coupling schemes. Circuit-QED is a natural solution to such a problem, as CPW cavities span many millimeters and may couple spin qubits over very long distances. This is
1.4 Coupling Spins to Microwave Photons

Coupling a single spin to a microwave frequency photon is a very challenging task. The intuitive approach, namely using the magnetic-dipole interaction between the single spin and the magnetic field of the microwave photon, yields a very small coupling rate. As an illustration, Figure 1.6 shows an experiment coupling an ensemble of spins to a CPW cavity. Here a single spin-photon coupling rate of $g_s/2\pi \approx 12$ Hz is found, and a total of $10^{12}$ spins are needed to reach the strong-coupling regime! Although recent efforts using lumped element low-inductance resonators have raised this coupling rate to around 1 kHz [23, 24], it is still relatively far from the MHz range which is desirable for fast quantum information applications. Furthermore, since $g_s \gg \kappa, \gamma_s$ is desirable...
for long-range qubit coupling, a small $g_s$ places highly stringent constraints on the required values of $\kappa$ and the spin decoherence rate $\gamma_s$ for such an application.

Another approach to spin-photon coupling which is particularly relevant to gate-defined QDs is a two-step process. First, a coherent electric-dipole coupling needs to be achieved between the electrons within the QDs and cavity photons. Second, a hybridization needs to be achieved between the electrons’ charge degrees of freedom and their spin degrees of freedom. This “spin-charge hybridization” can, for example, be achieved through exchange interactions within a DQD or a triple QD (TQD) \cite{25-27}. Another approach, which is adopted by the work in this thesis, is to place a single electron spin within a spatially inhomogeneous magnetic field \cite{28-32}. The cavity electric field perturbs the spatial distribution of the electron wavefunction, thereby generating an oscillating magnetic field which may flip the electron spin.

As introduced in later chapters, this indirect coupling scheme is also very challenging to implement. Gate-defined QD systems are often plagued by charge noise, which rapidly dephases the electron’s charge states and prevents coherent charge-photon coupling from being achieved. Similarly, due to the hybridization with charge states, the electron’s spin states also suffer from charge-noise-induced dephasing, preventing strong spin-photon coupling from being achieved as well. In this thesis, we develop a Si-based hybrid QD-cQED device architecture with low levels of charge noise, robust charge-photon coupling rates and large spin-charge hybridization. The combination of these features allow both strong charge-photon coupling and strong spin-photon coupling to be achieved with gate-defined QDs, opening the door to a new era of circuit-QED with semiconductor gate-defined QDs.
1.5 Outline of Contents

This thesis is structured as follows: In Chapter 2, we study the basic properties of undoped Si/SiGe heterostructures such as mobility, quantum lifetime and valley splitting at the two-dimensional level using a series of magneto-transport measurements. In Chapter 3, we present the design and fabrication efforts which eventually allow us to fabricate gate-defined QDs on top of such heterostructures and dipole-couple them to photons within CPW cavities. In Chapter 4, we explore charge-photon interaction in these devices, demonstrating strong-coupling between a single electron charge qubit and a single microwave photon. In Chapter 5, we engineer spin-charge hybridization using a micron-sized cobalt magnet placed over a DQD which allows the strong-coupling regime to be finally achieved between a single electron spin and a single microwave photon. In Chapter 6, we explore an interesting application of such hybrid cQED devices toward high-resolution spectroscopy of valley states within the DQD. In Chapter 7, we also study the coherence properties of hybrid valley-orbit states within the DQD using quantum interference measurements, finding a low level of charge noise sensitivity of such states which is reminiscent of superconducting transmon qubits.
Chapter 2

Magneto-Transport Study of Undoped Si/SiGe Heterostructures

The development of silicon quantum devices has gained considerable momentum due to reports of exceptionally long quantum coherence times of tens of minutes \[33\]. Its naturally abundant isotope, $^{28}\text{Si}$, carries zero nuclear spin, reducing hyperfine-induced spin dephasing \[14, 34, 35\]. Small spin-orbit coupling is also beneficial for spin qubits \[35\]. Following work in GaAs quantum dots, early experimental efforts were made towards fabricating Si quantum dots in modulation-doped Si/SiGe heterostructures, where the n-type dopant layer is separated from the Si quantum well (QW) by a setback distance ranging from 5 to 20 nm \[36, 39\]. Doped devices were challenging to operate in the few-electron regime, unstable \[39\], and sometimes suffered from gate leakage \[36, 38\].

It is now widely accepted that the elimination of the n-type dopant layer decreases the Coulomb disorder in the QW, and reduces hysteresis and gate leakage \[40, 41\]. Recent experiments on quantum dots made in undoped Si/SiGe QWs \[40, 43\] have
consistently reached the single-electron regime and demonstrated inhomogeneous spin dephasing times \( T_2^* = 360 \text{ ns} \) in naturally abundant Si, a substantial increase compared to GaAs spin qubits \[14, 43\]. Further improvement of the Si/SiGe QW system may be feasible if the remaining mobility limiting mechanisms are identified \[44, 45\].

The dominant scattering sources can be identified from measurements of the carrier mobility \( \mu \) as a function of 2DEG charge density \( n \), and measurements of the quantum lifetime \( \tau_q \) \[44, 46\]. For example, scattering from remote impurities \[44\] is predicted to result in a power-law dependence \( \mu \propto n^{1.5} \). Such experiments have been extensively performed for GaAs/AlGaAs heterostructures \[47–50\], Si MOSFETs \[51\], and doped Si/SiGe heterostructures \[52–54\]. Similar measurements on undoped Si/SiGe heterostructures are scarce \[54, 55\].

To thoroughly investigate the mobility limiting mechanisms in undoped Si/SiGe QWs, we report a series of systematic magnetotransport measurements. By examining 26 different heterostructures, we identify a strong correlation between background oxygen concentration in the QW and maximum mobility. These results indicate that significant enhancements in Si/SiGe mobility might be obtained through more careful control of background contamination during heterostructure growth.

### 2.1 Silicon Germanium Heterostructures

The samples were grown at Lawrence Semiconductor Research Laboratory using chemical vapor deposition [Fig. 2.1(a)]. Relaxed buffers of \( \text{Si}_{1-x}\text{Ge}_x \) are first grown on 6 inch diameter Czochralski process Si substrates with 10–20 ohm-cm resistivity, varying \( x \) from 0 to 0.3 over a thickness of 3 \( \mu \text{m} \). A 1 \( \mu \text{m} \) thick layer of \( \text{Si}_{0.7}\text{Ge}_{0.3} \) is grown on the virtual substrate before it is polished. The relaxed buffer substrate has threading dislocation densities on the order of \( 10^6/\text{cm}^2 \), as determined by cross-sectional transmission electron microscopy. The wafers are completed by growing a
225 nm thick Si$_{0.7}$Ge$_{0.3}$ layer, followed by a strained Si QW, a Si$_{0.7}$Ge$_{0.3}$ spacer layer and a protective Si cap. All heterostructure layers are grown at a temperature of 650 °C. The Si$_{1-x}$Ge$_x$ layers are grown with H$_2$SiCl$_2$ and GeH$_4$ at a rate of 60 – 65 nm/min. The Si Cap is grown with H$_2$SiCl$_2$ at a rate of 2 nm/min and the Si QW is grown with SiH$_4$ at a rate of 9 – 10 nm/min. We investigate heterostructures with Si cap thicknesses of 2 nm and 4 nm, Si$_{0.7}$Ge$_{0.3}$ spacer layer thicknesses $h = 20$ nm, 30 nm, 40 nm and 50 nm, and Si QW thicknesses of 5 nm, 8 nm and 11 nm.

Hall bars are fabricated on each of the 26 wafers, with the geometry shown in Fig. 2.1(b). We first use atomic layer deposition to grow an Al$_2$O$_3$ gate dielectric on top of the Si cap. We then evaporate Cr/Au on top of the Al$_2$O$_3$ to form a top gate. A positive dc bias is applied to the top gate to accumulate electrons in the QW.
and a 0.1 mV, 17 Hz ac voltage excitation is applied between the S and D ohmic contacts. The longitudinal voltage, $V_{xx}$, Hall voltage, $V_{xy}$, and source-drain current, $I_{SD}$, are simultaneously measured using standard ac lock-in techniques. The 2D longitudinal resistivity, $\rho_{xx} = (V_{xx}/I_{SD})(W/L)$, and Hall resistivity, $\rho_{xy} = (V_{xy}/I_{SD})$ are calculated from the measured voltages and currents. Density, $n$, and mobility, $\mu$, of carrier electrons are calculated according to the Hall formulas $n = B/(e\rho_{xy})$ and $\mu = (1/B)(\rho_{xy}/\rho_{xx})$.

Figure 2.1(c) displays a typical “turn-on” curve of the Hall bar devices. Zero current flow is observed below a threshold top gate voltage $V_T = 0.45$ V. For $V_G > V_T$, current starts to flow and we observe a linear increase in $n$ with a slope of $dn/dV_G = 3.96 \times 10^{11}/\text{cm}^2/\text{V}$. Using relative permittivities $\epsilon_r = 9$ for Al$_2$O$_3$ and $\epsilon_r = 13.1$ for Si$_{0.7}$Ge$_{0.3}$, we calculate $dn/dV_G = 4.00 \times 10^{11}/\text{cm}^2/\text{V}$, which is within 1% of the experimental value. At even higher values of $V_G$ (data not shown), electrons start to accumulate at the Al$_2$O$_3$/Si cap interface, screening the QW from any further increase in $V_G$. This causes a saturation of the electron density at a constant value of $8.0 \times 10^{11}/\text{cm}^2$ for $V_G > 2.5$ V [56].

2.2 Characterization at $T = 4.2$ K

Hall bars are first measured at $T = 4.2$ K and $B = 0.1$ T, below the onset of Shubnikov-de Haas (SdH) oscillations. Figure 2.2(a) shows the spacer layer thickness $h$ for each of the 26 wafers, along with the Si cap thickness and QW width. Recent studies of undoped Si/SiGe structures have shown that remote impurity scattering typically dominates in the low electron density regime, whereas both remote impurities and interface roughness dominate at higher electron densities [51, 52]. It is therefore helpful to examine electron mobilities in both regimes. Electron mobilities are plotted in Fig. 2.2(b) for $n_H = 7.0 \times 10^{11}/\text{cm}^2$ and in Fig. 2.2(c) for $n_L = 2.1 \times 10^{11}/\text{cm}^2$. 
Figure 2.2: (a) SiGe spacer layer thickness, $h$, for wafers 1–26. Wafers with solid (hollow) symbols have a 2 nm (4 nm) thick Si cap. QW thicknesses are represented by symbol shapes. Circles: 5 nm, rectangles: 8 nm, triangles: 11 nm. (b) $\mu_H$ is the $T = 4.2$ K mobility at $n_H = 7 \times 10^{11}$/cm$^2$. (c) $\mu_L$ is the $T = 4.2$ K mobility at $n_L = 2.1 \times 10^{11}$/cm$^2$. (d) $N_o$ is the concentration of oxygen atoms inside the QW.
Surprisingly, the mobilities show a nearly monotonic increase with wafer number, despite the large variation in heterostructure parameters throughout this series of wafers. On top of this trend, abrupt dips in the mobility are observed at Wafer No. 20 and 26.

Secondary ion mass spectrometry (SIMS) analysis was performed on each wafer to better understand the increase in mobility as a function of wafer number. In Fig. 2.2(d) we plot the concentration of oxygen atoms at the QW, $N_o$, for each wafer. For wafers 1 to 11, $N_o$ decreases from $8.7 \times 10^{18}/\text{cm}^3$ to the SIMS detection threshold of $1 \times 10^{17}/\text{cm}^3$. The steady decrease of oxygen content with increasing wafer number is likely due to reduced background in the reactor after the transfer of the wafers from the load lock to the growth chamber. The decrease in oxygen concentration is also correlated with the increase in mobility observed in Fig. 2.2(b–c). Wafer No. 26, which marks the beginning of a second cassette of wafers, shows an abrupt increase in $N_o$, which is also correlated with a drop in the mobility. The combination of mobility and SIMS data suggest that oxygen contamination is a mobility limiting factor in these undoped Si/SiGe heterostructures.

In addition to the correlation between $N_o$ and $\mu$, the data show that the heterostructure growth profile impacts the mobility of samples later in the growth series. As $h$ is increased from 40 to 50 nm for Wafers No. 15 and 16, we observe a corresponding increase in $\mu_H$ and $\mu_L$. For wafers 16–19, $h$ is constant and both $\mu_H$ and $\mu_L$ show very little variation. At Wafer No. 20, $h$ undergoes a large decrease from 50 to 20 nm, which is correlated with a large drop in mobility. For wafers 20–25, $h$ increases from 20 to 50 nm and we see that the mobilities also recover to the values obtained from Wafer No. 19. In contrast, for lower wafer numbers, the correlations between $h$ and mobility are weaker, suggesting that $N_o$ is the dominant mobility-limiting mechanism in these wafers. To obtain a better quantitative understanding of these correlations,
we perform detailed measurements on Wafer No. 5 and 16 at $T = 0.35$ K in order to contrast the properties of a low and high mobility wafer.

### 2.3 High Mobility Sample

Based on its high 4.2 K mobility, a Hall bar from Wafer No. 16 was cooled down in a $^3$He cryostat for further study. The oxygen content at the QW is $N_o = 5.0 \times 10^{17}$/cm$^3$. Figure 2.3 shows $\rho_{xx}$ and $\rho_{xy}$ as a function of $B$ up to 8 T, with $n = 2.17 \times 10^{11}$/cm$^2$. From the low field magnetotransport data we extract $\mu = 1.62 \times 10^5$ cm$^2$/Vs. We observe quantum Hall plateaus in $\rho_{xy}$ at consecutive integer filling factors $\nu$ for $B > 1.5$ T, which indicates that both spin and valley degeneracies are lifted. In addition, $\rho_{xx}$ displays clear zeros, ruling out parallel conduction paths. For $\nu > 6$, plateaus in $\rho_{xy}$ are no longer visible, although oscillations in $\rho_{xx}$ are visible up to $\nu = 24$.  

---

*Figure 2.3: Wafer No. 16. $\rho_{xx}$ (red) and $\rho_{xy}$ (black) as a function of $B$, with $V_G = 0.75$ V and $T = 0.35$ K. We observe clear quantum Hall plateaus in $\rho_{xy}$ at integer filling factors $\nu$ for $B > 1.5$ T.*
Figure 2.4 shows $\mu$ as a function of $n$ at five temperatures. At $T = 0.35$ K, $\mu(n)$ is not well described by a single exponent, an often observed feature in 2DEG systems [48]. Our data differ from previous work [54], where an exponent of $\alpha = 1.7$ is observed in the density range of $n = 0.6 - 4.5 \times 10^{11}$ cm$^{-2}$. For $n < 1 \times 10^{11}$/cm$^2$, the data roughly follow a $\mu \propto n^{1.5}$ scaling, consistent with scattering due to remote charged impurities [44]. At higher $n$, $\mu$ increases more slowly and nearly saturates when $n > 5 \times 10^{11}$/cm$^2$. The high density saturation likely arises from impurity charges located very near or inside the QW, which lead to values of $\mu$ that are only weakly dependent on $n$ [44]. It is notable that the mobility curves are temperature dependent at low densities, but all saturate to nearly the same high density value of 250,000 cm$^2$/Vs. Another feature of the higher temperature data is that the density dependence of $\mu(n)$ becomes stronger, though the curvature persists up to 4 K. At $T = 4$ K, the data approximately follow a $\mu \propto n^{1.5}$ trend for $n < 3 \times 10^{11}$/cm$^2$.

To further probe the scattering mechanisms that limit the mobility of Wafer No. 16, we measure low-field SdH oscillations in the longitudinal resistivity, $\rho_{xx}$. We subtract the slowly varying background $\rho_b$ from $\rho_{xx}$ as outlined by Coleridge et al. [49] yielding $\Delta \rho_{xx} = \rho_{xx} - \rho_b$. $\Delta \rho_{xx}$ is plotted against $1/B$ for three densities in Fig. 2.5(a). Clear periodic oscillations are observed, with a periodicity of 4 in $\nu$. This is consistent with the 2-fold spin degeneracy and 2-fold valley degeneracy at low fields. At higher fields $B > 0.7$ T, splitting of the peak in each period of the SdH oscillation becomes visible, due to increased Zeeman splitting. This splitting is examined in detail in Section VIII. We extract the amplitude of the oscillations in $\Delta \rho_{xx}$ at each period in $1/B$ [57] which is fit to:

$$\Delta \rho_{xx} = 4\rho_0 X(T) \exp\left(-\frac{\pi}{\omega_c \tau_q}\right), \quad (2.1)$$
Figure 2.4: Wafer No. 16. $\mu(n)$ for five different temperatures. The dashed lines show the predicted slopes for remote ionized impurity scattering and scattering due to impurities in the QW [44]. The charged impurity densities used to produce the two dashed lines are $3.7 \times 10^{12}/\text{cm}^2$ at a distance of 50 nm from the QW center (where the Al$_2$O$_3$/Si interface is) for remote impurity scattering, and $3.4 \times 10^9/\text{cm}^2$ in the QW center. 

where $\tau_q$ is the quantum lifetime of the electrons, $\rho_0$ is the zero-field resistivity, $X(T) = (2\pi^2k_B T / \hbar \omega_c) / \sinh(2\pi^2k_B T / \hbar \omega_c)$ is the temperature-damping factor, $\omega_c = eB/m^*$ is the cyclotron frequency, and $k_B$ is the Boltzmann’s constant. We use a constant effective mass $m^* = 0.2m_e$, where $m_e$ is the free electron mass, for all fits [58]. Figure 2.5(b) shows the results of such fits, known as Dingle plots. The slopes of the Dingle plots [50] are inversely proportional to the quantum lifetime $\tau_q$ and imply shorter quantum lifetimes at lower densities.

In Fig. 2.5(c), we compare the transport lifetime, $\tau_t$, and the quantum lifetime, $\tau_q$, across the electron density range $n = 1.8–6.8 \times 10^{11}/\text{cm}^2$. Values of $\tau_t$ are obtained from the mobility data in Fig. 2.4 via $\tau_t = \mu m^* / e$ and values of $\tau_q$ are obtained from analysis of the low field SdH oscillations. Both lifetimes show similar dependencies on $n$ and Dingle ratios, defined as $\tau_t / \tau_q$, range from 4 to 7 as shown in Fig. 2.5(d). In
Figure 2.5: Wafer No. 16. (a) $\Delta \rho_{xx}$ as a function of $1/B$ for three electron densities at $T = 0.4$ K. Traces have been offset by 150 $\Omega$ for clarity. (b) Dingle plots for the data in (a). Values of $\Delta \rho_{xx}$ used in this plot are the average of the maximum and minimum of each period of the SdH oscillations shown in (a). (c) $\tau_q$ and $\tau_t$ as functions of $n$ at $T = 0.4$ K. (d) Dingle ratio $\tau_t/\tau_q$ as a function of $n$ at $T = 0.4$ K. The dashed line is a guide to the eye.

comparison with previous work on GaAs/AlGaAs [50] and modulation-doped SiGe [52] where the Dingle ratios typically range from 10 to 20, the Dingle ratios measured for this undoped sample are sizably smaller, indicating that large angle scattering plays a more dominant role in this sample. Such a situation would arise when the distribution of impurities is more concentrated towards the location of the 2DEG [44]. This is unexpected in an undoped system where charged impurities are thought to reside mostly in the Al$_2$O$_3$/Si interface [54], $\sim$50 nm away from the 2DEG in this sample. Our interpretation of the possible cause for such distribution is the peak
in oxygen impurities at the 2DEG location observed in SIMS analysis. Ionization of a small fraction of these oxygen atoms would lead to a sizable amount of impurity charges inside the QW, which contribute to large angle scattering with a Dingle ratio near unity. The decreasing trend of the Dingle ratio at higher densities also differs from theoretical calculations based on a single dopant sheet \[46\]. This deviation is interpreted to be due to the mitigated contribution of remote impurity scattering to the overall momentum scattering rate at higher densities, since the scattering rate \( \tau^{-1}_t \propto n^{-1.5} \) for remote impurities but \( \tau^{-1}_i \propto n^{-0.1} \) for impurities inside the QW. At higher densities, scattering from impurities inside the QW becomes more dominant than remote impurities, which reduces the overall Dingle ratio.

The quantum lifetime \( \tau_q \) for this high mobility sample is also measured as a function of \( T \) at \( n = 6.7 \times 10^{11} / \text{cm}^2 \). Figure 2.6(a–b) displays low field SdH oscillations and the associated Dingle plots. The extracted values of \( \tau_q \) are plotted alongside \( \tau_t \), obtained by measuring the temperature-dependent mobility at this density [Fig. 2.6(c)]. The transport lifetime is relatively insensitive to temperature, while the quantum lifetime varies by nearly a factor of 4 from \( T = 0.4 \) K to 1.5 K. The resulting Dingle ratio, plotted in Fig. 2.6(d), increases almost linearly with temperature from 4 to 11. These data are in contrast with a single-particle description of electron scattering in 2DEGs \[45, 46, 59\], where the temperature dependence of both lifetimes is expected to be weak in the \( T \ll T_F \) regime, where \( T_F \) is the Fermi temperature (approximately 47 K at this density).

Arapov et al. \[60\] have recently measured an InGaA/GaAs double QW structure and report a similar, strong temperature dependence of \( \tau_q \) at \( T \ll T_F \), which the authors attribute to electron-electron interactions. A similar mechanism may explain the trends observed in this work. While electron-electron interactions may be significant, our analysis is based on the established single-particle model due to the fact that electron-electron interactions are expected to renormalize both \( \tau_q \) and \( m^* \).
Figure 2.6: Wafer No. 16. (a) $\Delta \rho_{xx}$ as a function of $1/B$ for $n = 6.7 \times 10^{11}/\text{cm}^2$ at four different temperatures. (b) Dingle plots for the data shown in (a). (c) $\tau_q$ and $\tau_t$ as a function of $T$ at $n = 6.7 \times 10^{11}/\text{cm}^2$. (d) Dingle ratio as a function of $T$. The dashed line is a guide to the eye.

The observed temperature dependence of $\tau_q$ in Fig. 2.6(c) could be due to a change in $\tau_q$ with temperature or an apparent change in $\tau_q$ due to a renormalized $m^*$. It is difficult to distinguish between these two possibilities based on the SdH data alone.

2.4 Low Mobility Sample

We next examine data from Wafer No. 5, which has a much lower maximum mobility of $\mu = 7.5 \times 10^4 \text{ cm}^2/\text{Vs}$ at 4.2 K. Wafer No. 5 has a 2 nm thick Si cap, a $h = 40$...
Figure 2.7: Wafer No. 5 (low mobility sample). (a) $\mu(n)$ at five different temperatures. The dashed line shows the prediction for scattering from impurities in the QW with a charged impurity density of $1.3 \times 10^{10}$/cm$^2$ at the QW center. (b) $\tau_q$ and $\tau_t$ plotted as a function of $n$, extracted from low field SdH oscillations. $T = 0.4$ K in this plot. (c) Dingle ratios $\tau_t/\tau_q$ obtained from the data in (b).
nm thick SiGe spacer layer, and a 5 nm wide Si QW. SIMS analysis shows a similar distribution of oxygen inside the SiGe spacer as the high mobility sample. However, the oxygen content in the QW is peaked at $N_o = 2.5 \times 10^{18}/\text{cm}^3$, which is five times higher than the high mobility sample.

Figure 2.7 shows the results of magnetotransport measurements on this low mobility sample. $\mu(n)$ is plotted in Fig. 2.7(a) and increases with $n$, although with a weaker dependence than the high mobility sample. $\mu(n)$ is also temperature dependent, and more strongly scales with $n$ at higher temperatures. At $T = 0.35$ K, $\mu(n)$ is nearly density-independent. As $T$ increases, $\mu(n)$ becomes more density-dependent and eventually reaches an approximate scaling of $\mu \propto n^{0.7}$ at $T = 4$ K. The smaller power-law exponent for this sample suggests that remote impurity scattering plays a less significant role compared to the high mobility sample. Instead, electron scattering is likely dominated by impurity charges situated inside the QW, which is consistent with the higher oxygen content observed in the SIMS data.

Figure 2.7(b) shows $\tau_q$ and $\tau_t$ for the low mobility sample at five different densities and with $T = 0.4$ K. Both lifetimes are shorter at lower electron densities, similar to the high mobility sample. Despite the factor of $\sim 3$ difference in $\tau_t$ between the two samples, the values of $\tau_q$ are very similar. This observation agrees well with recent theoretical results by Das Sarma et al. [63], who considered a two-impurity model and showed that increasing (decreasing) $\tau_t$ does not necessarily lead to increasing (decreasing) $\tau_q$ when there is more than one scattering mechanism. We also plot the density-dependent Dingle ratio $\tau_t/\tau_q$ of this sample in Fig. 2.7(c). $\tau_t/\tau_q$ ranges from 1.3 at high density to 2.3 at low density, significantly smaller than the high mobility sample. The smaller Dingle ratio observed in this sample implies that the underlying scattering events are even larger in angle compared to the high mobility sample, consistent with scattering from QW impurities.
2.5 Estimate of Defect Densities

Monroe et al. have carefully analyzed seven scattering mechanisms that are potentially relevant to the Si/SiGe materials system [44]. Among these mechanisms, alloy scattering, scattering due to strain modulation, scattering due to vicinal surfaces and scattering from threading dislocations are estimated to limit mobilities to above $10^7$ cm$^2$/Vs, two orders of magnitude higher than the mobilities measured in our samples. Interface roughness has been reported to be an important factor in a $\sim$ 500 nm deep Si/SiGe QW structure [55], but is expected to lead to a mobility that decreases with increasing density (a trend that is not observed in our data). We therefore limit our analysis to the two remaining scattering mechanisms: remote impurity scattering from charges located at the Al$_2$O$_3$/Si interface and scattering from oxygen related background charges in the QW.

We now compare the experimental data with these predictions, starting with the high mobility sample. Figure 2.4 shows $\mu(n)$ for Wafer No. 16. At low densities, $\mu(n)$ roughly follows the $\mu \propto n^{1.5}$ power law expected for remote impurity scattering, while for higher densities $\mu$ is a weak function of $n$. Superimposed on the data are dashed lines showing the expected scaling for remote impurity scattering and scattering from impurities in the QW calculated using theoretical results of Monroe et al. We set the distance between the remote impurities and the 2DEG to be $h = 50$ nm such that the remote impurities are Al$_2$O$_3$/Si interface charges, as reported by Li et al. [54]. The 2D remote impurity density is adjusted to $n_1 = 3.7 \times 10^{12}$/cm$^2$ to bring theory into agreement with the data. Similarly, for scattering from impurities in the QW we find reasonable agreement with the data when the 2D QW impurity density $n_2 = 3.4 \times 10^9$/cm$^2$. Dingle ratio data for Wafer No. 16 are plotted in Fig. 2.5(d) and show a linear decrease with $n$ over the entire density range. This is broadly consistent with a crossover from remote impurity scattering limited transport to local defect scattering-limited transport as $n$ increases.
In comparison, $\mu(n)$ is shown for the low mobility sample (Wafer No. 5) in Fig. 2.7(a). At $T = 0.35$ K, the mobility is weakly dependent on density over the entire density range, consistent with scattering from impurities in the QW. The dashed line shows the prediction for scattering from impurities in the QW taking $n_2 = 1.3 \times 10^{10} / \text{cm}^2$. We note that this defect density is 4 times higher than the high mobility sample, reminiscent of the factor of 5 difference between the oxygen contents in the QWs of the low and high mobility samples observed from SIMS data. It is also clear that the Dingle ratio is much less sensitive to density, with $\tau_t/\tau_q \sim 1 - 2$ over the entire density range. The small Dingle ratio is consistent with scattering from impurities in the QW.

### 2.6 Metal-to-Insulator Transition

For spin-based quantum information processing, quantum dots are typically operated in the few-electron regime [43]. One important gauge for the degree of disorder at low electron density is the critical electron density, $n_c$, for the metal-to-insulator transition (MIT) in 2DEGs. For silicon, a MIT has been observed in MOSFETs [64, 65], modulation doped Si/SiGe structures [66–68], an undoped Si/SiGe 2DEG structure [69], and an ambipolar Si-vacuum FET [70]. Values of $n_c$ vary greatly in Si/SiGe systems, ranging from 0.32 to $4.05 \times 10^{11} / \text{cm}^2$ [66–69].

The experimental signature for the MIT in 2DEG systems is a sign reversal in $d \rho / dT$, where $\rho$ is the resistivity of the system [64]. For $n > n_c$, $d \rho / dT > 0$ and the 2DEG displays metallic behavior. For $n < n_c$, $d \rho / dT < 0$ and the 2DEG behaves as an insulator. In Fig. 2.8, we plot $\rho_{xx}(T)$ for the high mobility sample at ten different densities below $n = 1.0 \times 10^{11} / \text{cm}^2$. We observe the following features in this data set:
Figure 2.8: Wafer No. 16. $\rho_{xx}(T)$ at $B = 0$ T for $n = 0.34, 0.36, 0.38, 0.40, 0.42, 0.46, 0.51, 0.59, 0.73$ and $0.96 \times 10^{11} / \text{cm}^2$ (from top to bottom). A metal-insulator transition occurs at $n_c = 0.46 \times 10^{11} / \text{cm}^2$.

1. At the lowest two densities $n = 0.34 \times 10^{11} / \text{cm}^2$ and $0.36 \times 10^{11} / \text{cm}^2$, $d\rho_{xx}/dT < 0$ throughout the measured temperature range. In addition, $\rho_{xx}$ appears to diverge exponentially at $T < 1$ K, indicative of a true insulating phase [71].

2. At the next three higher densities $n = 0.38, 0.40$ and $0.42 \times 10^{11} / \text{cm}^2$, $\rho_{xx}$ varies non-monotonically with temperature. While $d\rho_{xx}/dT < 0$ at $T = 4.2$ K, $d\rho_{xx}/dT > 0$ for a small, intermediate temperature range. We note that this behavior has also been observed by Lu et al. in another undoped Si/SiGe sample [69], and is common in Si MOSFET systems [72].

3. $d\rho_{xx}/dT > 0$ at 0.5 K for $n \geq n_c$, where $n_c = 0.46 \times 10^{11} / \text{cm}^2$.

4. At $n \geq n_c$, with the exception of $n = 0.51 \times 10^{11} / \text{cm}^2$, $d\rho_{xx}/dT > 0$ up to a crossover temperature $T_c$. For $T > T_c$, $d\rho_{xx}/dT < 0$. Furthermore, $T_c$ increases with increasing $n$. Das Sarma et al. interpreted this behavior as a quantum-to-classical crossover [45, 71, 73].
We note that the critical density is comparable to the lowest value of $0.32 \times 10^{11}/\text{cm}^2$ that has been reported in doped Si/SiGe structures [68], and a factor of 4 lower than the value of $1.9 \times 10^{11}/\text{cm}^2$ observed in a previous work on undoped Si/SiGe structure [69], indicating a very low level of disorder in our undoped sample. The critical density observed in our system lies at the lower end of the critical density spectrum [74] and within an order of magnitude of the lowest value of $n_c = 7.7 \times 10^9/\text{cm}^2$ in a GaAs system [75]. For a quantum dot of radius 29 nm fabricated on Wafer No. 16 [76], a critical density of $0.46 \times 10^{11}/\text{cm}^2$ corresponds to an electron occupancy of 1.2, demonstrating the feasibility of maintaining relatively low disorder potential in the few-electron regime.

2.7 Valley Splitting

Another important figure of merit for the Si/SiGe quantum well system is the valley splitting. Measurements of the valley splitting in Si MOSFET systems have been extensively performed. Most values range from 0.7 – 1.5 meV [77, 78], with one study reporting a value as large as 23 meV [79]. In comparison, the valley splitting in Si/SiGe systems tends to be smaller, ranging from 0.05 – 0.3 meV [80–82]. In this section, we determine $\Delta_v$ through careful analysis of the SdH oscillations in Wafer No. 16.

In Fig. 2.9 we plot $\rho_{xx}(B)$ with the electron density $n = 6.6 \times 10^{11}/\text{cm}^2$. SdH oscillations are observed above a field $B_{\text{eff}} = 0.38$ T and have a periodicity of 4 in $\nu$. For $B > B_s = 0.88$ T, we observe change in periodicity of the SdH oscillations, indicating that spin degeneracy has been lifted. The periodicity changes again beyond $B_v = 1.8$ T, consistent with the lifting of both spin and valley degeneracies. We have verified that the spin degeneracy is lifted before valley degeneracy using the tilted field method [80, 81].
Figure 2.9: Wafer No. 16. $\rho_{xx}(B)$ at $T = 0.4$ K. SdH oscillations are observed beyond $B_{\text{eff}} = 0.38$ T. Spin (valley) degeneracy is lifted at $B_s = 0.88$ T ($B_v = 1.8$ T).

The energy spectrum of 2D electrons in a perpendicular field is described by four characteristic energy scales. The first is the Zeeman splitting, $E_z = g\mu_B B$, where $\mu_B$ is the Bohr magneton and $g$ is the electronic $g$-factor. The second is $E_l = e\hbar B/m^* - E_z$, which is the Landau level spacing minus the Zeeman splitting. The third is the valley splitting, $\Delta_v$. Finally, clear SdH oscillations will only be observed when the Landau level spacing is greater than the Landau level broadening $\Gamma \approx \hbar/2\tau_q$. Spin splitting becomes visible when $E_z(B_s) \approx \Gamma$. Based on the effective field at which the SdH oscillations become visible, we estimate $\Gamma \approx E_l(B_{\text{eff}})$. We then have the relation $E_z(B_s) \approx E_l(B_{\text{eff}})$, allowing us to extract $g = 3.0 \pm 0.2$. The $g$-factor is in reasonable agreement with the value of $g = 2.9 \pm 0.1$ at $n = 5.9 \times 10^{11} /\text{cm}^2$ found in a previous study [81]. Based on this experimental value of $g$, we find $\Gamma \sim 150 \pm 10$ $\mu$eV. Finally, the valley degeneracy is lifted at the field for which $\Delta_v(B = 1.8$ T) $\sim 150$ $\mu$eV. The valley splitting is substantial and comparable to the two-electron singlet-triplet splitting that is measured in GaAs quantum dots [14], [35].
2.8 Conclusion

We have measured 26 wafers with different growth parameters to identify the dominant mobility limiting mechanisms in undoped Si/SiGe QW heterostructures. At 4.2 K we find correlations between mobility and oxygen content at the QW as well as the thickness of the top SiGe spacer. We also measured $\tau_t$ and $\tau_q$ for two Si/SiGe QW heterostructures across a wide density range at $T \sim 0.35$ K. Based on the density dependencies of the two lifetimes, we conclude that the mobility of high quality samples with low oxygen content at the QW is mostly limited by remote impurity charges, while the lower mobility samples with high oxygen content in the QW are limited by the impurity charges inside or very close to the QW, consistent with the correlations observed at 4.2 K. To further assess the merits of the high mobility heterostructure as a platform for spin-based quantum dots, we have measured a low critical density $n_c = 0.46 \times 10^{11}/\text{cm}^2$ for the MIT and a valley splitting $\Delta_v \sim 150 \mu\text{eV}$.

While we cannot rule out effects due to other impurities, our SIMS results suggest that significant improvements in the electron mobility may be obtained by reducing the level of oxygen content in the Si/SiGe heterostructure. Our data give no information about the microscopic scattering mechanism that results from the oxygen impurities. The literature suggests several possibilities: a) lattice strain due to interstitial and substitutional oxygen [83], b) thermal donor generation [84], and c) enhanced donor generation rate due to the presence of carbon impurities [85]. Further work is needed to identify the mechanism that leads to the correlation between oxygen concentration and mobility.

The SiGe spacer layer thickness can also be increased to reduce scattering from charged impurities at the surface of the wafer. This second approach has limitations for quantum dot devices, as it is desirable to have strong in-plane electrostatic confinement, which is harder to obtain in samples with deeper QWs. The magnetic fields at which the valley splitting was extracted corresponds to a cyclotron radius of $\sim 20$
nm, which is a realistic size for the lithographic patterning of quantum dots on Si. Therefore efforts should also be directed towards reducing the size of Si quantum dots to emulate high levels of magnetic confinement, which yielded large values of valley splitting in this work. Overlapping gate architectures may prove helpful to achieve tight electronic confinement in the relatively high effective mass Si/SiGe quantum well system. They are adopted for the experiments shown in the remainder of this thesis [76].
Chapter 3

Circuit QED with Gate-Defined Silicon Quantum Dots

In this chapter, we present the design, modeling and fabrication developments that eventually led to a hybrid cQED device coupling a superconducting co-planar waveguide (CPW) resonator to gate-defined silicon quantum dots. Hybrid devices of similar designs have previously been demonstrated in 1D systems such as InAs nanowires [86] and carbon nanotubes [87]. In 2D systems, hybrid cQED devices have been demonstrated with GaAs quantum wells [88]. A common problem afflicting such devices is the low quality factor of the superconducting resonator. While $Q$ values in the range of 100,000 to 1,000,000 are often demonstrated in cQED devices with superconducting qubits, cavities coupled to gate-defined quantum dots have $Q$ values ranging from a few hundred to a few thousand. Early generations of devices made on undoped Si/SiGe in our lab have quality factors much less than 1,000. Multiple iterations of design and fabrication were necessary to increase this value to a range (5,000 to 10,000) where cQED experiments may be performed with a chance of reaching the strong-coupling regime. Here we outline our efforts to understand and suppress the $Q$-limiting mechanisms in these hybrid devices.
3.1 Transmission Line Model of a Simple CPW Resonator

The basics of a CPW resonator are illustrated in Fig. 3.1. In the upper panel of Fig. 3.1(a), a bird’s eye view of a CPW resonator is sketched, comprising a center pin of width $a$ which is separated from the ground plane by an identical gap of width $s$ on the upper and lower edge. At the left edge and right edge of the center pin, a gap is created to form a planar capacitor which acts as a dielectric mirror. The lower panel of Fig. 3.1(a) shows the circuit model of the resonator, with the input/output capacitors denoted with capacitances $C_{in}$ and $C_{out}$. The center pin is modeled as a transmission line, having a characteristic impedance $Z_r$, a propagation constant $\beta$ and a length $l$. In Fig. 3.1(b), the cross-section of the CPW is shown, where a film of thickness $t$ which forms the CPW geometry rests on a dielectric substrate of thickness $h$.

While values of the coupling capacitances $C_{in}$ and $C_{out}$ are usually simulated with software packages such as Sonnet, the parameters related to the transmission line can be calculated easily. The characteristic impedance is given by:

$$Z_r = \frac{60\pi}{\sqrt{\epsilon_{eff}}} \left( \frac{K(k)}{K(k')} + \frac{K(k_3)}{K(k_3')} \right)^{-1},$$  \hspace{1cm} (3.1)

where an effective dielectric constant, $\epsilon_{eff}$ is defined as:

$$\epsilon_{eff} = \frac{1 + \epsilon_r K_0}{1 + K_0},$$  \hspace{1cm} (3.2)

where $K$ is the complete elliptical integral of the first kind. The various terms, determined by the CPW dimensions and the substrate dielectric constant $\epsilon_r$, are $K_0 = \frac{K(k')K(k_3)}{K(k)K(k_3')}$, $k = \frac{a}{b}$, $k_3 = \frac{\tanh\left(\frac{\pi a}{2h}\right)}{\tanh\left(\frac{\pi b}{2h}\right)}$, $k' = \sqrt{1-k^2}$ and $k_3' = \sqrt{1-k_3^2}$. Note the calculation here assumes a thin metal film so that $t \ll h$, which is true for the Nb.
film resonators used in this thesis. Also, the internal losses of the metal film and the dielectric substrate are both assumed to be zero, so that $Z_r = \sqrt{L_r/C_r}$ where $L_r$ and $C_r$ are the characteristic inductance and capacitance of the CPW. The propagation constant is related to $Z_r$ via $\beta = \omega L_r / Z_r$, where $\omega$ is the frequency of the signal and $L_r$ is given by \[ L_r = \frac{\mu K(k')}{4 K(k)}, \tag{3.3} \]

with $\mu$ being the magnetic permeability of the substrate.

To evaluate the performance of the resonator, a measurement of the scattering matrix $S$ is usually done with a network analyzer. For a $\lambda/2$ resonator shown in Fig. 3.1, the element of interest is the transmission, $|S_{21}|^2$. A handy way to calculate

\footnote{For more accurate calculations, a kinetic inductance, $L_K$, should be added to $L_r$, which causes a small (5%) correction to the CPW impedance. We use this correction for realistic calculations in Section 3.}
$|S21|^2$ is through the use of ABCD matrices [89], which we can write out for the circuit model in Fig. 3.1(a):

$$
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = 
\begin{pmatrix}
1 & \frac{1}{i\omega C_{in}} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos(\beta l) & iZ_r \sin(\beta l) \\
\frac{1}{Z_r} \sin(\beta l) & \cos(\beta l)
\end{pmatrix}
\begin{pmatrix}
1 & \frac{1}{i\omega C_{out}} \\
0 & 1
\end{pmatrix}.
$$

(3.4)

The resulting matrix can then be normalized by an external impedance $R_L$ (assumed to be 50 $\Omega$) and converted into $|S21|^2$ via [89] :

$$
|S21|^2 = \left| \frac{2}{A + B/R_L + C R_L + D} \right|^2.
$$

(3.5)

In Fig. 3.1(c), the calculated $|S21|^2$ through the resonator is plotted as a function of signal frequency $f$, using a set of circuit element values close to what we design in actual devices. It can be seen that the resonator is a highly selective band-pass filter, allowing perfect transmission at its fundamental resonance frequency of 7.5 GHz (higher order resonances also emerge from this circuit model and are not plotted) and heavily filtering transmission at other frequencies. A quality factor can be assigned to the resonator, $Q = f/\Delta f$, where $\Delta f$ is defined such that $|S21|^2 = 0.5$ (or $-3$ dB) at $f = f \pm \Delta f/2$. For this resonator, a simple analytic expression relating $Q$ to $C_{in}$ and $C_{out}$ can be found by Taylor-expanding $|S21|^2$, to leading order, in $\delta \omega = \omega_0 - \omega$, where $\omega_0$ is the resonance frequency and $\delta \omega$ is a small variation. The result is:

$$
Q = \frac{\pi}{2} \frac{1}{\omega_0^2 Z_r^2 (C_{in}^2 + C_{out}^2)}.
$$

(3.6)

Before concluding this section, we note that the simple resonator model does not capture any microwave loss in the resonator which, when present, reduces the peak of $|S21|^2$ to a value lower than 1 (or 0 dB). Microwave losses generally fall into two categories:
Resistive and dielectric losses, which may be modeled as a complex component in the propagation constant and are commonly referred to as a loss tangent $\tan \delta$ or an internal quality factor, $Q_{\text{int}}$.

- Radiative loss, which may be modeled as extra ports in the circuit model that couple to the resonator and split the signal.

As we will see in the next section, both of these loss mechanisms play a role in limiting the quality factors of resonators coupled to gate-defined Si QDs.

### 3.2 Basic Resonator Characterizations

As a first step towards enabling high quality factor resonators in a hybrid Si QD-cavity system, we fabricate Nb $\lambda/2$ CPW resonators on float-zone (FZ) Si wafers having resistivities in excess of 5000 $\Omega$-cm and (1 0 0) crystal orientation. The Si substrate is first cleaned, with continuous sonication, in acetone followed by isopropyl alcohol. An oxygen plasma etch ensues. Lastly, the Si chip is cleaned in Piranha etch solution ($H_2SO_4:H_2O_2$ 3:1) and then briefly etched in a dilute HF solution to remove native SiO$_2$ from the surface. A 15 nm thick layer of Al$_2$O$_3$ is then grown on the Si surface via atomic layer deposition. Lastly, a 50 nm thick Nb film is deposited on top of Al$_2$O$_3$ via DC sputtering at a rate of 8 Å/s. To define the resonator patterns, we spin a photo-resist mask on the chip and expose areas where the Nb is to be etched with a laser writer. The chip is then developed and etched in a Ar/SF$_6$ plasma. The resist film is then etched in an oxygen plasma to remove the cross-linked layer and subsequently stripped with acetone and isopropyl.

To separate $Q$ limitations imposed by the fabrication process from those imposed by resonator design or packaging, we fabricate resonators with four different designs on the same wafer and over the same fabrication run. The design variants are shown in Fig. 3.2. All resonators have an identical CPW center pin width of 10 $\mu$m and a
Figure 3.2: Designs for the four resonators fabricated on the same FZ Si wafer. XM1-20151015_FZ_A4: A bare resonator. Inset shows SEM image of the coupling capacitor which is the same for the input/output ends. XM1-20151015_FZ_B3: Resonator with air-bridges. Inset shows SEM image of a single air-bridge. XM1-20151015_FZ_C1: Resonator with a \( LC \)-filtered DC tap. Inset shows the inductor and part of the capacitor of the \( LC \)-filter. XM1-20151015_FZ_C2: Resonator with a \( LC \)-filtered DC tap and air-bridges. gap width of 6.2 \( \mu \)m, yielding a characteristic impedance of 54 \( \Omega \). The input/output coupling capacitors are also identical for all resonators, and dimensions measured in a scanning electron microscope (SEM) yield a design \( Q \) of 43,000. The differences in design are:

- **XM1-20151015_FZ_A4**: A bare resonator with no additional elements.

- **XM1-20151015_FZ_B3**: A bare resonator with 33 air-bridges distributed along the length of the resonator. The air-bridges are made from 300 nm thick aluminum films, spanning 25 \( \mu \)m and having a height of 3 \( \mu \)m.

- **XM1-20151015_FZ_C1**: A resonator with a DC tap located at the voltage node. An on-chip \( LC \)-filter, consisting of an inter-digitated capacitor with a capaci-
tance of \(C_f = 1 \text{ pF}\) and a spiral inductor of inductance \(L_f = 12 \text{ nH}\), is inserted to minimize microwave leakage through the DC-tap.

- **XM1-20151015_FZ_C2**: A resonator with \(LC\)-filtered DC-tap and air-bridges distributed along both the resonator and the DC tap.

The addition of the DC tap to the resonator center pin is a necessary for our hybrid device design and therefore included here for comparison. The purpose of having air-bridges on two resonators is to assess if the resonator \(Q\) is limited by the traditional practice of connecting CPW ground planes using wire-bonds, which have large inductances and are therefore ineffective shunts. Slot-line modes can be excited, which, if having frequencies in the vicinity of the cavity frequency, would have reduced the resonator \(Q\). Micro-fabricated air-bridges straddling the CPW center pin have much smaller inductances, can be precisely placed, and are therefore much more effective in suppressing slot-line modes and minimizing \(Q\) degradations due to packaging. The air-bridges are fabricated using a recipe modified from the process demonstrated by the Martinis group at Google [90], with the Al to Nb contact facilitated by a 10 nm thick Au layer. We have tested the DC continuity of the air-bridges by measuring their resistances at a temperature of 1.5 K, and values on the order of 0.1 Ω have been obtained for the six bridges tested.

All four resonators are measured in a dilution refrigerator at a base temperature of \(T = 20 \text{ mK}\). The transmission through each resonator, \(|S_{21}|^2\), is measured with a network analyzer. Fridge wiring details are presented in a later section of this chapter. For XM1-20151015_FZ_C1 and XM1-20151015_FZ_C2 which have DC taps, we bond the DC tap to a line on the circuit board which is grounded through a low-pass \(RC\)-filter having a resistance of 2.8 kΩ and a capacitance of 5.6 nF.

The transmission data, \(|S_{21}|^2\), for all four resonators, are shown in Fig. 3.3. The drive power for each resonator is adjusted to have about 100 photons in the cavity. It is striking that despite the differences in design, all resonators display an almost
Figure 3.3: Transmission data, $|S_{21}|^2$, for all four resonators on XM1-20151015_FZ, measured at $T = 20$ mK and with a drive power of $P_{in} = -128$ dBm, corresponding to $\sim 100$ photons stored in the cavity. The peaks of the $|S_{21}|^2$ data are not corrected for the attenuations/amplifications at various parts of the circuits.

identical $Q$ of 20,000 at this temperature and photon number. This suggests that the dominant loss mechanism in these resonators is likely related to the fabrication process rather than the center pin DC tap or packaging.

To further probe the resonator loss mechanism, we measure the $Q(P_{in}, T)$ of the resonators as functions of drive power $P_{in}$ and temperature $T$. For the power-dependent measurements, we keep $T = 20$ mK. For temperature-dependent measurements, we keep $P_{in} = -138$ dBm which corresponds to $\sim 10$ photons stored in the cavity. The results are plotted in Fig. 3.4. We observe that the $Q$ for all resonators increases monotonically and identically at higher drive powers, with a low-power plateau at $P_{in} < -140$ dBm and a high-power saturation at $P_{in} > -90$ dBm. In the case of varying temperature, the $Q$ rises monotonically and identically with $T$ up to $T = 0.4$ K, beyond which the $Q$ values of resonators with air-bridges decrease significantly toward nearly zero. The $Q$ values of the resonators without the air-bridges
Figure 3.4: (a) Resonator $Q$ as functions of $P_{\text{in}}$, at $T = 20$ mK. Dashed line shows fit to a loss model with TLS being the only loss source. (b) Resonator $Q$ as functions of $T$, at $P_{\text{in}} = -138$ dBm. Dashed line shows predicted temperature-dependent behavior based on the fit in panel (a).
rise at the same rate up to $T = 0.8$ K, beyond which the $Q$ of the resonator with the DC tap decreases to nearly 10,000, whereas the $Q$ of the resonator without the DC tap stays constant at a level of 28,000.

These observed power and temperature dependences (at $T \leq 0.4$ K) suggest that the dominant loss mechanism in all four resonators is likely due to fluctuating two-level systems (TLS), often attributed to defects at various metal/dielectric or dielectric/dielectric interfaces which can tunnel between different energy states by absorbing microwave photons from the cavity. At higher temperatures or drive powers, the TLS can be saturated and a corresponding increase in $Q$ is often observed. To understand whether TLS are indeed the underlying loss mechanism, we fit the measured $Q(P_{\text{in}}, T)$ to the following [91]:

$$1/Q(P_{\text{in}}, T) = 1/Q_{\text{TLS}}(P_{\text{in}}, T) + 1/Q_{\text{ext}},$$

where $Q_{\text{ext}}$ is the maximum $Q$ achievable by design, and $Q_{\text{TLS}}$ is an intrinsic $Q$ limit imposed by TLS loss, having the functional form:

$$Q_{\text{TLS}}(P_{\text{in}}, T) = Q_0 \sqrt{1 + \left(\frac{P_{\text{in}}}{P_c}\right)^\beta} \tanh\left(\frac{hf}{2k_B T}\right).$$

Here $Q_0$ is the lowest internal $Q$ in the case of completely unsaturated TLS, $P_c$ is a critical power and $\beta$ is a TLS-dependent exponent. The large number of free parameters makes extraction of meaningful information from the data difficult. We proceed objectively as follows:

- $Q_{\text{ext}}$ is a constant based on resonator dimensions. Using dimensions measured from SEM, we simulate this value to be $Q_{\text{ext}} = 43,000$ and use it for the fit.
- At $T = 20$ mK, $\tanh\left(\frac{hf}{2k_B T}\right) = 1$ and $Q_{\text{TLS}}(P_{\text{in}}) = Q_0 \sqrt{1 + \left(\frac{P_{\text{in}}}{P_c}\right)^\beta}$. At $P_{\text{in}} < -140$ dBm, we observe that $Q$ becomes nearly independent of $P_{\text{in}}$. This could
only happen when \( \left( \frac{P_{in}}{P_c} \right)^\beta \approx 0 \) and \( \sqrt{1 + \left( \frac{P_{in}}{P_c} \right)^\beta} \approx 1 \). We then have \( 1/Q = 1/Q_0 + 1/Q_{ext} \), leading to a lowest internal \( Q \) of \( Q_0 = 31,000 \).

Using these two considerations, the only free parameters left are \( P_c \) and \( \beta \), which are extracted to be \( P_c = -123 \) dBm and \( \beta = 0.41 \) by fitting the power-dependent data of the bare resonator, XM1-20151015_FZ_A4, in Fig. 3.4(a). The results of the fit are also used to predict the temperature dependent behavior of \( Q(P_{in}) = -138 \) dBm, which is plotted as the dashed line in Fig. 3.4(b). The results of the fit have reasonable agreement with the data across all powers and at \( T \leq 0.4 \) K.

Lastly, we comment on the \( Q \) behaviors at \( T > 0.4 \) K, which show marked departures from the TLS model predictions in all but the simplest resonator, XM1-20151015_FZ_A4. The deviations in the resonators with air-bridges, XM1-20151015_FZ_B3 and XM1-20151015_FZ_C2, are understood to be due to increasing quasi-particle populations in the Al air-bridges as \( T \) approaches the Al critical temperature of 1.2 K. Microwave losses in the air-bridges become more dominant compared to TLS losses, and we are unable to observe a clear resonance at low
drive powers beyond $T = 1.2$ K at which Al is in the normal (non-superconducting) state. We confirm this by measuring the $Q$ of XM1-20151015\_FZ\_B3 and XM1-20151015\_FZ\_C2 in a small magnetic field $B$, at $T = 20$ mK and $P_{in} = -138$ dBm. The field is aligned to be parallel to the plane of the resonator, but always has a normal component to the air-bridges due to the curvature of the Al film. The result is shown in Fig. 3.5. A dramatic decrease in $Q$ is observed in both resonators at $B > 6$ mT, which is close to the critical field $H_c = 10$ mT of Al but well below the critical field of Nb, which is on the order of 100 mT. The $Q$ of the samples without air-bridges, XM1-20151015\_FZ\_B3 and XM1-20151015\_FZ\_C2, remains unchanged in this magnetic field range. This confirms that losses due to air-bridges become more significant than TLS losses at high temperatures and moderate magnetic fields when Al approaches its normal state.

The difference in $Q$ between XM1-20151015\_FZ\_A4 and XM1-20151015\_FZ\_C1 at $T > 0.9$ K is not clearly understood from the experimental data. A plausible cause is due to the differences in the distribution of Al wire-bonds on the chip. This is suggested by the observation that the difference becomes only significant when the Al $T_c = 1.2$ K is approached, and the $Q$ of the DC-tapped resonator XM1-20151015\_FZ\_C1 becomes abruptly constant beyond this temperature. The precise nature of the interaction between wire-bonds and the resonator has not been widely studied and may be understood through further testing and simulations.

Before concluding this section, we remark that the dominant source of TLS fluctuators is likely the amorphous atomic-layer-deposited Al$_2$O$_3$ film underneath the Nb film. Bare resonators made without the Al$_2$O$_3$ layer have quality factors around 300,000. However, removing the Al$_2$O$_3$ layer, which serves as an etch stop layer for the Ar/SF$_6$ plasma, poses challenges for fabricating higher impedance resonators where the Nb patterns are sub-micron in size, which may be substantially damaged due
to over-etching into the underlying substrate. Future work employing different RIE gases with better Nb/Si selectivity may circumvent this issue.

3.3 Impact of Silicon Quantum Well on Resonator Quality Factors

The limitations on $Q$ by the additional dielectric layers used to create a Si-QW is not completely understood at the point of writing, partly due to the inadequate number of samples tested to draw a concrete conclusion. In GaAs hybrid QD/cavity systems which have seen early success [88], the QW beneath the resonator center pin has to be etched away since the resistive loss induced by the 2DEG electrons imposes a large constraint on achievable $Q$ of the resonator. Such a problem is not immediately apparent in accumulation-mode Si/SiGe heterostructures, where the QW is depleted of mobile electrons until a positive bias (usually $\geq 0.4$ V) is applied to the wafer surface. The first generation SiGe hybrid devices were designed with the QW present under the resonator center pin. It was found that at a temperature of 4.2 K, the quality factor of a bare resonator made on Si-QW degrades severely (from 1,000 to less than 100) once the DC tap voltage approaches 0.4 V. This behavior, however, went away when the same sample is cooled down to 10 mK. It is possible that $Q$-degradation due to the QW is only significant when the resonator is driven at higher powers or operated at higher temperatures. Nevertheless, to avoid potential complications, we etched away the Si-QW in later devices using RIE.

3.4 Impact of Microwave Leakage

A major impediment to achieving a $Q$ of even a few thousands in resonators coupled to Si QDs is the problem of microwave leakage from the resonator to proximal metallic
Figure 3.6: (a) Overall design for the first generation SiGe hybrid devices. The resonator has a design frequency of 7.5 GHz and a design $Q$ of 15,000. A “qubit notch” is opened at the voltage anti-node near both ends of the resonator, where quantum dots can be fabricated. Lower left inset shows optical image of a test DQD, which is not connected to the cavity. Lower right inset shows SEM image of the same DQD. (b) Conductance through the DQD, $g$, measured as functions of gate voltages on $P_1$ and $P_2$, $V_{P1}$ and $V_{P2}$, at $T = 4.2$ K. (c) Transmission through the resonator $|S21|^2$ at $T = 10$ mK and $P_{in} = -88$ dBm. A Lorentzian fit yields $Q = 1079$.

gates used for the electrostatic confinement of electrons in the underlying 2DEG. The electric dipole interaction between cavity photons and quantum dot electrons is usually achieved by connecting the resonator center pin, either capacitively or galvanically, to one of the gate electrodes. However, such a scheme inevitably forms large parasitic capacitances between the resonator and each one of the confinement gates, opening additional channels for photon leakage and thereby decreasing the $Q$. This problem is especially severe with Si QDs defined by overlapping gates which have very large parasitic capacitances between gate electrodes.

As an illustration of this problem, Figure 3.6 displays the design and characterization of an early generation of Si/SiGe hybrid devices. Large portions of the Nb ground plane are cut out at either end of the resonator (where the voltage anti-node...
Figure 3.7: (a) Test resonator design for simulating microwave leakage from resonator to QD confinement gates. The resonator has a design $Q$ of 150,000. Each of the five ports is bonded to a PCB trace which is connected to a microwave coaxial line in a 1.5 K system. Inset shows the Nb island inside the qubit notch, having similar shape as the overlapping Al gates in a real device. (b) Transmission data at $T = 1.5$ K, from Port 1 to each of the other 4 ports. The coaxial lines leading to ports 1 through 5 have the same attenuation, and the ports not under measurement are terminated with a 50 $\Omega$ cap.

is) to accommodate twelve wires made from Ti/Au, which contact the overlapping Al gates defining DQDs. To assess the functionality of the DQD without the complication introduced by the resonator, we first made DQDs that are not connected to the resonator center pin. Figure 3.6 (b) shows basic transport data from one such sample at $T = 4.2$ K. Voltages on the two plunger gates $P_1$ and $P_2$, $V_{P_1}$ and $V_{P_2}$, are swept and the conductance through the device is measured. Coulomb blockade is clearly observed and the two distinct slopes observed in the data indicate formation of a DQD. The resonator behavior for this batch of samples, on the other hand, is far from ideal. Even with the DQD disconnected from the resonator, the $Q$ was measured to be 1079 at $T = 10$ mK, as shown in Fig. 3.6 (c). When any of the Al DQD gates is connected to the resonator center pin, the resonance disappears altogether.

To determine if such a dramatic reduction in the $Q$ is indeed attributed to microwave loss into the QD gates, we made test resonators on FZ Si, altering the Nb etch
patterns to be similar to actual QD-cQED hybrid devices, where the resonator center pin is brought close to a Nb island having a similar shape as the overlapping Al gates in an actual hybrid device. The design is shown in Fig. 3.7(a). A transmission measurement is then made at $T = 1.5$ K to determine four S parameters: $|S_{21}|^2$, which is the transmission between resonator input/output, $|S_{31}|^2$, $|S_{41}|^2$, $|S_{51}|^2$, which are transmissions between resonator input and three selected Nb traces leading to the Nb island, mimicking the DC gates in an actual hybrid device. The data are shown in Fig. 3.7(b). It is clear that there is severe microwave leakage from the resonator center pin to the DC gates, as $|S_{31}|^2$ and $|S_{41}|^2$ are very close in magnitudes to $|S_{21}|^2$, and $|S_{51}|^2$ even lies 3 dB above $|S_{21}|^2$. The Q of this test resonator is 6600 at $T = 1.5$ K, much lower than the value of 20,000 attainable with bare resonators. This Q-degradation is expected to be significantly worse in a real device where the parasitic capacitance between the resonator center pin and the DQD gates is larger than this test design.

### 3.5 Modeling Microwave Leakage

Before delving into how to reduce microwave leakage in these devices, it is helpful to develop a simple circuit model. In a hybrid device, the photon field in the resonator is coupled to the electrons in the QD via dipole interactions. This interaction can be thought as a “quantum capacitance”, $C_q$, between the QD electrons and the resonator. The formation of the QD is enabled by a number of metal gates, which are also coupled to the resonator through parasitic capacitances. To see the influence of these metal gates on the resonator behavior, we use a 3-port circuit model shown in Fig. 3.8(a). While ports 1 and 2 still measure transmission through the resonator, port 3 is biased to a DC voltage to tune the electrostatic potential of the QD.
Figure 3.8: (a) Circuit model for a resonator coupled to a QD, and one additional port (port 3) for tuning the QD which is coupled to the resonator via a parasitic capacitance $C_p$. (b) $|S_{21}|^2$ and $|S_{31}|^2$ calculated from the circuit model, using the same resonator parameters as Fig. 3.1. The two additional capacitances are $C_p = 10$ fF and $C_q = 0$ fF. The $Q$ of the resonator is 2,800.

Figure 3.9: (a) Circuit model for two leakage gates, port 3 and port 4. (b) $|S_{21}|^2$, $|S_{31}|^2$ and $|S_{41}|^2$ calculated from the circuit model, using the same resonator parameters as Fig. 3.1. The parasitic capacitances are $C_{p1} = 10$ fF and $C_{p2} = 5$ fF. The $Q$ of the resonator is 2,280.
We first calculate the ABCD matrix for transmission through the resonator, $|S_{21}|^2$, which is:

$$
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
1 & \frac{1}{i\omega C_{in}} \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
\frac{1}{Z_3} + i\omega C_p & 1
\end{pmatrix} \begin{pmatrix}
\cos(\beta l) & iZ_r \sin(\beta l) \\
iZ_r \sin(\beta l) & \cos(\beta l)
\end{pmatrix} \begin{pmatrix}
1 & \frac{1}{i\omega C_{out}} \\
0 & 1
\end{pmatrix},
$$

(3.9)

where $Z_3$ is the output impedance of port 3, which we assume to also be 50 Ω. We have excluded any effect of the QD electrons by setting $C_q = 0$ (and will omit this circuit element in subsequent models), and used a parasitic capacitance of $C_p = 10$ fF.

The calculated $|S_{21}|^2$ is plotted in Fig. 3.8 (b). The peak of $|S_{21}|^2$ is now much lower than 0, and the $Q$ is dramatically reduced to 2,800, nearly two orders of magnitude lower than its design value of 100,000. We can also calculate the transmission from the resonator input, port 1, to the DC gate, port 3, by writing out the ABCD matrix for $|S_{31}|^2$:

$$
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
1 & \frac{1}{i\omega C_{in}} \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
\frac{1}{Z_3} + i\omega C_p + iZ_t \tan(\beta l) & 1
\end{pmatrix} \begin{pmatrix}
1 & \frac{1}{i\omega C_{out}} \\
0 & 1
\end{pmatrix},
$$

(3.10)

where $Z_t = R_L + \frac{1}{i\omega C_{out}}$. The calculated $|S_{31}|^2$ is plotted alongside $|S_{21}|^2$ in Fig. 3.8 (b), where it is seen that $|S_{31}|^2$ is 20 dB higher than $|S_{21}|^2$. The microwave photons in the resonator almost always leak out of the DC gates rather than the output port.

The limit on $Q$ due to microwave leakage is quickly exacerbated as the total number of DC gates increases. To see this, we solve for a 4-port model shown in Fig. 3.9, where two DC gates (ports 3 and 4) are coupled to the resonator via $C_{p1} = 10$ fF and $C_{p2} = 5$ fF. The $Q$ is further reduced to 2,280. Comparing $|S_{31}|^2$ and $|S_{41}|^2$, ...
we also see that the gate with higher parasitic capacitance (port 3) has a larger overall transmission.

For a SiGe hybrid device containing a triple QD on each side of the resonator, a total number of 24 DC gates are needed to fully control the QD potentials. Although efforts have been made to reduce the parasitic capacitances, details of the resonator and QD designs put a large constraint on how far they can be reduced. In the next section, we present another method of preserving resonator $Q$: the addition of microwave filters.

### 3.6 Reducing Microwave Leakage through On-chip Filters

Let us now consider the same circuits in Fig. 3.8 and Fig. 3.9 with an important modification: $LC$-filters having values of $L_f = 12$ nH and $C_f = 1$ pF are now inserted along each DC gate leading up the resonator. These circuit models are shown in Fig. 3.10 and Fig. 3.11.
Figure 3.11: (a) Circuit model for two leakage gates, ports 3 and 4, with LC-filters. (b) $|S21|^2$, $|S31|^2$ and $|S41|^2$ calculated from the circuit model, using the same resonator parameters as Fig. 3.9. The parasitic capacitances are $C_{p1} = 10 \text{ fF}$ and $C_{p2} = 5 \text{ fF}$. The $Q$ of the resonator is 96,300.

The resonator transmission $|S21|^2$ corresponding to the circuit in Fig. 3.10 is:

$$
\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{i\omega C_{in}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{i\omega C_{in}Z_{r} + Z_{3}} & 1 \\ \frac{1}{i\omega C_{p1}} & \frac{1}{i\omega L_{f}} \end{pmatrix} \begin{pmatrix} \cos(\beta l) & iZ_{r}\sin(\beta l) \\ \frac{i}{Z_{r}}\sin(\beta l) & \cos(\beta l) \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{i\omega C_{p1}} \\ 0 & 1 \end{pmatrix}.
$$

(3.11)

We then calculate the power leakage from the resonator to port 3, $|S31|^2$, by writing its ABCD matrix:

$$
\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{i\omega C_{in}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{i\omega C_{in}Z_{r} + Z_{3}} & 1 \\ \frac{1}{i\omega C_{p1}} & \frac{1}{i\omega L_{f}} \end{pmatrix} \begin{pmatrix} \frac{Z_{r} + iZ_{r}\tan(\beta l)}{Z_{r}Z_{r} + iZ_{r}\tan(\beta l)} & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{i\omega C_{p1}} \\ 0 & 1 \end{pmatrix}.
$$

(3.12)
The calculated \( |S21|^2 \) and \( |S31|^2 \) are both plotted in Fig. 3.10(b). We see that \( Q = 96,800 \) is almost back to the design value. At the same time, \( |S31|^2 \) is now suppressed to be 10 dB lower than \( |S21|^2 \).

Lastly, we calculate the transmissions for the 4-port model in Fig. 3.11. With an \( LC \)-filter inserted for each DC gate, the leakage curves \( |S31|^2 \) and \( |S41|^2 \) are both suppressed to be below \( |S21|^2 \), and we see a high \( Q \) of 96,300 is also preserved in this model.

3.7 A cQED Architecture for Gate-Defined Si Quantum Dots

Finally, equipped with what we have learned from early generations of devices and basic modeling, we present a complete hybrid Si QD-cQED device architecture in this section. The devices are fabricated on Si/SiGe heterostructures grown by chemical vapor deposition \[39, 92\]. A 3 \( \mu \)m thick linearly graded \( Si_{1-x}Ge_x \) relaxed buffer substrate is grown on top of a Si wafer (resistivity > 5000 \( \Omega \)-cm). The buffer is chemically and mechanically polished before the growth of a 170 – 375 nm thick \( Si_{0.7}Ge_{0.3} \) layer, an 8 nm thick Si quantum well (QW), a 50 – 60 nm thick \( Si_{0.7}Ge_{0.3} \) spacer and a 2 – 4 nm thick Si cap. Wafers grown under similar conditions have maximum mobilities \( \mu = 650,000 \) cm\(^2\)/Vs and can support electron densities up to \( n = 8 \times 10^{11}/\text{cm}^2 \).

The cavity fabrication process is designed to achieve two major goals: protection of the Si QW from the reactive ions used to etch the Nb cavity and the reduction of internal photon losses introduced by two-level system (TLS) defects at the heterostructure interfaces. The Si QW in the area under the cavity center pin is first removed through a 70 nm deep reactive ion etch to minimize the internal photon loss. A 30 nm thick \( Al_2O_3 \) film, which serves as an etch stop for cavity fabrication,
Figure 3.12: (a) Optical image of an example silicon hybrid cQED device. A Si DQD is placed at each voltage anti-node of the cavity. LC-filters reduce leakage of cavity photons through dc biasing lines. (b) A cross-section (not to scale) taken along the horizontal dashed line in (a) shows the overlapping Al gates that define the DQD and the Si/SiGe heterostructure layers. To minimize internal losses the quantum well is removed in areas beneath the cavity center pin [cross-section through the vertical dashed line in (a)]. (c) False-color scanning electron microscope (SEM) image of a DQD.

is then grown over the entire substrate using atomic layer deposition. Next, a 50 nm thick Nb film is deposited via dc sputtering. The cavity and filter patterns, shown in Fig. 3.12(a), are defined with a second reactive ion etch step using a SF$_6$/Ar plasma. A hydrofluoric acid etch then removes the Al$_2$O$_3$ film in the area where the DQD is to be defined. The resulting cross-sections of the device are schematically illustrated in Fig. 3.12(b).

Accumulation-mode DQDs are defined using an overlapping gate architecture. A scanning electron microscope image of the DQD is shown in Fig. 3.12(c). For the
measurements presented in this thesis, electrons are only accumulated in one of the
DQDs and the other DQD does not contribute to the cavity response. The first Al
layer, shaded in purple, consists of two large gates G1 and G2 that selectively screen
the electrostatic potentials of the upper Al layers and form a quasi-one-dimensional
transport channel. The second Al layer, shaded in pink, consists of two plunger gates
P1 and P2 that are used to tune the chemical potentials of the DQD, as well as source
(S) and drain (D) accumulation gates. P1 is connected to the cavity center pin and
capacitively couples the DQD to the time-dependent voltage $V_C(t)$ of the cavity. A dc
tap is placed at the voltage node of the cavity and used to dc-bias gate P1. The third
Al layer, shaded light green, consists of three tunnel barrier gates. Gate B2 tunes
the interdot tunnel coupling ($t_c$), gate B1 tunes the dot 1–source reservoir coupling,
while gate B3 tunes the dot 2–drain reservoir coupling.

To minimize microwave leakage, we insert an $LC$-filter in each dc bias line. An
$LC$-filter is also used to dc bias the cavity, in contrast to previous devices that used
a spiral inductor on the dc tap and no filter along gate bias lines [86]. Figure 3.13(a)
shows the image of a single filter, which consists of a long interdigitated capacitor
with $C_f \approx 1 \text{ pF}$ and a spiral inductor with $L_f \approx 13 \text{ nH}$. The overall dimensions of a
filter are $700 \text{ µm}$ by $200 \text{ µm}$.  

To evaluate the attenuation of the filter, we measure its transmission $|S_{21}|^2$ at
a temperature $T = 1.5 \text{ K}$. The data, shown in Fig. 3.13(b), display a clear roll-off
with frequency $f$. Oscillations with a frequency spacing of $\sim 500 \text{ MHz}$ are also seen
throughout the data range. The oscillations may be due to reflections at the wire
bonds connecting the filter to the circuit board, parasitic modes introduced by discon-
tinuities in the ground plane, and parasitic capacitances/inductances that result from
the relatively large size of the filter components. On average, 20 dB of attenuation is
obtained around the cavity center frequency $f_c = 7.67 \text{ GHz}$ [Fig. 3.13(c)].
Figure 3.13: (a) Left: Optical image of a compact LC-filter, showing the spiral inductor and a portion of the capacitor. Middle: Zoomed-in view of the capacitor (red outline)/inductor (blue outline). Right: Circuit model for the LC-filter. (b) Measured LC-filter transmission $|S_{21}|^2$ (red) and ABCD matrix predictions (black). (c) Cavity transmission, $|S_{21}|^2$, measured with the DQD in Coulomb blockade. The black line is a fit to a Lorentzian with $Q = 5400$.

For comparison, $|S_{21}|^2$, as calculated using the ABCD matrix approach, is plotted in Fig. 3.13(b) [89]. The theory predicts a filter attenuation of 24 dB at $f = 7.67$ GHz. With the exception of the oscillations in the data, the overall transmission through the filter is in good agreement with theory. The undesired oscillations may be suppressed by using air-bridges to better connect distinct regions of the cavity ground plane, and improved circuit board designs to minimize the impedance of the wire-bonds [90].
The incorporation of $LC$-filters into the cavity design results in a significant increase in the cavity quality factor compared to earlier generation devices. Figure 3.13(c) shows the normalized cavity transmission $|S_{21}|^2$ as a function of $f$ with the DQD configured in Coulomb blockade at $T = 10$ mK. The input power $P_{in} \approx -130$ dBm corresponds to an intra-cavity photon number $n \approx 3$. A fit to a Lorentzian function yields $Q = 5,400$, corresponding to a total photon loss rate $\kappa/2\pi = f_c/Q = 1.4$ MHz. Using the Sonnet EM simulation program, we estimate the cavity input and output coupling rates to be $\kappa_{in}/2\pi = \kappa_{out}/2\pi = 0.4$ MHz. The remaining loss rate of 0.6 MHz may be attributed to a combination of internal loss due to the dielectric layers under the cavity and remnant microwave leakage through the $LC$-filters. The internal loss may be reduced by etching away Al$_2$O$_3$ and Si$_{0.7}$Ge$_{0.3}$ in the gap between the cavity center pin and the ground plane where the electric field intensity is large. Microwave leakage can be further suppressed through improved filter designs such as multi-pole $LC$-filters and band-stop filters [93].
Chapter 4

Strong Coupling of a Single Electron in Silicon to a Single Photon

In cavity quantum electrodynamics (CQED), light-matter interactions can lead to the coherent hybridization of the quantum degrees of freedom of a photonic cavity and a two-level atom [2]. A hallmark of CQED physics is the strong coupling regime, where the coherent coupling rate between the two-level atom and the cavity photon $g$ exceeds the photon loss rate $\kappa$ and the atomic decoherence rate $\gamma$. Achieving strong coupling is highly relevant in quantum information science, as cavity photons can be used to mediate long-range qubit interactions [10, 12]. Strong coupling was first observed in atomic systems using alkali atoms [3, 94] and later with optically addressed quantum dots [5, 6] and superconducting qubits in the circuit quantum electrodynamics (cQED) architecture [7].

Qubits based on electrons confined in semiconductor quantum dots are the focus of intense research efforts due to their scalability and potential for long coherence. Silicon is a particularly attractive host material for quantum dot qubits, owing to its
exceptional spin coherence times [33, 95] and highly established fabrication technologies leading to the achievement of high-fidelity single-qubit [33, 96] and two-qubit [19] logic gates. A universal challenge for quantum dot devices is charge noise, which is a dominant dephasing mechanism in both charge and spin qubits [14, 88]. Charge noise has hindered a large number of attempts to coherently couple quantum dot devices to microwave frequency photons [86, 88, 97]. Here we report the observation of strong coupling between a single electron in a Si double quantum dot (DQD) and microwave frequency photons in a superconducting cavity, facilitated by highly suppressed charge noise in our device architecture [98]. These experiments pave the way toward long-range coupling of semiconductor qubits in one of the most technologically relevant material systems in the semiconductor industry, as well as the coherent coupling of a single electron spin to a single microwave photon [99], which is the subject of the next chapter.

4.1 Microwave Measurement Circuit

To measure the transmission through the cavity, we use a homodyne detection circuit outlined in Fig. 4.1. A coherent microwave tone at a fixed frequency $f_d$ is first split by a microwave splitter where the signal at port 2 drives the LO port of an IQ-mixer. The signal at port 1 is attenuated by $-60$ dB of room temperature attenuators, phase-shifted by a mechanical phase shifter (which is precisely controlled by a stepper motor), and further attenuated by two $-30$ dB attenuators placed at the 3 K and 10 mK stages of the dilution refrigerator before reaching the input port of the resonator.

The transmitted resonator signal first passes through a circulator before being coupled to a strong pump tone at a frequency $f_p = 8.102$ GHz through a directional coupler. The coupled signal then passes through a Josephson traveling wave parametric amplifier (JTWPA) [100], which provides $+30$ dB of gain to the cavity.
Figure 4.1: Circuit diagram used for transmission measurement of the resonator.
output signal with a noise temperature of $T_N \approx 520$ mK. The cavity signal is further amplified by a high electron mobility transistor (HEMT) amplifier placed at the 3K stage of the dilution refrigerator having a gain of $+35$ dB and $T_N = 4$ K, followed by a room temperature amplifier with another $+30$ dB of gain. Finally, the cavity signal is passed to the RF port of the IQ mixer and down-converted to a DC signal with in-phase ($I$) and quadrature ($Q$) components. Both components are low-pass filtered, amplified and measured through a digital multimeter. The $I$ and $Q$ components of the homodyne signal are also calibrated to yield the amplitude and phase of the transmitted signal by driving the mechanical phase shifter with the stepper motor.

### 4.2 Charge-Photon Interaction

The hybrid silicon-cQED device used in this chapter [101] has the same design as the device shown previously in Fig. 3.12. The cavity has a center frequency $f_c = 7.684$ GHz, loaded quality factor $Q_c = 7460$, and photon loss rate $\kappa/2\pi = 1.0$ MHz.

We first demonstrate that the electromagnetic field of the cavity is sensitive to charge dynamics in the DQD. The cavity is driven with a coherent microwave tone having a fixed frequency $f = f_c$ and power $P \approx -130$ dBm (corresponding to $\sim 3$ intracavity photons). The amplitude $A$ and phase $\phi$ of the transmitted signal are extracted using a traveling wave parametric amplifier [100] and homodyne demodulation [86]. Figure 4.2(a) shows the cavity transmission amplitude, $A/A_0$, as a function of the DQD plunger gate voltages $V_{P1}$ and $V_{P2}$. Here $A_0$ is a normalization constant that is set such that the maximum value of $A/A_0$ is unity with the DQD configured in Coulomb blockade [86]. A DQD stability diagram is clearly visible in these data, with charge stability islands labeled as $(N_1, N_2)$ where $N_1$ ($N_2$) is the number of electrons in dot 1 (2). At the boundaries of charge stability islands, electron tunneling events
Figure 4.2: (a) DQD charge stability diagram extracted from measurements of $A/A_0$ as a function of $V_{P1}$ and $V_{P2}$, with fixed drive frequency $f = f_c$. (b) $A/A_0$ in the vicinity of the $(3, 2) \leftrightarrow (2, 3)$ interdot charge transition [red circle in (a)], after the DQD is tuned such that $2t_C/h \approx f_c$. Dashed lines mark the boundaries of the stability diagram. (c) Eigenenergies $E_{JC}$ of the Jaynes-Cummings Hamiltonian describing the cQED system for three different values of $t_C$, calculated with $g_c/2\pi = 6.7$ MHz (solid lines) and 0 (dashed lines). (d) $A/A_0$ as a function of $\epsilon$ for the values of $t_C$ shown in (c). Black lines are fits to cavity input-output theory.
damp the electromagnetic field in the cavity, resulting in reduced cavity transmission amplitudes [86, 88].

A central figure of merit in CQED systems is the cooperativity $C = \frac{2g^2}{\kappa\gamma}$, which can physically be interpreted as the ratio of the coherent coupling rate $g$ to the incoherent coupling rates $\kappa$ and $\gamma$. To estimate this parameter, we focus on an interdot charge transition at which the total number of electrons in the DQD is fixed. Here a single excess electron functions as a charge qubit described by the Hamiltonian $H_a = \frac{1}{2} f_a \sigma_z$, where $\sigma_z$ is a Pauli matrix. The qubit transition frequency is $f_a = \sqrt{\epsilon^2 + 4t_C^2}/h$, where $\epsilon$ is the DQD energy level detuning, $t_C$ is the interdot tunnel coupling and $h$ is Planck’s constant [35]. The cavity is governed by the Hamiltonian $H_c = hf (a^\dagger a + \frac{1}{2})$, where $a^\dagger (a)$ is the photon creation (annihilation) operator. In addition, the qubit and the cavity are coupled through an interaction Hamiltonian $H_{int} = h(g_c/2\pi) \sin \theta (a^\dagger \sigma^- + a \sigma^+) \sigma^+ \sigma^-$ is the qubit creation (annihilation) operator, $g_c$ is the charge-photon electric-dipole coupling rate and $\sin \theta = 2t_C/\sqrt{\epsilon^2 + 4t_C^2}$ [86, 88]. The total Hamiltonian of the system is the Jaynes-Cummings Hamiltonian $H_{JC} = H_a + H_c + H_{int}$. We perform measurements at the $(3, 2) \leftrightarrow (2, 3)$ interdot charge transition [Fig. 4.2(b)] where the minimum qubit frequency is most closely matched to the cavity frequency $2t_C/h \approx f_c$. Strongly reduced cavity transmission amplitudes $A/A_0 \approx 0$ are observed near the interdot transition [see Fig. 4.2(d)]. These data give preliminary evidence for highly coherent charge-photon interactions and a large cooperativity.

To determine the charge-photon coupling rate $g_c$, we measure $A/A_0$ as a function of $\epsilon$. Qualitatively, when the qubit-cavity detuning $\Delta/2\pi = f_a - f_c \approx 0$, the eigenenergies $E_{JC}$ of the cQED system (in the single-excitation manifold) are $E_{JC}/h \approx f_c \pm \sin \theta (g_c/2\pi)$ [Fig. 4.2(b)]. The large detuning between the drive frequency $f = f_c$ and $E_{JC}/h$ results in a strong reduction in the cavity transmission amplitude. Figure 4.2(d) shows measurements of $A/A_0$ as a function of $\epsilon$ for three values of $t_c$. With $2t_c/h = 7.72$ GHz, the qubit frequency exceeds the cavity frequency
for all values of $\epsilon$ [Fig. 4.2(c)] and a single minimum in $A/A_0$ is observed at $\epsilon = 0$, where $\Delta$ is smallest. At lower values of $t_c$, the minimum qubit frequency becomes smaller than the cavity frequency and we observe two minima in $A/A_0$ at values of $\epsilon$ where $\Delta \approx 0$ [dashed lines in Fig. 4.2(c)]. Input-output theory is used to account for decoherence effects and to quantitatively fit the measured transmission amplitudes [1, 86, 97], with $g_c$ as a free parameter. Inputs to the model are the measured photon loss rate $\kappa/2\pi = 1.0$ MHz and a charge qubit decoherence rate $\gamma_c/2\pi = 2.6$ MHz (see next section). All three data sets are in excellent agreement with theory, with a best-fit charge-photon coupling rate $g_c/2\pi = 6.7 \pm 0.2$ MHz [7].

### 4.3 Single Electron Charge Coherence

Past attempts to reach the strong coupling regime with semiconductor quantum dots have been impeded by background charge fluctuations, which cause temporal fluctuations of the DQD energy levels, resulting in rapid decoherence [86, 88]. We now
show that the combination of high quality Si/SiGe heterostructures and a new overlapping gate architecture allow for a significant reduction in charge noise. In these measurements the interdot tunnel coupling is set to $2t_C/h = 7.82$ GHz such that the device is in the dispersive regime with $\Delta \gg g_c$. The cavity phase response $\Delta \phi$ is measured at $f = f_c$ while gate P1 is driven by an additional microwave tone at frequency $f_s$. The phase response is expected to be $\Delta \phi = \pm \tan^{-1}(2g_c^2/\kappa \Delta)$, with the $-(+)$ sign corresponding to the charge qubit being in the ground (excited) state \[102\]. When $f_s \approx f_a(\epsilon)$, the microwave excitation at $f_s$ will increase the excited state population $P_x$ of the charge qubit and lead to more positive values of $\Delta \phi$ \[102\]. In Fig. 4.3(a) we plot $\Delta \phi$ as a function of $\epsilon$ and $f_s$. The frequency dispersion relation of the charge qubit, corresponding to the resonance condition $f_s = f_a(\epsilon)$, is visible on top of the slowly varying phase response of the cavity. To determine the decoherence rate $\gamma_c$, we focus on the phase response near $\epsilon = 0$. Here the charge qubit is at a “sweet spot” and the energy level separation is first order insensitive to charge noise \[103, 104\]. Figure 4.3(b) shows $\Delta \phi$ as a function of $f_s$ with $\epsilon = 0$. The data are fit to a Lorentzian function centered at the qubit frequency $f_a = 7.82$ GHz, in agreement with theory \[87, 102\]. The full-width-at-half maximum (FWHM) of the Lorentzian function is $5.2 \pm 0.2$ MHz, corresponding to a charge decoherence rate of $\gamma_c/2\pi \approx 2.6$ MHz. This decoherence rate is two to three orders of magnitude lower than previously reported values \[88, 103, 105, 106\] and is pivotal to achieving strong charge-photon coupling in our system, which we demonstrate in the next section.

4.4 Charge-Photon Vacuum Rabi Splitting

In the strong coupling regime, the coherent hybridization of quantum states involving light and matter leads to the emergence of two clearly resolvable normal modes in the cavity transmission spectrum, separated by the vacuum Rabi frequency $2g_c/2\pi$ \[1\] \[2\].
Figure 4.4: (a) Cavity transmission spectrum $A/A_0$ as a function of $f$ and $\epsilon$ with $2t_c/h = f_c = 7.68$ GHz. The system eigenenergies are overlaid on the data for the case of no coupling $g_c/2\pi = 0$ (dashed lines) and $g_c/2\pi = 6.7$ MHz (solid lines). (b) $A/A_0$ as a function of $f$ at $\epsilon = 6$ $\mu$eV and 0. Dashed lines are predictions from cavity input-output theory.
We search for vacuum Rabi splitting with the device tuned to $2t_c/h = 7.68 \text{ GHz}$, such that the cavity is in resonance with the qubit (at the sweet spot $\epsilon = 0$). The cavity transmission spectrum $A/A_0$ is plotted as a function of $f$ and $\epsilon$ in Fig. 4.4(a). At $\epsilon = 6 \mu eV$, the transmission spectrum $A/A_0$ is close to that of a bare cavity due to a large qubit-cavity frequency detuning $\Delta$ [red curve, Fig. 4.4(b)]. However, in the range $-2 \mu eV$ to $2 \mu eV$, two maxima emerge in $A/A_0$ at frequencies corresponding to the eigenenergies of the system (overlaid on data). In particular, we observe a pair of distinct peaks with equal heights in the cavity transmission spectrum at $\epsilon = 0$, where $f_a = f_c$ [blue curve, Fig. 4.4(b)]. The peaks are separated by $2g_c/2\pi = 13.4 \text{ MHz}$, consistent with the value of $g_c/2\pi = 6.7 \text{ MHz}$ extracted from the data in Fig. 4.2(d). The observed normal mode splitting indicates that the strong coupling regime has been reached. A high cooperativity $C = g_c^2/\kappa\gamma_c = 17$ is also achieved by the charge-photon cQED system.

## 4.5 Conclusion

We have demonstrated, for the first time, strong coupling between a semiconductor charge qubit and a microwave frequency photon. Beyond enabling a new platform for quantum optics experiments such as generation of single photon states [107], coherent charge-photon interaction in a Si DQD-cQED device is a critical step toward strong spin-photon coupling (the subject of the next chapter). Shortly after the submission and publication of this work [101], strong charge-photon coupling was also achieved in a group at ETH using a high-impedance resonator [108], ushering in a new era of strong-coupling cQED with gate-defined quantum dot qubits.

It is also important to emphasize another exciting finding of this charge-photon experiment, namely the charge qubit decoherence rate of $\gamma_c/2\pi = 2.6 \text{ MHz}$. Such a rate corresponds to a coherence time of $T_2 \approx 60 \text{ ns}$, well beyond the best reported...
values for semiconductor DQD charge qubits [88, 103, 105, 106, 109, 110]. More recently, the ETH group also reported a GaAs DQD charge qubit with comparable coherence times [111]. These results indicate that charge coherence times approaching 100 ns are in fact possible in gate-defined semiconductor QDs, boding well for spin qubits in these systems which are often limited by charge noise due to spin-charge hybridization [17]. On the hand, as the readers may perceive upon reading subsequent chapters, this remarkable degree of charge coherence is not always attained. Further materials research is needed to uncover the physical origin of charge decoherence in such devices.
Chapter 5

A Coherent Spin-Photon Interface in Silicon

Solid-state electron spins and nuclear spins are quantum mechanical systems that can be highly isolated from environmental noise, a virtue that endows them with coherence times as long as hours and establishes solid-state spins as one of the most promising quantum bits (qubits) for constructing a quantum processor [13, 33, 95]. On the other hand, this degree of isolation comes at a cost, as it poses difficulties for the spin-spin interactions needed to implement two-qubit gates. To date, most approaches have focused on achieving spin-spin coupling through the exchange interaction or the much weaker dipole-dipole interaction [14, 112, 113]. Among existing classes of spin qubits, electron spins in gate-defined Si quantum dots (QDs) have the advantages of scalability due to mature fabrication technologies and low dephasing rates owing to isotopic purification [114]. Currently, Si QDs are capable of supporting fault-tolerant control fidelities for single-qubit gates and high fidelity two-qubit gates based on exchange [19, 21, 96, 115]. Coupling of spins over long distances has been pursued through physical displacement of electrons [116, 119], as well as “super-exchange” via an intermediate quantum dot [120]. The recent demonstration of strong-coupling between
the charge state of a quantum dot electron and a single photon has also raised the prospect of spin-photon strong coupling, which could enable photon-mediated long-distance spin entanglement [101, 108, 121]. Spin-photon coupling may be achieved by coherently hybridizing spin qubits with photons trapped inside microwave cavities, similar to cavity quantum electrodynamics (CQED) with atomic systems and circuit QED (cQED) with solid-state qubits [3, 7, 8, 94, 101, 108, 122]. Such an approach, however, is extremely challenging: the small magnetic moment of a single spin leads to magnetic-dipole coupling rates ranging from 10 – 150 Hz, which are far too slow compared with electron spin dephasing rates to enable a coherent spin-photon interface [22, 23, 122–125].

In this chapter, we resolve this outstanding challenge by using spin-charge hybridization to couple the electric field of a single photon to a single spin in Si [28–30, 32, 122]. We measure spin-photon coupling rates $g_s/2\pi$ up to 11 MHz, nearly five orders of magnitude higher than typical magnetic-dipole coupling rates. These values of $g_s/2\pi$ exceed both the photon decay rate $\kappa/2\pi$ and the spin decoherence rate $\gamma_s/2\pi$, firmly anchoring our spin-photon system in the strong-coupling regime [23, 123, 125].

Our coupling scheme consists of two stages of quantum state hybridization: First, a single electron is trapped within a gate-defined Si double quantum dot (DQD) having a large electric-dipole moment. A single photon confined within a microwave cavity hybridizes with the electron charge state through electric-dipole interaction [86, 88]. Second, a micromagnet placed over the DQD hybridizes electron charge and spin by producing an inhomogeneous magnetic field [28, 30, 32]. The combination of the electric-dipole interaction and spin-charge hybridization gives rise to a large effective spin-photon coupling rate. At the same time, the relatively low level of charge noise in the device ensures that the effective spin decoherence rate $\gamma_s$ remains
below the coherent coupling rate $g_s$, a crucial criterion which has hampered a previous effort to achieve spin-photon strong coupling [97].

Beyond the demonstration of a coherent spin-photon interface, we also show that our device architecture is capable of single-spin control and readout. Single-spin rotations are electrically driven [115, 126] and the resulting spin state is detected through a dispersive phase shift in the cavity transmission, which reveals Rabi oscillations [86].

### 5.1 Spin-Photon Interface

The device enabling spin-photon strong coupling is shown in Fig. 5.1(a) and contains two gate-defined DQDs fabricated using an overlapping aluminum gate stack [Fig. 5.1(b)]. The gates are electrically biased to create a double-well potential that confines a single electron in the underlying natural-Si quantum well [Fig. 5.1(c)]. A plunger gate (P2) on each DQD is connected to the center pin of a half-wavelength Nb superconducting cavity with a center frequency $f_c = 5.846$ GHz and quality factor $Q_c = 4,700$ ($\kappa/2\pi = f_c/Q_c = 1.3$ MHz), hybridizing the electron charge state with a single cavity photon through the electric-dipole interaction [86, 88, 101, 108, 121]. Since the spin-photon coupling rate $g_s$ is directly proportional to the charge-photon coupling rate $g_c$ [25, 26, 28–32, 122], we have modified the cavity dimensions [inset of Fig. 5.1(a)] to achieve a high characteristic impedance $Z_r$ and therefore a high $g_c$ ($g_c \propto \sqrt{Z_r}$) [108]. To hybridize the trapped electron’s charge state with its spin state, a Co micromagnet is fabricated near the DQD which generates an inhomogeneous magnetic field. For our device geometry, the magnetic field due to the Co micromagnet has a component along the z-axis, $B_z^M$, which is approximately constant for the DQD and a component along the x-axis which takes on a different average value of $B_{x,L}^M$ ($B_{x,R}^M$) for the left (right) dot, as illustrated by Fig. 5.1(c). The relatively large
Figure 5.1: (a) Optical image of the microwave cavity. Inset shows an optical image of the cavity center pin (0.6 µm) and vacuum gap (20 µm). (b) False-color SEM of a DQD. The Co micromagnet location is indicated by the orange dashed lines. (c) Schematic cross-sectional view of the DQD device. (d) DQD 1 stability diagram as measured by $A/A_0$ near the $(1,0) \leftrightarrow (0,1)$ interdot transition. $V_{B2}$ denotes the voltage on gate B2 which controls the interdot tunnel coupling $t_C$. (e) $A/A_0$ as a function of $\epsilon$ with $V_{B2} = 710$ mV, and a fit to cavity input-output theory. (f) $2t_C/h$ as a function of $V_{B2}$ for DQD 1.
field difference $B^M_{x:L} - B^M_{x:L} = 2B^M_x$ leads to spin-charge hybridization which, when combined with charge-photon coupling, gives rise to spin-photon coupling [29, 32].

We first characterize the strength of charge-photon interaction, since this sets the scale of the spin-photon interaction rate. For simplicity, only one DQD is active at a time for all of the measurements presented in this chapter. The cavity is driven by a coherent microwave tone at frequency $f = f_c$ and power $P \approx -133$ dBm (corresponding to $\sim 0.6$ photons in the cavity, based on ac-Stark shift measurements of the spin qubit frequency in the dispersive regime [102]). The normalized cavity transmission amplitude $A/A_0$ is displayed in Fig. 5.1(d) as a function of the voltages $V_{P1}$ and $V_{P2}$ on gates P1 and P2 of DQD 1, which reveals the location of the $(1, 0) \leftrightarrow (0, 1)$ interdot charge transition [86, 88, 101, 108, 121]. Here $(N_1, N_2)$ again denotes a charge state with the number of electrons in the left (P1) and right (P2) dot being $N_1$ and $N_2$, respectively. The charge-photon coupling rate is quantitatively estimated by measuring $A/A_0$ as a function of the DQD level detuning $\epsilon$ [Fig. 5.1(e)]. By fitting the data to cavity input-output theory using $\kappa/2\pi = 1.3$ MHz, we find $g_c/2\pi = 40$ MHz and $2t_C/h = 4.9$ GHz, where $t_C$ is the interdot tunnel coupling and $h$ is Planck’s constant [86, 97, 101]. A charge decoherence rate $\gamma_c/2\pi = 35$ MHz is also estimated from the fit and independently confirmed using microwave spectroscopy with $2t_C/h = 5.4$ GHz [101, 102, 108]. Fine control of the DQD tunnel coupling, which is critical for achieving spin-charge hybridization [29], is shown in Fig. 5.1(f) where $2t_C/h$ is plotted as a function of the voltage $V_{B2}$ on the interdot barrier gate B2. A similar characterization of DQD 2 yields $g_c/2\pi = 37$ MHz and $\gamma_c/2\pi = 45$ MHz at the $(1, 0) \leftrightarrow (0, 1)$ interdot charge transition. Due to the higher impedance of the resonator, the values of $g_c$ measured here are significantly larger than in previous Si DQD devices [98, 101], which is helpful for achieving spin-photon strong coupling. In general, there are device-to-device variations in $\gamma_c$ [98, 101]. It is unlikely the slightly higher charge decoherence rate is a result of the new cavity design since the Purcell
Figure 5.2: (a) $A/A_0$ as a function of the cavity drive frequency $f$ and an externally applied magnetic field $B_{z}^{\text{ext}}$ for DQD 1. Insets show data from DQD 2 at the same values of $t_C$ and $\epsilon$, and plotted over the same range of $f$. $B_{z}^{\text{ext}}$ ranges from -94 mT to -91.1 mT (91.1 mT to 94 mT) for the left (right) inset. (b) $A/A_0$ as a function of $f$ for DQD 1 at $B_{z}^{\text{ext}} = 90.3$ mT and 92.2 mT. (c) $A/A_0$ as a function of $f$ for DQD 2 at $B_{z}^{\text{ext}} = 91.1$ mT and 92.6 mT.

decay rate $[125]$ is estimated to be $\Gamma_c/2\pi \approx 0.02$ MHz $\ll \gamma_c/2\pi$. Excited valley states are not visible in the cavity response of both DQDs, suggesting they have negligible population $[127]$. We therefore exclude valleys from the analysis below.

### 5.2 Single Spin-Photon Strong Coupling

We now demonstrate strong-coupling between a single electron spin and a single photon, as evidenced by the observation of vacuum Rabi splitting. Vacuum Rabi
splitting occurs when the transition frequency of a two-level atom $f_a$ is brought into resonance with a cavity photon of frequency $f_c$ \[3\] [7]. Light-matter hybridization leads to two vacuum-Rabi-split peaks in the cavity transmission. For our single spin qubit, the transition frequency between two Zeeman-split spin states is $f_a \approx E_Z/h$, where the Zeeman energy $E_Z = g\mu_B B_{\text{tot}}$, and the approximate sign is due to spin-charge hybridization which slightly shifts the qubit frequency. Here $g$ is the electron $g$-factor, $\mu_B$ is the Bohr magneton and $B_{\text{tot}} = \sqrt{(B_{x,L}^M + B_{x,R}^M)^2 + (B_z^M + B_z^{\text{ext}})^2}$ is the total magnetic field. To bring $f_a$ into resonance with $f_c$, we vary the external magnetic field $B_z^{\text{ext}}$ along the $z$-axis while measuring the cavity transmission spectrum $A/A_0$ as a function of the drive frequency $f$, as shown in Fig. 5.2(a). Vacuum Rabi splittings are clearly observed at $B_z^{\text{ext}} = -91.2$ mT and $B_z^{\text{ext}} = 92.2$ mT, indicating that $E_Z/h = f_c$ at these field values and the single spin is coherently hybridized with a single cavity photon. These measurements are performed on DQD 1, with $2t_C/h = 7.4$ GHz and $\epsilon = 0$. The dependences of $g_s$ on $\epsilon$ and $t_C$ are thoroughly investigated in the next section [31]. Based on $g = 2$ for Si, we estimate an intrinsic field of \~120 mT added by the micromagnet, comparable to values found in a previous experiment using a similar Co micromagnet design [115].

To further verify the achievement of spin-photon strong coupling, we plot the cavity transmission spectrum at $B_z^{\text{ext}} = 92.2$ mT in Fig. 5.2(b). The two normal mode peaks are separated by the vacuum Rabi frequency $2g_s/2\pi = 11.0$ MHz, giving an effective spin-photon coupling rate $g_s/2\pi = 5.5$ MHz. The photon decay rate at finite magnetic field is extracted by the linewidth of $A/A_0$ at $B_z^{\text{ext}} = 90.3$ mT where $E_Z/h$ is largely detuned from $f_c$, yielding $\kappa/2\pi = 1.8$ MHz. A spin decoherence rate $\gamma_s/2\pi = 2.4$ MHz, with contributions from both charge decoherence and magnetic noise from the $^{29}$Si nuclei, is extracted from microwave spectroscopy in the dispersive regime with $2t_C/h = 7.4$ GHz and $\epsilon = 0$, confirming that the strong-coupling regime $g_s > \gamma_s, \kappa$ has been reached. It is remarkable that the spin-photon coupling rate obtained here is

77
more than four orders of magnitude larger than currently achievable rates using direct magnetic-dipole coupling to lumped element superconducting resonators \[23, 24\].

The local magnetic field generated using Co micromagnets is very reproducible, as evidenced by examining the other DQD in the cavity. Measurements on DQD 2 show vacuum Rabi splittings at \(B_{\text{ext}}^z = \pm 92.6 \text{ mT}\) [insets to Fig. 5.2(a)]. The spin-photon coupling rate and spin decoherence rate are determined to be \(g_s/2\pi = 5.3 \text{ MHz}\) and \(\gamma_s/2\pi = 2.4 \text{ MHz}\) respectively [Fig. 5.2(c)]. These results are highly consistent with DQD 1, which we focus on for the rest of this chapter.

### 5.3 Electrical Control of Spin-Photon Coupling

For quantum information applications, it is desirable to rapidly turn qubit-cavity coupling on for quantum state transfer, and off for qubit state preparation. Fast control of the coupling rate is often accomplished by quickly modifying the qubit-cavity detuning \(f_a - f_c\). Practically, such tuning can be achieved by varying the qubit transition frequency \(f_a\) with voltage or flux pulses [10, 12], or by using a tunable cavity [108]. These approaches are not directly applicable for control of the spin-photon coupling rate since \(f_a\) primarily depends on magnetic fields that are difficult to vary on nanosecond timescales. In this section, we show that control of the spin-photon coupling rate may be achieved electrically by tuning \(\epsilon\) and \(t_C\) [26, 30].

We first investigate the \(\epsilon\) dependence of \(g_s\). Figure 5.3(a) shows measurements of \(A/A_0\) as a function of \(B_{\text{ext}}^z\) and \(f\) for \(\epsilon = 0, 20 \mu\text{eV}\) and \(40 \mu\text{eV}\). At \(\epsilon = 20 \mu\text{eV (~4.8 GHz)}\), vacuum Rabi splitting is observed at \(B_{\text{ext}}^z = 92.1 \text{ mT}\) with a spin-photon coupling rate \(g_s/2\pi = 1.0 \text{ MHz}\) that is significantly lower than the value of \(g_s/2\pi = 5.5 \text{ MHz}\) obtained at \(\epsilon = 0\). At \(\epsilon = 40 \mu\text{eV (~9.7 GHz)}\), only a small dispersive shift is observed in the cavity transmission spectrum at \(B_{\text{ext}}^z = 91.8 \text{ mT}\), suggesting further decrease in \(g_s\). These observations are qualitatively understood.
Figure 5.3: (a) $A/A_0$ as a function of $f$ and $B_{z}^{\text{ext}}$ at $\epsilon = 0$, $\epsilon = 20 \mu\text{eV} \ (\sim 4.8 \text{ GHz})$ and $\epsilon = 40 \mu\text{eV} \ (\sim 9.7 \text{ GHz})$, with $2t_c/h = 7.4 \text{ GHz}$. Insets show $A/A_0$ as a function of $f$ at values of $B_{z}^{\text{ext}}$ indicated by the white dashed lines. (b) Spin-photon coupling rate $g_s/2\pi$ and spin decoherence rate $\gamma_s/2\pi$ as functions of $2t_c/h$, with $\epsilon = 0$. The dashed lines show theory predictions. (c) DQD energy levels as a function of $\epsilon$, calculated with $B_{z}^{\text{ext}} + B_{z}^{\text{M}} = 209 \text{ mT}$, $B_{z}^{\text{M}} = 15 \text{ mT}$ and $2t_c/h = 7.4 \text{ GHz}$. Here $B_{z}^{\text{M}}$ denotes the magnetic field produced by the Co magnet that is parallel to $B_{z}^{\text{ext}}$, and $B_{z}^{\text{M}}$ is related to the strength of the inhomogeneous magnetic field perpendicular to $B_{z}^{\text{ext}}$. 

79
by considering that at $\epsilon = 0$ the electron is delocalized across the DQD and forms molecular bonding(anti-bonding) charge states $|\sim\rangle(|+\rangle)$ [Fig. 5.3(c)]. In this regime, the cavity electric field leads to a displacement of the electron wavefunction on the order of $\sim 1$ nm [29]. Consequently, the electron spin experiences a large oscillating magnetic field, resulting in a substantial spin-photon coupling rate. In contrast, with $|\epsilon| \gg t_C$, the electron is localized within one dot and it is natural to work with a basis of localized electronic wavefunctions $|L\rangle(|R\rangle)$ where $L(R)$ corresponds to the electron being in the left(right) dot [Fig. 5.3(c)]. In this effectively single-dot regime, the displacement of the electron wavefunction by the cavity electric field is estimated to be of order $\sim 3$ pm for a single-dot orbital energy $E_{\text{orb}} = 2.5$ meV [76], greatly suppressing the spin-photon coupling mechanism [29]. The large difference in the effective displacement lengths between the single-dot and double-dot regimes also implies an approximate two orders of magnitude improvement in the spin-photon coupling rate at $\epsilon = 0$ compared to $|\epsilon| \gg t_C$. Alternatively, the reduction of $g_s$ may be interpreted as a result of suppressed hybridization between the $|\sim, \uparrow\rangle$ and $|+\rangle, \downarrow\rangle$ states due to their growing energy difference at larger $|\epsilon|$, as evident from Fig. 5.3(c) (see discussion below). These measurements highlight the important role of charge hybridization in the DQD.

Additional electric control of $g_s$ is enabled by voltage-tuning $t_C$ [Fig. 5.1(f)]. Figure 5.3(b) shows $g_s/2\pi$ and $\gamma_s/2\pi$ as a function of $2t_C/h$ at $\epsilon = 0$, as extracted from vacuum Rabi splitting measurements and microwave spectroscopy of the electron spin resonance (ESR) transition linewidth. Both rates rapidly increase as $2t_C/h$ approaches the Larmor precession frequency $E_Z/h \approx 5.8$ GHz, and a spin-photon coupling rate as high as $g_s/2\pi = 11.0$ MHz is found at $2t_C/h = 5.2$ GHz. These trends are consistent with the DQD energy level spectrum shown in Fig. 5.3(c) [29, 31, 32]. Here $\uparrow (\downarrow)$ denotes an electron spin that is aligned(anti-aligned) with $B_{\text{ext}}^z$. With $2t_C/h \gg E_Z/h$ and $\epsilon = 0$, the two lowest energy levels are $|\sim, \downarrow\rangle$ and $|\sim, \uparrow\rangle$ and
the electric-dipole coupling to the cavity field is small. As $2t_C$ is reduced and made comparable to $E_Z$, the ground state remains $|-, \downarrow\rangle$ but the excited state becomes an admixture of $|-, \uparrow\rangle$ and $|+, \downarrow\rangle$ due to the magnetic field gradient $B_{x,R}^M - B_{x,L}^M = 2B_x^M$ and the small energy difference between the states. The quantum transition that is close to resonance with $E_Z$ is now partially composed of a change in charge state from $-$ to $+$, which responds strongly to the cavity electric field and gives rise to larger values of $g_s$. For $2t_C/h < E_Z/h$, a decrease in $t_C$ increases the energy difference between $|-, \uparrow\rangle$ and $|+, \downarrow\rangle$ which reduces their hybridization and results in a smaller $g_s$.

We note that hybridization with charge states increases the susceptibility of the spin to charge noise and relaxation, resulting in an effective spin decoherence rate $\gamma_s$ that is a strong function of $t_C$ as well [see Fig. 5.3(b)] [29, 31, 32]. Theoretical predictions of $g_s$ and $\gamma_s$ as a function of $2t_C/h$, based on measured values of $g_c$ and $\gamma_c$ [Fig. 5.1(e)], are in good agreement with the data [Fig. 5.3(b)] (see Appendix A for a description of the theory) [31]. The electric control of spin-photon coupling demonstrated here allows the spin qubit to quickly switch between regimes with strong coupling to the cavity, and idle regimes where the spin-photon coupling rate and susceptibility to charge decoherence are small.

5.4 Quantum Control and Dispersive Readout of a Single Spin

The preceding measurements demonstrate the ability to coherently couple a single electron spin to a single photon, potentially enabling long-range spin-spin couplings [10, 12]. For the device to serve as a building block of a quantum processor, it is also necessary to deterministically prepare, control, and read out the spin state of the trapped electron. We first induce spin transitions by driving gate P1 with a continuous microwave tone of frequency $f_s$ and power $P_s = -106$ dBm. When $f_s \approx E_Z/h$,
Figure 5.4: (a) Cavity phase response $\Delta \phi$ at $f = f_c$ when gate P1 is continuously driven at a variable frequency $f_s$ and power $P_s = -106$ dBm. A background phase response, obtained by measuring $\Delta \phi(B_{z}^{ext})$ in the absence of a microwave drive on P1, is subtracted from each column of the data to correct for slow drifts in the microwave phase. (b) Electron spin resonance (ESR) line as measured in $\Delta \phi(f_s)$ at $2t_C/h = 11.1$ GHz, $\epsilon = 0$, $B_{z}^{ext} = 92.18$ mT and $P_s = -123$ dBm. Dashed line shows a fit to a Lorentzian with full-width-at-half-maximum (FWHM) $\gamma_s/\pi = 0.81 \pm 0.04$ MHz. (c) Schematic showing the experimental sequence for coherent spin control and measurement. (d) $\Delta \phi$ as a function of $\tau_B$, showing single-spin Rabi oscillations. The excited state population of the spin qubit $P^\uparrow$ is indicated on the right y-axis. Solid line is a guide to the eye.
the excited state population of the spin qubit $P_\uparrow$ increases and the ground state population $P_\downarrow$ decreases. In the dispersive regime, where the qubit-cavity detuning $\Delta/2\pi \approx E_Z/h - f_c$ satisfies $|\Delta/2\pi| \gg g_s/2\pi$, the cavity transmission experiences a phase response $\Delta \phi \approx \tan^{-1}(2g_s^2/\kappa \Delta)$ for a fully saturated ($P_\uparrow = 0.5$) qubit \cite{101, 102}. It is therefore possible to measure the spin state of a single electron by probing the cavity transmission. As a demonstration, we spectroscopically probe the ESR transition by measuring $\Delta \phi$ as a function of $f_s$ and $B_{ext}^z$ in Fig. 5.4(a). These data are acquired with $2t_C/h = 9.5$ GHz and $\epsilon = 0$. The ESR transition is clearly visible as a narrow feature with $\Delta \phi \neq 0$ that shifts to higher $f_s$ with increasing $B_{ext}^z$. $\Delta \phi$ also changes sign as $B_{ext}^z$ increases, consistent with the sign change of the qubit-cavity detuning $\Delta$ when the Larmor precession frequency $E_Z/h$ exceeds $f_c$. The nonlinear response in the small region around $B_{ext}^z = 92$ mT is due to the breakdown of the dispersive condition $|\Delta/2\pi| \gg g_s/2\pi$.

Next, we demonstrate coherent single-spin control and dispersive spin state readout. For these measurements, we choose $\epsilon = 0$ and $2t_C/h = 11.1$ GHz to minimize the spin decoherence rate $\gamma_s$ [Fig. 5.4(b)]. Here the spin-photon coupling rate $g_s/2\pi = 1.4$ MHz [Fig. 5.3(b)]. The external field is fixed at $B_{ext}^z = 92.18$ mT, ensuring the system is in the dispersive regime with $\Delta/2\pi = 14$ MHz $\gg g_s/2\pi$. A measurement of $\Delta \phi(f_s)$ in the low power limit [Fig. 5.4(b)] yields a Lorentzian line shape with a full-width-at-half-maximum of 0.81 MHz, corresponding to a low spin decoherence rate $\gamma_s/2\pi = 0.41$ MHz \cite{101, 102}. Qubit control and measurement are achieved using the pulse sequence illustrated in Fig. 5.4(c): Starting with a spin-down state $|\downarrow\rangle$ at $\epsilon = 0$, the DQD is pulsed to a large detuning $\epsilon' = 70 \mu$eV ($\sim 17$ GHz) which decouples the spin from the cavity. A microwave burst with frequency $f_s = 5.874$ GHz, power $P_s = -76$ dBm, and duration $\tau_B$ is subsequently applied to P1 to drive a spin rotation \cite{86, 115, 126}. The DQD is then adiabatically pulsed back to $\epsilon = 0$ for a fixed measurement time $T_M$ for dispersive readout. To reinitialize the qubit, we
Figure 5.5: Time-averaged cavity phase response $\Delta \phi(f_s)$ as a function of wait time $T_M$, measured using the pulse sequence illustrated in Fig. 5.4(c). The microwave burst time is fixed at $\tau_B = 80$ ns. Dashed line shows a fit to the function $\phi_0 + \phi_1(T_1/T_M)(1 - \exp(-T_M/T_1))$, yielding a spin relaxation time $T_1 \approx 3.2 \mu s$. The experimental conditions are the same as Fig. 5.4(d).

choose $T_M = 20 \mu s \gg T_1(\epsilon = 0)$, where $T_1(\epsilon = 0) = 3.2 \mu s$ is the spin relaxation time measured at $\epsilon = 0$. Figure 5.4(d) displays the time-averaged $\Delta \phi$ as a function of $\tau_B$, obtained with an integration time of 100 ms for each data point. We observe coherent single spin Rabi oscillations with a Rabi frequency $f_R = 6$ MHz. In contrast with readout approaches that rely on spin-dependent tunneling [115, 126, 128], our dispersive cavity-based readout is in principle quantum non-demolition [8]. The readout scheme is also distinct from a previous work employing a cavity-coupled InAs DQD, which detects spin state through Pauli blockade rather than spin-photon coupling [86]. In addition to enabling single spin-photon coupling, our device is capable of preparing, controlling, and dispersively reading out single spins.

The conversion of $\Delta \phi$ to the excited state population of the spin qubit $P_\uparrow$ is done by first considering the theoretically expected cavity phase response, which
is $\phi_\uparrow = \tan^{-1}(2g_s^2/\kappa_\Delta) = 9.6^\circ$ when the spin qubit is in the excited state, and $\phi_\downarrow = -\tan^{-1}(2g_s^2/\kappa_\Delta) = -9.6^\circ$ when the spin qubit is in the ground state [9, 102]. Since our measurement is averaged over $T_M \gg T_1$, spin relaxation during readout will reduce the phase contrast observed in the experiment. We then measure the spin relaxation time $T_1$ by fixing the microwave burst time at $\tau_B = 80$ ns, which corresponds to a $\pi$-pulse on the spin qubit. The cavity phase response $\Delta \phi$ is then measured as a function of $T_M$ for $T_M > 5 \mu s > T_1$ [Fig. 5.5]. The result is fit to a function of the form $\Delta \phi = \phi_0 + \phi_1(T_1/T_M)[1 - \exp (-T_M/T_1)]$ to extract $T_1 = 3.2 \mu s$, where $\phi_0$, $\phi_1$ are additional fitting parameters [86]. We have ignored the effects of the cavity ringdown time $1/\kappa \approx 90$ ns and the $\pi$-pulse time of 80 ns from the fit, since both of these times are much shorter than the measurement time $T_M$. The phase contrast resulting from the fit, $\phi_1 \approx 17.7^\circ$, is close to the maximum contrast expected at this spin-photon detuning, $\phi_\uparrow - \phi_\downarrow = 19.2^\circ$. Based on this value of $T_1$, we convert the measured phase response into the excited state population via $P_\uparrow = (1/2)(1 + \Delta \phi/\phi_\uparrow)$, where $\phi_\uparrow = \phi_\uparrow(T_1/T_M)[1 - \exp (-T_M/T_1)] = 1.5^\circ$ is the reduced phase response due to spin relaxation during the readout time $T_M = 20 \mu s$. The converted spin population $P_\uparrow$ shown in Fig. 5.4(d) has a visibility of $\sim 70\%$, which may be improved by performing single shot measurements in the future [9].

### 5.5 AC-Stark Shift

The narrow linewidth $\gamma_s/\pi$ of the single-spin qubit also allows the number of intracavity photons, $n_{ph}$, to be resolved accurately via the AC-stark shift effect. Fig. 5.6 shows the center frequency of the ESR signal $f_{\text{ESR}}$ (measured by $\Delta \phi$) as a function of the cavity drive power $P$. It is observed that the $f_{\text{ESR}}$ shifts to lower values as $P$ increases. This is due to AC-stark shift which changes the qubit frequency by $(2n_{ph}g_s^2/\Delta)/2\pi$ for every additional photon in the cavity [102]. Through a linear fit
Figure 5.6: ESR resonance frequency $f_{\text{ESR}}$ measured using the cavity phase response $\Delta \phi$ in the dispersive regime, plotted as a function of the estimated power at the input port of the cavity $P$. The device is configured with $g_s/2\pi = 2.4$ MHz and spin-photon detuning $\Delta/2\pi \approx -18$ MHz. Dashed line shows a fit to $f_{\text{ESR}} = f_{\text{ESR}}(P = 0) + (2n_{\text{ph}}g_s^2/\Delta)/2\pi$, where $n_{\text{ph}}$ is the average number of photons in the cavity and plotted as the top x-axis. The experiments are conducted with $P \approx -133$ dBm (0.05 fW), which corresponds to $n_{\text{ph}} \approx 0.6$.

of the data to a functional form $f_{\text{ESR}} = f_{\text{ESR}}(P = 0) + (2n_{\text{ph}}g_s^2/\Delta)/2\pi$, the photon number $n_{\text{ph}}$ is determined to high accuracy.

5.6 Prospect for Long-Range Two-Qubit Coupling

The coherent spin-photon interface may be readily applied to enable spin-spin coupling across the cavity bus. Here we evaluate two possible schemes for implementing such a coupling, both of which have been demonstrated with superconducting qubits [10] [12]. The first approach uses direct photon exchange to perform quantum state transfer between two qubits [10]. The transfer protocol starts by tuning qubit 1 into
resonance with the unpopulated cavity for a time $1/4g_s$, at the end of which the state of qubit 1 is completely transferred to the cavity. Qubit 1 is then rapidly detuned from the cavity and qubit 2 is brought into resonance with the cavity for a time $1/4g_s$, at the end of which the state of qubit 1 is completely transferred to qubit 2. Therefore, the time required for a quantum state transfer across the cavity is $1/2g_s$. Since the decay of vacuum Rabi oscillations occurs at a rate $\kappa/2 + \gamma_s$, the threshold for coherent state transfer between two spin qubits is $2g_s/(\kappa/2 + \gamma_s) > 1$. The ratio $2g_s/(\kappa/2 + \gamma_s)$ is plotted as a function of $2t_C/h$ in Fig. 5.7(a). It is seen that $2g_s/(\kappa/2 + \gamma_s) > 1$ for all values of $2t_C/h$, indicating that spin-spin coupling via real photon exchange is achievable and may be implemented at any value of $t_C$. For our present device parameters, the regime $2t_C/h \approx 6$ GHz, where spin-charge hybridization is large and the ratio $2g_s/(\kappa/2 + \gamma_s)$ reaches a maximum of 3.5, seems more advantageous for such a coupling scheme.

The second approach to spin-spin coupling uses virtual photon exchange [12]. In this scheme, both spin qubits would operate in the dispersive regime, with an effective coupling rate $J = g_s^2(1/\Delta_1+1/\Delta_2)/2$, where $\Delta_1$ and $\Delta_2$ are the qubit-cavity detunings.
for qubit 1 and qubit 2, respectively. Assuming both qubits operate with an equal detuning $\Delta_{1,2} = 10g_s$ to minimize Purcell decay, $J = g_s/10$. For coherent spin-spin interaction, $J > \gamma_s$ needs to be satisfied, leading to the condition $g_s/\gamma_s > 10$. In Fig. 5.7(a), we plot the ratio $g_s/\gamma_s$ as a function of $2t_C/h$, observing a maximum of $g_s/\gamma_s \approx 4$ at $2t_C/h \approx 10$ GHz. Since the dominant spin mechanism is likely hyperfine-induced dephasing by the $^{29}$Si nuclei in this regime (the decoherence rate $\gamma_s/2\pi \approx 0.4$ MHz is close to the decoherence rates commonly found with single-spin qubits in natural-Si [126]), transitioning to isotopically purified $^{28}$Si host materials is likely to lead to an order of magnitude reduction in $\gamma_s/2\pi$, as demonstrated recently [17]. Such an improvement will allow virtual-photon-mediated spin-spin coupling to be implemented in our device architecture as well.

Lastly, we note that both coupling approaches will benefit significantly from larger values of the charge-photon coupling rate $g_c$, which is achievable through the development of higher impedance cavities [108, 129]. The present superconducting cavity is estimated to have an impedance between 200 $\Omega$ and 300 $\Omega$. Increasing this value to $\sim 2$ k$\Omega$, possible through using NbTiN as the superconducting material, will lead to another factor of three increase in $g_c$ and therefore $g_s$. This may allow the $g_s/\gamma_s > 100$ regime to be accessed, where high-fidelity two-qubit gates can be implemented between distant spins. Improvements in the fidelity of cavity-mediated two-qubit gates, particularly in the case of real photon exchange, can also be sought through improving the quality factor of the cavity (thereby reducing $\kappa$). This is achievable through implementing stronger gate line filters [98] and the removal of lossy dielectrics such as atomic-layer-deposited Al$_2$O$_3$ underneath the cavity.
5.7 Conclusion

In conclusion, we have realized a coherent spin-photon interface where a single spin in a Si DQD is strongly coupled to a microwave photon through the combined effects of the electric-dipole interaction and spin-charge hybridization. Spin-photon coupling rates up to 11 MHz are measured in the device, exceeding magnetic-dipole coupling rates by nearly five orders of magnitude. The spin decoherence rate is strongly dependent on the interdot tunnel coupling $t_C$ and ranges from 0.4 – 6 MHz, possibly limited by a combination of charge noise, charge relaxation and remnant nuclear field fluctuations. All-electric control of spin-photon coupling and coherent manipulation of the spin state are demonstrated, along with dispersive readout of the single spin which lays the foundation for quantum non-demolition readout of spin qubits. These results may enable the construction of an ultra-coherent spin quantum computer having photonic interconnects and readout channels, with capacity for surface codes, “all-to-all” connectivity, and easy integration with other solid-state quantum systems such as superconducting qubits \cite{8, 10, 12, 130, 132}. 
Chapter 6

High-Resolution Valley Spectroscopy of Silicon Quantum Dots

Since spin qubits are often indirectly manipulated through electrical means to achieve fast control [14, 43], a precise knowledge of the other quantum degrees of freedom governing QD electrons is of critical importance for improving the quantum gate fidelities. In III-V semiconductors such as GaAs, these relevant quantum degrees of freedom include charge and orbital states, which are reproducibly defined by QD lithographic dimensions and may be spectroscopically probed through a variety of techniques, such as photon assisted tunneling and pulsed-gate spectroscopy [133, 134].

In silicon, electrons possess an additional quantum degree of freedom. The conduction band of bulk Si has six degenerate minima (termed valleys) [135]. In Si/SiGe heterostructures, the four in-plane valleys are raised in energy compared to the two out-of-plane valleys through the strain in the Si quantum well [58, 114]. The relatively small energy splitting between the two low-lying valley states has been observed to contribute to spin relaxation [18, 126], but may potentially also be harnessed to
make charge-noise-insensitive qubits [136]. This valley splitting has been found to vary substantially within the range of 35 – 270 μeV in Si/SiGe QD devices [76, 137], posing an urgent challenge to the reproducibility and scalability of spin qubits based on Si/SiGe heterostructures. A first step toward controlling valley splitting is the development of an experimental method to accurately and efficiently determine its value.

In this chapter, we demonstrate cavity-based spectroscopy of valley states in Si/SiGe DQDs using a hybrid circuit quantum electrodynamics (cQED) device architecture [98, 101]. Charge transitions involving excited valley states generate observable “fingerprints” in the stability diagram of a cavity-coupled Si/SiGe DQD, as predicted by a recent theory [138]. The occupation of the valley states can be increased by raising the device temperature or by applying a finite source-drain bias, \( V_{SD} \), across the DQD. Such a cavity-based valley detection scheme is highly efficient since it eliminates the need for a magnetic field. Our approach yields information on the valley splitting, intra- and inter-valley tunnel couplings, and is therefore an attractive alternative to conventional magnetospectroscopy and photon assisted tunneling [42, 138].

### 6.1 Cavity-Based Detection of Valley States

The hybrid Si/SiGe cQED device is shown in Fig. 6.1(a). The cavity is a half-wavelength (\( \lambda/2 \)) Nb transmission line resonator with a center frequency \( f_c = 7.796 \) GHz, loaded quality factor \( Q_c = 2480 \), and photon loss rate \( \kappa/2\pi = 3.1 \) MHz. A DQD is defined in a Si quantum well near a voltage anti-node of the cavity [Fig. 6.1(c)]. Three overlapping layers of Al gates are patterned on top of an undoped Si/SiGe heterostructure to achieve tight electronic confinement, similar to device used in Chapter 3. The Si/SiGe heterostructure consists of a 4 nm thick Si cap, a 50 nm thick Si\(_{0.7}\)Ge\(_{0.3}\)
Figure 6.1: (a) Optical image of the device. (b) Schematic representation of the experiment and the DQD energy levels. $E_L$ and $E_R$ denote the valley splittings in the left and right dots, and $\epsilon$ is the interdot level detuning. (c) Tilted angle false-colored scanning electron microscope image of the DQD [red outline in (a)]. (d) DQD charge stability diagram acquired by measuring the cavity transmission amplitude $A/A_0$ as a function of the plunger gate voltages $V_{LP}$ and $V_{RP}$, with fixed interdot barrier gate voltage $V_{MB} = 150$ mV.

spacer layer, a 8 nm thick Si quantum well, and a 225 nm thick Si$_{0.7}$Ge$_{0.3}$ layer grown on top of a Si$_{1-x}$Ge$_x$ relaxed buffer substrate.

Figure 6.1(b) shows a schematic representation of the equilibrium configuration of the DQD energy levels. In contrast with previous III/V semiconductor DQD-cQED devices, where two charge states interact with the cavity photons, a total of four charge states are involved in the charge-cavity interaction in this work due to the presence of valley states in Si [86, 88]. Electric dipole coupling between DQD electrons and cavity photons is maximized by connecting gate RP to the cavity center pin [88, 98].

Readout of the DQD charge states is performed in a dilution refrigerator (with base lattice temperature $T_{lat} = 10$ mK) by driving the cavity with a coherent microwave
tone at frequency $f = f_c$ and power $P_m \approx -128$ dBm (the average intra-cavity photon number $n_c \approx 1$). A small probe power is chosen so as to not perturb the DQD energy levels from thermal equilibrium. The cavity output field is amplified and demodulated to yield the normalized transmission amplitude $A/A_0$ and phase $\Delta \phi$ response. Figure 6.1(d) shows $A/A_0$ as a function of plunger gate voltages $V_{LP}$ and $V_{RP}$, revealing a few-electron DQD charge stability diagram. Charge stability islands are labeled with $(N_L, N_R)$, with $N_L$ and $N_R$ being the total number of electrons in the left dot and the right dot.

We now focus on the $(1, 0) \leftrightarrow (0, 1)$ interdot charge transition. Figure 6.2(a) shows $A/A_0$ as a function of $V_{LP}$ and $V_{RP}$, with $V_{MB} = 323$ mV. A clear reduction in $A/A_0$, to a minimum value of $\sim 0.8$ (green arrows), is seen along the interdot charge transition where $\epsilon = 0$. Parallel to this central minimum, two additional minima in $A/A_0$ are also visible (red and blue arrows). The observed cavity response is strikingly different from previously reported devices, where $A/A_0$ exhibited either a single minimum at $\epsilon = 0$ for $2t_c/h > f_c$, or two minima with similar depths at values of $\epsilon$ where $\sqrt{\epsilon^2 + 4t_c^2}/h = f_c$ when $2t_c/h < f_c$ [86–88, 101, 106]. Here $t_c$ is the interdot tunnel coupling. These additional features suggest the presence of higher-lying avoided crossings in the DQD energy level diagram that lead to a non-zero electric susceptibility at finite values of $\epsilon$.

A qualitative understanding of the data can be obtained considering the full DQD energy diagram shown in Fig. 6.2(b) [138]. Here the DQD is modeled as a four-level system consisting of the left dot ground state $|L\rangle = |(1, 0)\rangle$, left dot excited state $|L'\rangle = |(1', 0)\rangle$, right dot ground state $|R\rangle = |(0, 1)\rangle$ and right dot excited state $|R'\rangle = |(0, 1')\rangle$. For large detuning $|\epsilon| > 100$ $\mu$eV, valley states within the same dot are separated by the respective valley splitting, $E_L$ and $E_R$. For small detuning $|\epsilon| < 100$ $\mu$eV, the four states are hybridized by the intra-valley tunnel coupling $t$ ($t$ is equivalent to the interdot tunnel coupling $t_c$ in single valley systems, such as
InAs) and the inter-valley tunnel coupling $t'$, giving rise to a total of four avoided crossings [138]. The strong minimum in $A/A_0$ at $\epsilon = 0$ is predominantly due to the avoided crossing involving the DQD ground states $|L\rangle$ and $|R\rangle$ (green arrows), similar to previous work [86, 88, 98, 101, 106]. The two minima in $A/A_0$ at $\epsilon \neq 0$ are due to the avoided crossings involving states $|L\rangle - |R'\rangle$ (red arrows) and $|L'\rangle - |R\rangle$ (blue arrows), located at $\epsilon \approx \pm 50 \mu eV$. The lower visibility of these two minima arises from the smaller thermal population of the excited states. The $|L'\rangle - |R'\rangle$ avoided crossing is expected to have no appreciable contribution to the cavity response due to the negligible population of the two highest-lying states. Moreover, for $E_L \approx E_R$, the $|L'\rangle - |R'\rangle$ avoided crossing occurs near $\epsilon = 0$ and its response would be masked by the $|L\rangle - |R\rangle$ avoided crossing. The temperature dependence of $A/A_0$ will be examined in more detail in the next section.

A more quantitative data set is obtained by measuring $A/A_0$ as a function of $\epsilon$ for several values of $V_{MB}$, which primarily tunes $t$ and $t'$ [Fig. 6.2(c)]. With $V_{MB} = 327$ mV, the “side minima” have depths comparable to the central minimum at $\epsilon = 0$. When $V_{MB}$ is lowered to 323 mV, the central minimum becomes deeper, whereas the side minima remain relatively unchanged. As $V_{MB}$ is further lowered to 318 mV, the central minimum is split into two minima due to the fact that the $|L\rangle - |R\rangle$ transition frequency is now equal to $f_c$ at both $\epsilon = 7 \mu eV$ and $\epsilon = -7 \mu eV$ [86, 88, 101].

The theory developed in Ref. [138] is used to analyze these data. Starting from the four-level system shown in Fig. 6.2(b), the electric susceptibility $\chi$ is calculated as a function of $\epsilon$, and used to predict the cavity response $A/A_0$ with cavity input-output theory [86]. We simultaneously fit the three data sets shown in Fig. 6.2(c) to theory, assuming that changes in $V_{MB}$ modify $t$ and $t'$ but leave $E_L$ and $E_R$ unchanged [138]. The fits to the data are plotted as black lines in Fig. 6.2(c) and are in excellent agreement with the experimental data with best fit valley splittings $E_L = E_R = 51 \pm 3 \mu eV$, similar to values found in previous devices [76]. The uncertainties for the valley
Figure 6.2: (a) Cavity transmission amplitude $A/A_0$ plotted as a function of $V_{LP}$ and $V_{RP}$ near the $(1, 0) \leftrightarrow (0, 1)$ interdot charge transition with $T_{\text{lat}} = 10 \text{ mK}$. The black arrow denotes the detuning parameter $\epsilon$. (b) DQD energy level diagram with $E_L = E_R = 51 \mu\text{eV}$, $t = 20.9 \mu\text{eV}$ and $t' = 15.0 \mu\text{eV}$. The grey dashed lines show the uncoupled energy levels ($t = t' = 0$). (c) $A/A_0$ measured as a function of $\epsilon$ for three values of $V_{MB}$, and fits to theory (black lines).
splittings, estimated based on the linewidths of the side minima, are significantly smaller than those found through magnetospectroscopy, which are typically on the order of 10 \( \mu \text{eV} \) [76] [137]. The equal valley splitting of the DQD is intentionally achieved through device tuning, and data showing \( E_L \neq E_R \) are included in the last section of this chapter. Best fit values of \( t \) and \( t' \) are listed in Fig. 6.2(c), which both decrease at lower \( V_{MB} \) and show an approximately constant ratio of \( t/t' = 1.4 \) [138].

Other inputs to the theory are the charge-cavity coupling rate \( g_0/2\pi = 19 \text{ MHz} \), total charge decoherence rate \( \gamma/2\pi = 30 \text{ MHz} \), and the electron temperature \( T_e = 135 \text{ mK} \) (at the cryostat base temperature \( T_{\text{lat}} = 10 \text{ mK} \)). Low frequency charge noise is accounted for in the model by smoothing \( A/A_0 \) using a Gaussian with standard deviation \( \sigma_\epsilon = 1.5 \mu \text{eV} \) [86]. Fits to the cavity phase response (not shown) yield similar results. We also note that the excited states observed here are unlikely to be orbital excited states (orbital energies are around 3 meV in these devices [76]), or spin states due to the single electron occupancy of the DQD and zero external magnetic field.

### 6.2 High-Temperature Enhancement of the Valley State Visibility

Since only occupied electronic states will contribute to the electric susceptibility \( \chi \), the visibility of the valley-induced features is likely limited by the relatively low electron temperature \( k_B T_e \ll E_L \approx E_R \), where \( k_B \) is Boltzmann’s constant [138]. This visibility may therefore be improved by raising the temperature of the DQD. Figure 6.3(a) shows the DQD stability diagram taken at \( T_{\text{lat}} = 250 \text{ mK} \). In comparison with Fig. 6.2(a), the side minima (red and blue arrows) are notably more visible relative to the central minimum. All three minima are also broader. These observations are expected since by raising temperature, the thermal population of the DQD ex-
cited states is increased and the ground state population is decreased. This leads to a smaller $\chi$ for the $|L\rangle - |R\rangle$ avoided crossing, but a larger $\chi$ for the $|L'\rangle - |R\rangle$ and $|L\rangle - |R'\rangle$ avoided crossings [138, 139]. As a result, the difference between the depths of the side minima and the central minimum in $A/A_0$ is reduced. On the other hand, background charge noise is also expected to increase at higher temperatures, leading to broadening of the observed features [140].

Close agreement between theory and experiment is demonstrated by the plots in Fig. 6.3(b), where we show $A/A_0$ as a function of $\epsilon$ for $T_{\text{lat}} = 10, 250, \text{and } 400 \text{ mK}$. The temperature dependence of the cavity response is theoretically modeled by taking into account the Fermi-Dirac distribution of the electrons in the leads and the Bose-Einstein distribution of the phonon bath, which are assumed to be in equilibrium with the base temperature of the cryostat $T_{\text{lat}}$, except for $T_{\text{lat}} = 10 \text{ mK}$ where the best fit is found with an electron temperature $T_e = 135 \text{ mK}$. The temperature-dependent charge noise used in the model is $\sigma_\epsilon = 1.5 \mu\text{eV}$ for $T_{\text{lat}} = 10 \text{ mK}$, $\sigma_\epsilon = 5 \mu\text{eV}$ for

---

**Figure 6.3:** (a) $A/A_0$ plotted as a function of $V_{\text{LP}}$ and $V_{\text{RP}}$ with $V_{\text{MB}} = 323 \text{ mV}$ and $T_{\text{lat}} = 250 \text{ mK}$. (b) $A/A_0$ as a function of $\epsilon$ for three values of $T_{\text{lat}}$, with $V_{\text{MB}} = 323 \text{ mV}$. Black solid lines are theory predictions. Data and fit for $T_{\text{lat}} = 10 \text{ mK}$ [Fig. 6.2(c)] are reproduced here for direct comparison.
$T_{\text{lat}} = 250 \text{ mK}$ and $\sigma_\epsilon = 9 \ \mu\text{eV}$ for $T_{\text{lat}} = 400 \ \text{mK}$. The charge decoherence rate $\gamma/2\pi$ has little impact on theoretical predictions in Fig. 6.3(b) and is fixed at 30 MHz. In comparison with the data at $T_{\text{lat}} = 10 \ \text{mK}$, we conclude that raising device temperature is an effective method for improving the visibility of the side minima associated with excited valley states.

### 6.3 Valley State Visibility under Finite Source-Drain Bias

To improve the visibility of the valley states without increasing temperature, we may repopulate the valley states via a finite source-drain bias $V_{\text{SD}}$. The lower inset of Fig. 6.4(a) shows the current through the DQD, $I$, as a function of $V_{\text{LP}}$ and $V_{\text{RP}}$ with $V_{\text{SD}} = 0.3 \ \text{mV}$. These data exhibit characteristic finite bias triangles (FBT), as expected for charge transport in a DQD [35]. The main panel of Fig. 6.4(a) shows $A/A_0$ measured over the same range of gate voltages, which varies appreciably along the directions defined by the blue, green and red arrows. In the region between the FBTs, sequential single electron transport is forbidden due to Coulomb blockade. Here $A/A_0$ as a function of $\epsilon$ is little changed from the $V_{\text{SD}} = 0$ data [see Fig. 6.2(a)]. Within each FBT, electronic transport leads to non-equilibrium populations of the DQD electronic states and $A/A_0$ is strongly altered [87]. To give a specific example, we observe an enhancement of the absolute visibility of the $|L'\rangle - |R\rangle$ avoided crossing (blue arrow) in the lower FBT [Fig. 6.4(a)].

We can qualitatively understand the nonequilibrium cavity response by considering the transport process within the lower FBT. At $\epsilon \approx -50 \ \mu\text{eV}$ [Fig. 6.4(b)], the relevant states involved are labeled in the figure, with $(|L'\rangle \pm |R\rangle)/\sqrt{2}$ being hybridized valley-orbit states extended over the two dots. The charge transport cycle starts with the $|0,0\rangle$ state, from which an electron tunnels from the right reservoir at
Figure 6.4: (a) $A/A_0$ measured with $V_{MB} = 323$ mV and $V_{SD} = 0.3$ mV. Upper inset: Theory. Lower inset: Measured DQD current, $I$. (b, c) DQD energy level diagrams at $V_{RP} = 622$ mV for (b) $\epsilon \approx -50$ µeV and (c) $\epsilon \approx 50$ µeV. (d, e) $A/A_0$ measured at (d) $V_{RP} = 624$ mV and (e) $V_{RP} = 622$ mV. Black solid lines are fits to theory.
a rate $\Gamma_R$ into one of the three states available in the right dot. Relaxation processes within the DQD (black curly arrows labeled with rate $\gamma$) lead to one of the three states available in the left dot, from which the electron may tunnel with a rate $\Gamma_L$ onto the left lead. This transport cycle results in an increased population difference between the hybridized valley-orbit states compared to equilibrium and thus a high visibility of the $|L'\rangle - |R\rangle$ avoided crossing. In contrast, at the opposite detuning $[\epsilon \approx 50 \mu eV$, see Fig. 6.4(c)], an electron may relax into state $|R\rangle$ where it will remain stuck. Consequently, the population difference between the hybridized valley-orbit states $(|L\rangle \pm |R'\rangle)/\sqrt{2}$ is not increased and no enhancement in the visibility of the $|L\rangle - |R'\rangle$ avoided crossing is observed.

To quantitatively model the nonequilibrium cavity response, we solve for the steady state occupation probability $\rho_k$ of each DQD eigenstate $|k\rangle$ (see Appendix B for details). Using a Lindblad master equation approach [141], we derive a set of rate equations for $\rho_k$: $0 = \dot{\rho}_k = \sum_{j \neq k} (\Gamma_{kj}\rho_j - \Gamma_{jk}\rho_k) + \sum_{j \neq k} (\tau_{kj}\rho_j - \tau_{jk}\rho_k)$. Here $\Gamma_{jk} = \sum_{v=\pm,l=L,R} \Gamma_l \left(|\langle j|c_{l,v}|k\rangle|^2 + |\langle k|c_{l,v}|j\rangle|^2\right) n_{jk}^{(l)}$ denotes the transition rate from state $k$ to state $j$ due to an electron tunneling on or off the DQD, with $\Gamma_l$ being the tunneling rate to lead $l$, $c_{l,v}$ the annihilation operator for electrons in dot $l$ and valley $v$ and the $n_{jk}^{(l)}$ factors account for the finite temperature in the source-drain leads. The sum $\sum_{j \neq k} (\tau_{kj}\rho_j - \tau_{jk}\rho_k)$ describes decay processes between states with the same total number of electrons, where the decay rate $\tau_{jk} \sim \gamma$ for states with one electron in the DQD. The resulting values of $\rho_k$ are then used to calculate $A/A_0$ via cavity input-output theory [138].

The experimental data in Figs. 6.4(d, e) are fit to theory, yielding best fit tunneling rates $\Gamma_L = 62$ MHz, $\Gamma_R = 132$ MHz and $\gamma = 188$ MHz. In the upper inset of Fig. 6.4(a), we have calculated $A/A_0$ over the same gate voltage range as the data using these parameters and a capacitance matrix based model of the DQD [133]. The discrepancy between data and theory at $\epsilon = -50 \mu eV$ may be due to the simplicity
of the theoretical model, which takes the same decay rate $\gamma$ for all transitions and assumes energy independent tunneling rates $\Gamma_L$ and $\Gamma_R$. We also note that two factors in the DQD tunneling rates facilitate the improved visibility of the $|L\rangle$ - $|R\rangle$ avoided crossing: First, $\gamma$ is comparable to $\Gamma_L$ and $\Gamma_R$, ensuring the populations of the $|L\rangle$ and $|R\rangle$ states neither relax to thermal equilibrium due to slow loading/unloading from the leads, or become nearly equal due to fast loading/unloading. Second, the loading rate $\Gamma_R$ is larger than $\Gamma_L$ such that the DQD spends negligible time in the empty state $|0\rangle$, which has zero charge susceptibility.

Finally, we find an enhancement in the absolute visibility of the $|L\rangle$ - $|R'\rangle$ avoided crossing (red arrows) in the upper FBT with $V_{SD} = -0.3$ mV. Figure 6.5(a) shows the cavity transmission amplitude when a negative source-drain bias $V_{SD} = -0.3$ mV is applied to the device of the main text. Similar to the forward bias case $V_{SD} = 0.3$ mV
[Fig. 6.4(a)], $A/A_0$ is little changed from its thermal equilibrium behavior in the region between the FBTs (where $V_{RP} \approx 622.5$ mV), due to Coulomb blockade [Fig. 6.5(b)]. Within each FBT, electronic transport leads to nonequilibrium populations of DQD states which strongly alter $A/A_0$. Under this bias direction, we observe enhancement in the visibility of the $|L\rangle - |R'\rangle$ avoided crossing (red arrows) in the upper FBT [where $V_{RP} \approx 624.5$ mV, Fig. 6.5(b)]. By fitting to theory (a detailed description is provided in Appendix B), we obtain best fit tunneling rates $\Gamma_L = 25$ MHz and $\Gamma_R = 151$ MHz for negative bias.

6.4 Data with Unequal Valley Splittings

In this section, we show data for Si/SiGe DQD devices with unequal valley splittings, $E_L \neq E_R$. In Fig. 6.6(a), the cavity transmission amplitude $A/A_0$ near the $(1,0) \leftrightarrow (0,1)$ interdot charge transition, obtained from the device of the main text in a different cooldown, is plotted. Here the valley-induced dispersive features, marked by blue and red arrows, remain visible. Unlike Fig. 6.2(a), the two side minima are not symmetrically placed about the central (green) minimum in this case, with the blue minimum being closer to the green minimum than the red minimum. Such an asymmetry is indicative of unequal valley splittings $E_L \neq E_R$, which are determined by quantitatively fitting the detuning dependence of cavity transmission amplitude, $A/A_0(\epsilon)$ [Fig. 6.6(b)] [138]. The best fit parameters obtained are $E_L = 25.5 \mu$eV, $E_R = 58.5 \mu$eV, $t = 22.5 \mu$eV and $t' = 14.7 \mu$eV [Fig. 6.6(c)]. The other fitting parameters are $g_0/2\pi = 30$ MHz, $\gamma/2\pi = 70$ MHz, $T_e = 155$ mK and $\sigma_e = 2.2 \mu$eV.

We also observe valley signature in a separate Si/SiGe DQD device with a cavity center frequency $f_c = 7.747$ GHz and loaded quality factor $Q_c = 2379$. The cavity transmission amplitude for this device, shown in Fig. 6.6(d), displays only one additional minimum (brown arrows). By fitting to $A/A_0(\epsilon)$ [Fig. 6.6(e)], valley splittings
Figure 6.6: (a – c) Data from a different cooldown of the same device in Fig. 6.2 showing: (a) $A/A_0$ as a function of $V_{LP}$ and $V_{RP}$ near the $(1, 0) \leftrightarrow (0, 1)$ interdot charge transition with $T_{lat} = 10$ mK. (b) $A/A_0$ measured as a function of $\epsilon$, and fit to theory (black lines). (c) DQD energy level diagram calculated using the best fit parameters of (b). (d – f) Same set of data taken from a different device.
of $E_L = 21 \ \mu eV$, $E_R = 71 \ \mu eV$ and tunnel couplings of $t = 14.5 \ \mu eV$, $t' = 1.5 \ \mu eV$ are obtained for this device, with other parameters being $g_0/2\pi = 22$ MHz, $\gamma/2\pi = 150$ MHz, $T_e = 200$ mK and $\sigma_e = 2.5 \ \mu eV$. By plotting the DQD energy level diagram using these parameters in Fig. 6.6(f), it is revealed that the brown minimum in Fig. 6.6(d) in fact arises from the $|L'\rangle - |R'\rangle$ avoided crossing, which is not observed in the device of the main text due to equal valley splittings $E_L = E_R$. The large difference in the inter-valley tunnel coupling $t'$ between this device and the device of the main text may arise from different degrees of interfacial disorder at the Si quantum well [142].

6.5 Conclusion

In conclusion, we observe dispersive features in the cavity response of a hybrid Si/SiGe DQD-cQED device that arise from the valley degree of freedom in Si. The cavity response is sensitive to the valley splitting in each dot, the inter- and intra-valley tunnel couplings, and the time-averaged occupation of the levels [138]. The relative occupation of the DQD energy levels can be driven out of equilibrium by increasing temperature or applying a source-drain bias, thereby increasing the visibility of the valley states. These measurements constitute an efficient method for accurate spectroscopy of valley states in Si/SiGe heterostructures and are applicable to other Si devices, such as Si metal-oxide-semiconductor (MOS) quantum dots. Rapid and accurate measurements of the valley splitting may accelerate progress toward understanding, and ultimately controlling valley splittings in these highly relevant quantum devices.
Chapter 7

Electrically Protected Valley-Orbit States in Silicon

Environmental fluctuations are pervasive in biological and condensed matter systems [143, 144]. Electrical fluctuations, commonly referred to as charge noise, typically fall off with frequency $f$ with a spectrum that is close to $1/f$, and can induce uncontrollable evolution of quantum systems [145]. These fluctuations are often the leading source of decoherence for solid-state qubits. For example, early superconducting qubits were highly sensitive to charge noise [146, 147]. Even semiconductor spin qubits are sensitive to electrical noise, which limits the fidelity of single spin rotations in isotopically enriched $^{28}$Si [17] and two-spin gate operations based on the exchange interaction [13, 140]. It is therefore of critical importance to reduce charge noise or find ways to mitigate its impact on qubit coherence.

Developing electrically protected qubits has been a recurring theme in solid-state quantum computation, both for superconducting and semiconductor qubits. Superconducting qubits have improved performance when operated at “sweet spots” where the qubit transition energy is first-order insensitive to the level detuning [104]. Optimization of the ratio of the Josephson energy $E_J$ to the charging energy $E_c$ has led to
the development of transmon qubits that are highly insensitive to charge noise [148].

To date, approaches to mitigating the detrimental impact of charge noise on semiconductor qubits include dynamic decoupling [140], device operation at sweet spots [103, 149, 150], and hybrid qubits [151], where higher lying states in the qubit energy level spectrum lead to flat bands with an energy separation that is largely insensitive to charge fluctuations.

Here we demonstrate a DQD charge qubit that utilizes valley-orbit mixing in Si to achieve a qubit transition energy with reduced sensitivity to charge noise. The bulk conduction band of Si has six equivalent valleys. Strain in Si/SiGe heterostructures partially lifts the six-fold valley degeneracy by raising the energy of the four in-plane valleys [58]. The electric field at the quantum well interface hybridizes the two low-lying ±z-valleys, yielding a valley splitting in the range of 10–300 µeV [127, 137, 152]. In Si double quantum dots (DQDs), hybridization of orbital and valley degrees of freedom leads to valley-orbit couplings that exceed 10 µeV [127, 152]. We show here that valley-orbit coupling gives rise to hybrid valley-orbit states that have a transition frequency that only weakly depends on the DQD level detuning – an attribute that protects the valley-orbit states from charge decoherence in a manner analogous to superconducting transmon qubits [11]. These results highlight the important role the valley d.o.f. can play in the development of charge-noise-insensitive Si qubits.

7.1 Strongly Coupled DQD-cQED Device with Low-Lying Valley States

The device used in this experiment consists of a Si/SiGe DQD that is electric-dipole-coupled to a microwave cavity having a center frequency $f_c = 7.796$ GHz and photon decay rate $\kappa/2\pi = 3.3$ MHz [Figs. 7.1(a)–7.1(b)] [101, 127]. The Hamiltonian govern-
Figure 7.1: (a) Optical image of the superconducting cavity. (b) Tilted-angle false-color scanning electron microscope image of the DQD. (c) Experiment schematic: The DQD is probed by a cavity photon with energy $hf$ and interferometry is performed by periodically driving the detuning parameter $\epsilon(t)$. (d) Cavity transmission amplitude $A/A_0$ as a function of $V_{P1}$ and $V_{P2}$ near the $(1,0) \leftrightarrow (0,1)$ interdot transition. (e) Left panel: $A/A_0$ as a function of $\epsilon$ and $f$. Right panel: $A/A_0$ as a function of $f$ at $\epsilon = 0$. Dashed line is a fit to cavity input-output theory with $2g_{VO}/2\pi = 16.0$ MHz.
ing the DQD charge states and valley d.o.f. is \[138:\]

\[
H_0 = \begin{pmatrix}
\frac{\epsilon}{2} + E_L & 0 & t & t' \\
0 & \frac{\epsilon}{2} & -t' & t \\
t & -t' & -\frac{\epsilon}{2} + E_R & 0 \\
t' & t & 0 & -\frac{\epsilon}{2}
\end{pmatrix}.
\]

(7.1)

The Hamiltonian is written in a basis spanned by the left dot ground (excited) state \(|L\rangle (|L'\rangle)\) and the right dot ground (excited) state \(|R\rangle (|R'\rangle)\). Here \(E_L (E_R)\) is the valley splitting of the left (right) dot, \(t (t')\) is the intra-valley (inter-valley) tunnel coupling and \(\epsilon\) is the DQD level detuning [Fig. 7.1(c)]. We note here that the intra-valley tunnel coupling is commonly referred to as the interdot tunnel coupling \(t_c\) in single valley DQDs, such as GaAs or InAs.

Values of \(E_L, E_R, t\) and \(t'\) are measured through the dispersive interaction between the DQD and cavity photons \[127\]. We first identify the \((1,0) \leftrightarrow (0,1)\) interdot charge transition by measuring the normalized cavity transmission amplitude \(A/A_0\) as a function of the left (P1) and right (P2) plunger gate voltages \(V_{P1}\) and \(V_{P2}\) [Fig. 7.1(d)] \[101\]. Here \((N_1, N_2)\) denotes a charge state with \(N_1\) \((N_2)\) electrons in the left (right) dot. Vacuum Rabi splitting, with a frequency \(2g_{VO}/2\pi = 16.0\) MHz, is observed in the cavity transmission spectrum at \(\epsilon = 0\) \[7, 101\], where the charge state transition frequencies are first-order insensitive to fluctuations in \(\epsilon\) [Fig. 7.1(e), left panel] \[103\]. Here a charge decoherence rate \(\gamma_0/2\pi = 4.1\) MHz is extracted using microwave spectroscopy, indicating that the device is in the strong-coupling regime \(g_{VO} > [\gamma_0, \kappa] \[101\]. By fitting \(A(f)/A_0\) at \(\epsilon = 0\) [Fig. 7.1(e), right panel] and the results of microwave spectroscopy (see next section) to cavity input-output theory \[138\], we obtain \(E_L = 37.5\) \(\mu\)eV, \(E_R = 38.3\) \(\mu\)eV, \(t = 24.3\) \(\mu\)eV, and \(t' = 11.2\) \(\mu\)eV. The comparable magnitudes of \(t, t', E_L, E_R\) have an important implication: the
DQD energy levels are strongly influenced by hybridization of the valley and orbital d.o.f.

### 7.2 Microwave Spectroscopy of Valley States

In the previous chapter, we showed that the thermal populations of excited valley states allow high-lying avoided crossings to be probed by the cavity [127]. The DQD of this work has a sizably lower electron temperature $T_e \approx 50$ mK which renders transitions between excited states invisible in the DQD stability diagram [Fig. 7.1(d)]. This constraint may be overcome through raising the temperature of the device or ap-
plying a finite source-drain voltage \cite{127}. Here we demonstrate an additional method for measuring valley splittings that is based on a pump-and-probe scheme.

In this measurement, a second microwave drive is applied to the plunger gate P1 with a frequency $f_s$ and an estimated power $P_s \approx -110\, \text{dBm}$. The resulting cavity transmission $A/A_0$ is shown in the upper panel of Fig. 7.2(a) as a function of $\epsilon$ and $f_s$. In addition to the minimum at $\epsilon = 0$, we observe two other minima at $\epsilon = 32\, \mu\text{eV}$ and $\epsilon = -30\, \mu\text{eV}$ for $f_s = 8.31\, \text{GHz}$ and $f_s = 8.24\, \text{GHz}$, respectively. These features are understood by considering the DQD energy level shown in the lower panel of Fig. 7.2(a). For convenience, we denote the energy of each DQD charge state, in ascending order, as $E_0$, $E_1$, $E_2$ and $E_3$. At $\epsilon = 32\, \mu\text{eV} (-30\, \mu\text{eV})$, the transition frequency between the 1st and the 2nd excited states, $(E_2 - E_1)/\hbar$, is nearly equal to the cavity frequency $f_c$. When $f_s$ equals $(E_1 - E_0)/\hbar$, the electron is pumped from the ground state to the 1st excited state by the second microwave drive. The increased population of the 1st excited state, $p_1$, allows the transition between the 1st and 2nd excited states to become visible in the cavity transmission.

The cavity transmission amplitude $A(\epsilon)/A_0$ at $f_s = 8.31\, \text{GHz}$, $8.24\, \text{GHz}$ and $8.15\, \text{GHz}$ may be fit to cavity input-output theory involving all four charge states of the DQD \cite{138}. Instead of describing the level populations by a Boltzmann distribution, we use an expression for $p_1$ which results from the pump \cite{153}:

$$p_1 = \frac{1}{2} \frac{T_1 \Delta_1^2}{\frac{1}{T_2} + T_2 \delta\omega^2 + T_1 \Delta_1^2}. \quad (7.2)$$

Here $T_1$ and $T_2$ are the relaxation and decoherence times of the quantum transition between the two lowest charge states. $\delta\omega/2\pi = (E_1 - E_0)/\hbar - f_s$ is the detuning between the transition frequency and the pump. $\Delta_1 \propto P_s$ is related to the power of the pump. Since $T_1$ is not explicitly measured in this experiment, the product $T_1 \Delta_1^2$ is left as a fitting parameter. The results of the fit are plotted as black lines in Fig. 7.2(b),

\[110\text{]}

\[127\text{]}

\[138\text{]}

\[153\text{]}

7.3 Landau-Zener-Stückelberg-Majorana Interferometry of Valley-Orbit States

The valley-orbit nature of DQD charge states is more clearly visualized through Landau-Zener-Stückelberg-Majorana (LZSM) interferometry [154][160]. LZSM inter-
ferometry is performed by periodically driving the detuning parameter $\epsilon$, which in the time-domain sweeps the system through avoided crossings in the energy level diagram. The St"{u}ckelberg phase accumulated between avoided crossing traversals leads to a quantum interference pattern – a “fingerprint” that sensitively depends on the system’s Hamiltonian [153–157].

We probe the energy level structure of the Si DQD in the time-domain by varying $\epsilon$ sinusoidally in time such that $\epsilon(t) = \epsilon_0 + eV_{ac} \cos (2\pi f_g t)$, where $f_g$ denotes the frequency of the drive and $t$ in this expression denotes time. We set $V_{B2} = 410$ mV for these measurements (resulting in $t = 25.4 \mu eV$ and $t' = 11.8 \mu eV$), such that the device is in the dispersive regime with $E_{VO}/h - f_c \geq 55$ MHz $\gg g_{VO}/2\pi$ [101, 127].

With these values of $t$ and $t'$ the cavity-induced Purcell decay rate is estimated to be $\Gamma_P/2\pi < 0.1$ MHz $\ll \gamma_0/2\pi$ such that charge decoherence is dominated by internal noise in the device, not coupling to the cavity mode [125]. In the dispersive regime, a change in the DQD charge state population leads to dispersive shift in the cavity transmission $A/A_0$ [161, 162].

The LZSM interference pattern for this device is shown in Fig. 7.3(a), where we plot the steady-state cavity transmission amplitude $A/A_0$ as a function of $\epsilon_0$ and $eV_{ac}$ with $f_g = 50$ MHz. As $eV_{ac}$ is increased from zero, we observe $>20$ interference fringes in a V-shaped region bounded by $eV_{ac} \approx \epsilon_0$. Within each interference fringe, a series of minima with $A/A_0 < 1$ are observed, indicating changes in the time-averaged population of the DQD charge states due to the evolution of the St"{u}ckelberg phase [158–160]. No interference fringes are observed outside of the V-shaped region, as here the detuning parameter is no longer swept through the avoided crossings near $\epsilon = 0$ [Fig. 7.3(b)].

The overall LZSM interference pattern observed in our device significantly deviates from previous work on superconducting and semiconductor qubits, where the interference fringes have an arc-like shape that are symmetric with respect to $\epsilon = 0$.
Instead, the interference fringes have the symmetric arc-like structure for $eV_{ac} < 60 \mu eV$, but become increasingly more asymmetric at larger $eV_{ac}$ and resemble a harp. Moreover, we observe clear quantum interference fringes with $f_g = 50$ MHz, which is roughly 200 times slower than GaAs charge qubit driving frequencies [159].

The LZSM interference pattern may be qualitatively understood by considering the DQD energy level diagram [Fig. 7.3(b)]. For $|\epsilon| \gg E_{L,R}$, the DQD eigenstates are the unhybridized charge states $|L\rangle, |L'\rangle, |R\rangle, |R'\rangle$. For smaller values of $\epsilon$, the charge and valley d.o.f. hybridize through the tunnel couplings $t$ and $t'$ to form valley-orbit states. The quantum transition between the two lowest-lying energy states has a frequency $E_{VO}/h$, which is plotted in Fig. 7.3(c). For $|\epsilon| \leq E_{L,R}$, $E_{VO}/h$ has a quadratic dispersion relation as found in conventional two-level charge qubits [159, 160]. In contrast, for $|\epsilon| \gg E_{L,R}$, $E_{VO}$ approaches $E_L$ (for $\epsilon < 0$) or $E_R$ (for $\epsilon > 0$). Driving the DQD with $eV_{ac} \leq E_{L,R}$ therefore results in the arc-like interference patterns often associated with driven two-level systems [153]. However, once $eV_{ac} \gg E_{L,R}$ the energy difference between the ground state and first excited state is primarily set by single-dot valley splittings that are different for both dots, leading to an asymmetric interference pattern.

To quantitatively compare the data to theory, we employ input-output theory, which provides the cavity response $A/A_0$ as a function of the DQD susceptibility $\chi$ [161]. If the DQD is driven, a proper time-average of $\chi$ is required and can be derived within Floquet theory [163]. At the low drive frequency $f_g$ used in this experiment, the Floquet states are approximated by adiabatic solutions of the Schrödinger equation (see Appendix. C). Theoretical predictions for $A/A_0$ are shown in the inset of Fig. 7.3(a). The excellent agreement between experiment and theory confirms the energy level structure of the DQD charge states. An alternative interpretation of the LZSM interference pattern based on dressed states is given in Appendix. C.
The data in Fig. 7.3(a) also yield information on the quantum coherence of the two lowest valley-orbit states away from $\epsilon = 0$. For constructive interference of the St"uckelberg phase, consecutive passages through the avoided crossing must occur within the coherence time of the system. As such, LZSM interferences are observable only if the time-averaged decoherence rate $\bar{\gamma} = \frac{1}{T} \int_0^T \gamma[\epsilon(t)] \, dt$ satisfies $\bar{\gamma} \lesssim f_g$ where $T = 1/f_g$ is the period of the drive and $t$ here denotes time \[153, 160\]. For typical semiconductor charge qubits, charge dephasing rates $\gamma_\phi$ are several GHz at $\epsilon \neq 0$ and LZSM interferometry must be performed at high drive frequencies $f_g \geq 2.5$ GHz to see an interference pattern \[159, 160\]. In contrast, we observe a clear quantum interference pattern with $f_g = 50$ MHz, which indicates long-lived charge coherence even when far detuned from $\epsilon = 0$.

A primary factor contributing to the long coherence times of the valley-orbit states is evident from the relatively flat dispersion relation $E_{VO}(\epsilon)$. Based on the qubit parameters, $|dE_{VO}/d\epsilon|$ has a maximum value of 0.08 at $\epsilon = 30 \mu eV$ and asymptotes to zero at large $|\epsilon|$. In contrast, the dispersion relation of a conventional charge qubit $E_{CQ}(\epsilon)$ yields $|dE_{CQ}/d\epsilon| \approx 1$ at finite $\epsilon$ [Fig. 7.3(c)]. Here and in Fig. 7.5(b), we have assumed $t_c = 25.4 \mu eV$ for the charge qubit such its minimum energy splitting is the same as the valley-orbit qubit. The Landau-Zener transition probability as the system is swept across the avoided crossing at $\epsilon \approx 0$ will therefore be similar for both cases, allowing for more direct comparison. Based on the energy-level structure, charge-noise-induced fluctuations in $\epsilon$ will lead to markedly smaller fluctuations in the energy splitting between the valley-orbit states and lower decoherence rates \[103, 140, 147, 160\].
Figure 7.4: (a) Histograms showing the out-of-phase component of the cavity output field, $Q$, sampled at a frequency of 250 kHz over 1 sec. at $\epsilon = -8 \mu$eV (cyan) and $\epsilon = -60 \mu$eV (red). Black lines are fits to Gaussian functions. Inset shows $Q(\epsilon)$ and the colored dots represent the locations where the histograms are taken. $eV_{ac} = 0$ for these measurements. (b) Power spectral density of the noise in $\epsilon$, $S_\epsilon(f)$, with a fit to $S_\epsilon(f) \approx S_0[(1 \text{ Hz})/f]^\beta$. $S_\epsilon(f)$ is too small to be resolved beyond $f = 1 \text{ kHz}$ using this method.

### 7.4 Charge Noise Spectroscopy

To evaluate whether $|dE_{VO}/d\epsilon|$ is sufficient to support the observed level of charge coherence, we explicitly measure the detuning noise \[^{164}\]. The out-of-phase component of the cavity output field, $Q$, is first recorded as a time-series at $\epsilon = -8 \mu$eV where $Q$ is strongly dependent on $\epsilon$ with a sensitivity $|dQ/d\epsilon| = C$ [inset to Fig. 7.4(a)]. The data, shown as a histogram in Fig. 7.4(a), have a Gaussian profile with standard deviation $\delta_{\text{Sens}}$. To separate the noise in $Q$ due to fluctuations in $\epsilon$ from background noise in the measurement setup, we also sample $Q$ at $\epsilon = -60 \mu$eV, where $dQ/d\epsilon \approx 0$. These data [Fig. 7.4(a)] have a standard deviation $\delta_{\text{Ref}}$. The standard deviation of the detuning fluctuations is then $\delta_\epsilon = \frac{1}{C} \sqrt{\delta_{\text{Sens}}^2 - \delta_{\text{Ref}}^2} = 0.87 \mu$eV (corresponding to a maximum fluctuation of about 17 MHz in $E_{VO}/h$).

Discrete Fourier transforms of the time-series also allow the power spectral density of the detuning noise $S_\epsilon(f)$ to be determined [Fig. 7.4(b)]: we perform fast Fourier
transforms on twenty equivalent time traces of $Q$ taken at $\epsilon = -8 \mu eV$. Each time trace is sampled at a rate of 250 kHz and over the duration of one second. The coefficients $f_i$ of the discrete Fourier spectrum are then squared $|f_i|^2$ for each of the twenty time traces and averaged, which give a PSD $S_{\text{sens}}$. The same procedure is repeated at $\epsilon = -60 \mu eV$ to obtain a reference PSD, $S_{\text{Ref}}$. The PSD of noise in $\epsilon$ is then extracted according to

$$S_{\epsilon} = \frac{1}{C^2} \left( S_{Q \text{sens}} - S_{Q \text{Ref}} \right).$$

At low frequencies the power spectrum scales as $S_{\epsilon}(f) \approx S_0[(1 \text{ Hz})/f]^{\beta}$ with $S_0 = 0.11 \mu eV^2/\text{Hz}$ and $\beta \approx 1.4$. Using this noise spectrum we calculate a maximum dephasing rate $\gamma_{\phi}/2\pi \approx 6 \text{ MHz}$ at $|\epsilon| = 30 \mu eV$, which is indeed below the drive frequency $f_g = 50 \text{ MHz}$. Converting to units of electron charge, we find $S_c = (e/E_c)^2 S_{\epsilon} = 3.8 \times 10^{-9} e^2/\text{Hz}$ at a $f = 1 \text{ Hz}$, where $E_c \approx 5.4 \text{ meV}$ is the charging energy of this device.

## 7.5 A Charge-Noise-Insensitive Valley-Orbit Qubit

The valley-orbit states may serve as the basis states of a highly controllable and coherent hybrid qubit. First, the orbital nature of the hybrid qubit allows fast manipulation using microwaves, as evidenced by its strong-coupling to a single photon in the cavity [Fig. 7.1(e)]. Second, unlike conventional two-level charge qubits, the valley-orbit qubit may be operated away from $\epsilon = 0$ without significant loss of coherence due to its relatively flat energy bands and small charge noise sensitivity [11].

As a demonstration, we measure $A(\epsilon_0, eV_{\text{ac}})/A_0$ with $f_g = 100 \text{ MHz}$ [Fig. 7.5(a)]. Here we observe clearly resolved interference minima that show no perceptible decay in intensity as the drive amplitude $eV_{\text{ac}}$ increases, further confirming that the decoherence rate of the valley-states does not increase appreciably at large DQD detunings. To compare with theoretical expectations, we calculate $A(\epsilon_0, eV_{\text{ac}})/A_0$ using experimental parameters [see the left panel of Fig. 7.5(b)] and find excellent agreement
Figure 7.5: (a) $A/A_0$ as a function of $\epsilon_0$ and $eV_{ac}$, with $f_g = 100$ MHz. (b) Theoretically predicted LZSM interference patterns obtained from a 4-level valley-orbit model (left) and a standard two-level charge qubit model (right). The color-scale is the same as in panel (a). (c) $A/A_0$ as a function of $eV_{ac}$, taken along the dashed lines in panels (a) and (b). The experimental data and calculation based on valley-orbit states are plotted along the bottom $x$-axis, whereas the calculation based on a charge qubit is plotted along the top $x$-axis.

with the data. In comparison, the interference patterns calculated for a conventional charge [right panel of Fig. 7.5(b)] decay rapidly as $eV_{ac}$ increases due to fast decoherence at large DQD detunings. These simulations are more directly compared by plotting $A/A_0$ along a line connecting the center of a resonance minimum within each fringe [Fig. 7.5(c)].

### 7.6 Conclusion

In conclusion, we observe LZSM interference patterns in a cavity-coupled Si DQD charge qubit when the DQD detuning is driven at frequencies as low as 50 MHz. Analysis of the interference patterns reveals that the basis states for the DQD charge qubit are hybridized valley-orbit states rather than pure orbital states. Compared to
conventional charge qubits, and other recently developed hybrid qubits that all rely on operation at sweet spots to maintain coherence \[103\] \[151\] \[152\], qubits formed by the valley-orbit states have a small sensitivity to charge noise, even at arbitrary detunings. Analogous to superconducting transmon qubits, the charge noise insensitivity of the valley-orbit states arises from their flat energy level structure \[11\]. Deterministic control of valley splittings, perhaps made possible through further material research efforts, has the potential of turning the valley d.o.f. in Si into a powerful resource for reducing the charge noise sensitivities of silicon qubits.
Chapter 8

Outlook

Achieving the strong-coupling regime between single microwave photons and gate-defined semiconductor quantum dots, particularly with their spin degrees freedom and in silicon, has been a long-sought goal of quantum information science. We have finally made such a demonstration in this thesis. The implication of this newly developed experimental platform is profound: spin qubits can now be coupled and read out purely through microwave photons just like superconducting qubits, all the while retaining $T_1$ times orders of magnitude longer compared to superconducting qubit systems (due to the ability to quickly switch between single- and double-dot regimes as demonstrated in Chapter 5). The combination of these features makes the hybrid Si DQD-cQED architecture a highly attractive way to build a quantum processor, as it circumvents the trade-off between lifetime and connectivity that has been a major roadblock until now.

On the other hand, such optimism needs to be accompanied by a keen awareness that much work is still ahead of us to improve the hybrid cQED system to the level of cooperativity (about 10,000) often seen in state-of-the-art superconducting qubits [165]. Fortunately, a clear path exists toward such a goal: To raise spin-photon coupling rates, one may transition into higher impedance cavities using high kinetic
inductance materials such as NbTiN and lumped element resonator designs. To reduce the decoherence rate of the spin qubits, isopotically purified $^{28}\text{Si}$ heterostructures may be used to minimize dephasing effects from nuclear bath fluctuations. Further materials research may also clarify the sources of charge noise in our samples and allow the low charge noise seen in the strongly coupled charge-photon samples to be consistently achieved in future devices. Lastly, improvement in the cavity quality factor may also be realized by removing the amorphous $\text{Al}_2\text{O}_3$ layer underneath the resonator, implementing stronger filters, and eventually transitioning to multi-layer wiring using flip-chip techniques [166].
Appendix A

Theory of Spin-Photon Coupling in DQDs

A.1 Input-Output Theory for Cavity Transmission

We briefly summarize the theoretical methods used to fit the cavity transmission $A/A_0$ shown in Fig. 5.2(b) and in Fig. 5.2(c). For a detailed description of the theory, we refer the reader to Ref. [31]. We start from the Hamiltonian describing the DQD

$$H_0 = \frac{1}{2} \left( \epsilon \tau_z + 2 t_c \tau_x + B_z \sigma_z + B_x^M \sigma_x \tau_z \right),$$

(A.1)

where $\tau_x$ and $\tau_z$ are Pauli operators acting on the orbital charge states of the DQD electron, $\sigma_x$ and $\sigma_z$ are Pauli operators acting on the spin states of the electron, $B_z = B_z^{\text{ext}} + B_z^M$ denotes the total magnetic field along the $z$-axis, and $B_x^M = (B_{x,R}^M - B_{x,L}^M)/2$ is half the magnetic field difference of the DQD in the $x$-direction. In the theoretical model, we have assumed that the average magnetic field in the $x$-direction $(B_{x,R}^M + B_{x,L}^M)/2 = 0$, which is a good approximation given the geometry of the micromagnet and its alignment with the DQD. We then add the electric dipole coupling to the
cavity with the Hamiltonian

$$H_1 = g_c (a + a^\dagger) \tau_z,$$  \hspace{1cm} (A.2)

where $a$ and $a^\dagger$ are the cavity photon operators. The electric dipole operator can be expressed in the eigenbasis $\{|n\rangle\}$ of $H_0$ as

$$\tau_z = \sum_{n,m=0}^{3} d_{nm} |n\rangle \langle m|.$$ \hspace{1cm} (A.3)

In the Heisenberg picture, we then write the quantum Langevin equations for the operators $a$ and $\sigma_{nm} = |n\rangle \langle m|,$

$$\dot{a} = i\Delta_0 a - \frac{\kappa}{2} a + \sqrt{\kappa_1} a_{in,1} + \sqrt{\kappa_2} a_{in,2} - ig_c e^{i\omega_R t} \sum_{n,m=0}^{3} d_{nm} \sigma_{nm},$$ \hspace{1cm} (A.4)

$$\dot{\sigma}_{nm} = -i \left( E_m - E_n \right) \sigma_{nm} - \sum_{n',m'} \gamma_{nm,n'm'} \sigma_{n'm'} + \sqrt{2} \gamma F$$

$$- ig_c \left( a e^{-i\omega_R t} + a^\dagger e^{i\omega_R t} \right) d_{nm} p_{nm},$$ \hspace{1cm} (A.5)

where $\Delta_0 = \omega_R - \omega_c$ is the detuning of the driving field ($\omega_R = 2\pi f$) relative to the cavity frequency ($\omega_c = 2\pi f_c$) and $p_{nm} = p_n - p_m$ the population difference between levels $n$ and $m$ ($p_n$ can, e.g., be assumed to be a Boltzmann distribution in thermal equilibrium). This description is equivalent to a more general master equation approach in the weak-driving regime where population changes in the DQD can be neglected. Furthermore, $\kappa_1$ and $\kappa_2$ are the photon decay rates at ports 1 and 2 of the cavity, and $a_{in,1}$ is the input field of the cavity which we assume to couple through port 1 only ($a_{in,2} = 0$). The quantum noise of the DQD $F$ will be neglected in what follows. The superoperator $\gamma$ with matrix elements $\gamma_{nm,n'm'}$ represents decoherence processes which include charge relaxation and dephasing due to charge noise (these processes also imply some degree of spin relaxation and dephasing due to spin-charge hybridiza-
tion via $B_x^M$). Our goal is to relate the incoming parts $a_{in,i}$ of the external field at the ports to the outgoing fields $a_{out,i} = \sqrt{\kappa_i} a - a_{in,i}$. The transmission $A = \bar{a}_{out,2}/\bar{a}_{in,1}$ through the microwave cavity is then computed using a rotating-wave approximation (RWA) to eliminate the explicit time-dependence in Eqs. (A.4) and (A.5), by solving the equations for the expected value of these operators in the stationary limit ($\bar{a}$, $\bar{\sigma}_{n,m}$),

$$A = \frac{-i\sqrt{\kappa_1\kappa_2}}{-\Delta_0 - i\kappa/2 + g_c \sum_{n=0}^{2} \sum_{j=1}^{3-n} d_{n,n+j} \chi_{n,n+j}}$$

(A.6)

with the single-electron partial susceptibilities $\chi_{n,n+j} = \bar{\sigma}_{n,n+j}/\bar{a}$.

### A.2 Theoretical Models for Spin-Photon Coupling and Spin Decoherence

Here we present a brief derivation of the analytical expressions for the spin-photon coupling rate $g_s$ and spin decoherence rate $\gamma_s$. A more extensive discussion of spin-photon coupling and spin decoherence specific to our device architecture is presented in Ref. [31]. We focus on the $\epsilon = 0$ regime used in Fig. 5.3(b). Accounting for spin-charge hybridization due to the field gradient $B_x^M$, the relevant eigenstates of the DQD Hamiltonian Eq. (A.1) are $|0\rangle \approx |-,\downarrow\rangle$, $|1\rangle \approx \cos \frac{\Phi}{2} |-,\uparrow\rangle + \sin \frac{\Phi}{2} |+,\downarrow\rangle$, $|2\rangle \approx \sin \frac{\Phi}{2} |-,\uparrow\rangle - \cos \frac{\Phi}{2} |+,\downarrow\rangle$ and $|3\rangle \approx |+,\uparrow\rangle$. Here we have introduced a mixing angle $\Phi = \tan^{-1} (\frac{g_{\mu_B} B_x^M}{2\kappa_c - g_{\mu_B} B_z})$. The dipole transition matrix element for the primarily spin-like transition between $|0\rangle$ and $|1\rangle$ is given by $d_{01} \approx -\sin \frac{\Phi}{2}$, and the dipole transition matrix element for the primarily charge-like transition between $|0\rangle$ and $|2\rangle$ is given by $d_{02} \approx \cos \frac{\Phi}{2}$. The transition between $|0\rangle$ and $|3\rangle$ is too high in energy (off resonance) and is therefore excluded from our model. The spin-photon coupling rate is given by $g_s = g_c|d_{01}| = g_c|\sin \frac{\Phi}{2}|$, in agreement with previous theory works [29, 32].
To calculate the effective spin decoherence rate $\gamma_{s}^{(c)}$ arising from charge decoherence, we first construct the operators $\sigma_{01} = |0\rangle\langle 1| \approx \cos \frac{\Phi}{2}\sigma_s + \sin \frac{\Phi}{2}\sigma_\tau$ and $\sigma_{02} = |0\rangle\langle 2| \approx \sin \frac{\Phi}{2}\sigma_s - \cos \frac{\Phi}{2}\sigma_\tau$. Here $\sigma_s = |-, \downarrow\rangle\langle -, \uparrow|$ and $\sigma_\tau = |-, \downarrow\rangle\langle +, \downarrow|$ are lowering operators for the electron spin and charge respectively. Assuming the electron charge states have a constant decoherence rate $\gamma_c = \gamma_1/2 + \gamma_\phi$, where $\gamma_1$ is the charge relaxation rate and $\gamma_\phi$ is a dephasing rate due to charge noise [4], the equations of motion for these operators are:

$$\dot{\sigma}_{01} = \gamma_c \left(-\sin^2 \frac{\Phi}{2}\sigma_{01} + \sin \frac{\Phi}{2}\sigma_{02}\right),$$  \hspace{1cm} (A.7)

$$\dot{\sigma}_{02} = \gamma_c \left(\sin \frac{\Phi}{2}\sigma_{01} - \cos^2 \frac{\Phi}{2}\sigma_{02}\right).$$  \hspace{1cm} (A.8)

Combined with charge-photon coupling, the overall equations of motion (A.4) and (A.5) in a rotating frame with a drive frequency $f \approx f_c$ assume the form

$$\dot{a} = i\Delta_0 a - \frac{\kappa}{2} a + \sqrt{\kappa_1 a_{in,1}} - ig_c (d_{01}\sigma_{01} + d_{02}\sigma_{02}),$$  \hspace{1cm} (A.9)

$$\dot{\sigma}_{01} = -i\delta_1 \sigma_{01} - \gamma_c \sin^2 \frac{\Phi}{2}\sigma_{01} + \frac{\sin \Phi}{2}\sigma_{02} - ig_c d_{10},$$  \hspace{1cm} (A.10)

$$\dot{\sigma}_{02} = -i\delta_2 \sigma_{02} - \gamma_c \cos^2 \frac{\Phi}{2}\sigma_{02} + \frac{\sin \Phi}{2}\sigma_{01} - ig_c d_{20}.$$  \hspace{1cm} (A.11)

The $\delta_1$ and $\delta_2$ terms are defined as $\delta_1/2\pi = (E_1 - E_0)/h - f$ and $\delta_2/2\pi = (E_2 - E_0)/h - f$, where $E_{0,1,2}$ corresponds to the energy of the $|0\rangle$, $|1\rangle$, or $|2\rangle$ state. Steady-state solutions to the above equations give the electric susceptibility of the spin qubit transition $\chi_{01} = \frac{\sigma_{01}}{a} = \frac{g_s}{\delta_1 - \gamma_s^{(c)}}$, where we have identified a charge-induced spin decoherence rate $\gamma_{s}^{(c)} = \gamma_c \left[\delta_2 \sin^2 \frac{\Phi}{2} + \delta_1 \cos^2 \frac{\Phi}{2}\right]/\delta_2$. To account for spin dephasing due to fluctuations of the $^{29}$Si nuclear spin bath, we express the total spin decoherence rate assuming a Voigt profile: $\gamma_s = \gamma_{s}^{(c)}/2 + \sqrt{(\gamma_{s}^{(c)}/2)^2 + 8(\ln 2)(1/T_{2,nuclear}^{s})^2}$, where
$T_{2,\text{nuclear}} \approx 1 \mu s$ is the electron spin dephasing time due to nuclear field fluctuations.

In fitting to the data of Fig. 5.3(b), we use the experimentally determined values of $g_c/2\pi = 40$ MHz and $\gamma_c/2\pi = 35$ MHz, along with a best fit field gradient $B_x^M = 15$ mT. For every $t_c$, the fit value for $B_z$ is adjusted such that the spin qubit frequency $(E_1 - E_0)/\hbar$ matches the cavity frequency $f_c$ exactly. The slight discrepancy between theory and experiment for $\gamma_s$ may be due to the frequency dependence of $\gamma_c$, changes in $\gamma_c$ with $B_z^{\text{ext}}$, or other decoherence mechanisms not captured by this simple model. To resolve such a discrepancy, a complete measurement of $\gamma_c$ as functions of $2t_c/\hbar$ and the external field $B_z^{\text{ext}}$ is needed, which will be the subject of future work.

The complete theory presented in Ref. 31 also allows $g_s/2\pi$ to be calculated for finite values of $\epsilon$. Using $2t_c/\hbar = 7.4$ GHz, we estimate $g_s/\hbar = 2.3$ MHz at $\epsilon = 20$ $\mu$eV (\textasciitilde4.8 GHz), close to the value $g_s/\hbar = 1.0$ MHz measured at this DQD detuning [Fig. 5.3(a)].

In this theoretical model, we have ignored Purcell decay of the spin qubit through the cavity 125. This is justified since $\gamma_s$ at every value of $t_c$ is measured with a large spin-cavity detuning $\Delta \approx 10g_s$. The expected Purcell decay rate of the spin qubit is $\Gamma_p/2\pi = \left[\kappa g_s^2/(\kappa^2/4 + \Delta^2)\right]/2\pi \approx 0.02$ MHz, well below the measured values of $\gamma_s/2\pi$. We also note that, at least in the $2t_c \gg E_z$ limit, spin decoherence at $\epsilon = 0$ is dominated by noise-induced dephasing rather than energy relaxation. This is because at $2t_c/\hbar = 11.1$ GHz, the spin decoherence rate $\gamma_s/2\pi = 0.41$ MHz corresponds to a coherence time $T_2 = 0.4 \mu s \ll 2T_1 = 6.4 \mu s$. 
Figure A.1: Calculated cavity transmission spectra (black solid lines) superimposed on top of the experimentally measured vacuum Rabi splittings shown in Fig. 5.2(b) and Fig. 5.2(c). The calculations are produced with $g_c/2\pi = 40$ MHz (37 MHz), $\kappa/2\pi = 1.8$ MHz, $\gamma_c/2\pi = 105$ MHz (120 MHz), $B_z = B_z^{\text{ext}} + B_z^{\text{M}} = 209$ mT, $B_x^{\text{M}} = (B_{x,R}^{\text{M}} - B_{x,L}^{\text{M}})/2 = 15$ mT and $2t_c/h = 7.4$ GHz for DQD 1 (DQD 2). For comparison, $A(f)/A_0$ simulated for a two-level charge qubit having a decoherence rate $\gamma_c/2\pi = 2.4$ MHz coupled to a cavity with $\kappa/2\pi = 1.8$ MHz at a rate $g_c/2\pi = 5.5$ MHz is shown in panel (a) with thermal photon numbers $n_{\text{th}} = 0.02$ (black dashed line) and $n_{\text{th}} = 0.5$ (red dashed line).

A.3 Asymmetric Line-Shapes of Spin-Photon Vacuum Rabi Splitting

In contrast to charge-photon systems [7, 101, 108], the two resonance modes in the vacuum Rabi splittings of Fig. 5.2(b) and Fig. 5.2(c) show slightly unequal widths. This effect can be seen by comparing the observed spectrum of DQD 1 with the expected behavior of an equivalent two-level charge qubit strongly coupled to a cavity, calculated using a master equation simulation with thermal photon number $n_{\text{th}} = 0.02$ [black dashed line in Fig. A.1(a)]. The unequal widths are unlikely a result of a large thermal photon number in the cavity, as the transmission spectrum calculated with $n_{\text{th}} = 0.5$ (orange dashed line) clearly does not fit the experimental data [167].

Instead, the observed asymmetry likely arises from the dispersive interaction between the cavity and the primarily charge-like transition between $|0\rangle$ and $|2\rangle$, which
results in three-level dynamics that is more complicated than the two-level dynamics characterizing charge-photon systems. A more complete treatment of this effect is given in Ref. [31]. Here we compare the experimental observation with theory by calculating $A(f)/A_0$ using $g_c/2\pi = 40$ MHz (DQD 1) and 37 MHz (DQD 2), $\gamma_c/2\pi = 105$ MHz (DQD 1) and 130 MHz (DQD 2), $\kappa/2\pi = 1.8$ MHz, tunnel couplings $2t_c/h = 7.4$ GHz, $B^M_x = 15$ mT and $B_z = 209.6$ mT. The results are shown as black solid lines alongside experimental data in Fig. A.1. The agreement between experiment and theory is very good for both devices. In particular, the asymmetry between the vacuum Rabi modes is also seen in the theoretical calculations. The larger values of $\gamma_c$ used in the theoretical calculations may again be due to the frequency dependence of $\gamma_c$ or changes in $\gamma_c$ with $B^e_z$. Further experiments are needed to resolve this difference.
Appendix B

Theory for Cavity Response under Finite Bias

In order to describe the DQD with valley states theoretically, we begin with the Hamiltonian $H$ of the system. We consider a basis with 0, 1 or 2 spinless electrons and two valleys in each dot with 11 basis functions: $|0\rangle$, $|L\rangle$, $|L'\rangle$, $|R\rangle$, $|R'\rangle$, $|RR'\rangle$, $|LL'\rangle$, $|LR\rangle$, $|L'R'\rangle$, $|L'R\rangle$, and $|L'R'\rangle$, where 0 denotes an empty dot, $L$ ($R$) an electron in the left (right) dot in the lower valley, and $L'$ ($R'$) in the higher valley. In the relevant parameter regime, double occupation of each dot is suppressed, and since we apply no external magnetic field, we can neglect the spin degree of freedom and assume that electrons with spin up and spin down behave the same way. To obtain the diagonal elements of $H$ in this basis, we turn to the capacitor model of the DQD, as shown in Fig. B.1 [133].
The electrostatic potential in the dots $\tilde{V}_L$ and $\tilde{V}_R$ can then be obtained by

$$\begin{pmatrix} \tilde{V}_L \\ \tilde{V}_R \end{pmatrix} = \frac{1}{C_L \tilde{C}_R - C_M^2} \begin{pmatrix} \tilde{C}_R & C_M \\ C_M & \tilde{C}_L \end{pmatrix} \times$$

$$\begin{pmatrix} -|e|(n_L + n'_L) + C_L V_L + C'_L V'_L + C'_{RL} V'_R \\ -|e|(n_R + n'_R) + C_R V_R + C'_R V'_R + C'_{LR} V'_L \end{pmatrix},$$

where $e$ is the elementary charge, $V_L$ and $V_R$ are the potentials of the left and right leads, $V'_L$ and $V'_R$ are the potentials of the left and right gates, and

$$\begin{align*}
\tilde{C}_L &= C_L + C_M + C'_L + C'_{RL} \quad \text{and} \\
\tilde{C}_R &= C_R + C_M + C'_R + C'_{LR}
\end{align*}$$

(B.2a) (B.2b)
are the sums of the capacitances connected to the left and right dot respectively. The electrostatic potential energy of the system is

\[ U(n_L, n'_L, n_R, n'_R) = \frac{1}{2} \tilde{C}_L \tilde{V}_L^2 + \frac{1}{2} \tilde{C}_R \tilde{V}_R^2 - C_M \tilde{V}_L \tilde{V}_R. \]  

(B.3)

The capacitor model relies on the knowledge of the seven capacitance values included in Fig. B.1. These can all be obtained (see Ref. [133]) from the finite bias triangles (FBT) [lower inset of Fig. 6.4(a) and upper inset of Fig. 6.5(a)] if we also know the charging energy, which is measured to be 5.4 meV for both dots. The values of the best fit capacitances are listed in Table B.1.

The sum of the electrostatic potential energy and the contribution from the yet unknown valley splittings fully determines the diagonal elements of \( \mathcal{H} \). The energy term corresponding to the excited valley states equals \( E_L n'_L + E_R n'_R \), where \( E_L \) and \( E_R \) are the valley splittings in the quantum dots.

The off-diagonal elements of \( \mathcal{H} \) allow coherent coupling between basis states with the same total electron number. In the 4 × 4 one-electron block of \( \mathcal{H} \), we allow interdot transitions between electronic states within the same valley with the matrix element \( t \)
and with a different matrix element $\pm t'$ between opposite valleys, similar to Ref. [138]

\[
\begin{pmatrix}
0 & 0 & t & -t' \\
0 & 0 & t' & t \\
t & t' & 0 & 0 \\
-t' & t & 0 & 0 \\
\end{pmatrix}
\begin{array}{c}
|L\rangle \\
|L'\rangle \\
|R\rangle \\
|R'\rangle \\
\end{array}
\]  
\tag{B.4}

The $6 \times 6$ two-electron block takes the following form:

\[
\begin{pmatrix}
0 & 0 & t' & t & t & -t' \\
0 & 0 & t' & t & t & -t \\
t' & t' & 0 & 0 & 0 & 0 \\
t' & t' & 0 & 0 & 0 & 0 \\
t & t & 0 & 0 & 0 & 0 \\
-t & -t & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{array}{c}
|RR\rangle \\
|LL\rangle \\
|LR\rangle \\
|L'R\rangle \\
|LR'\rangle \\
|L'R'\rangle \\
\end{array}
\]  
\tag{B.5}

For the relation between the $t$ and $t'$ transition amplitudes and for the sign in front of $t'$ in the one-electron block, see the Appendix of Ref. [138]. In the two-electron block, the signs in front of the tunneling amplitudes can be obtained in the second quantized formalism by observing the fermionic anticommutation rules.

We assume that incoherent transitions can occur between the eigenstates of $\mathcal{H}$, both internally and by electron hopping between the DQD and the leads. These incoherent interactions with the environment can be taken into account with the Lindblad master equation:

\[
\dot{\rho} = -\frac{i}{\hbar}[\mathcal{H},\rho] + \mathcal{D}(\rho),
\]  
\tag{B.6}
where $\hbar$ is the reduced Planck constant, $\rho$ is the density matrix, and the dissipative part $\mathcal{D}(\rho)$ consists of the following terms:

$$
\mathcal{D}(\rho) = \sum_{v=\pm, l=\{L,R\}} \Gamma_l \left( c_{lv}^\dagger \rho c_{lv} - \frac{1}{2} \rho c_{lv}^\dagger c_{lv}^\dagger - \frac{1}{2} c_{lv} c_{lv}^\dagger \rho \right) + \sum_{f,i} \left( b_{fi} \rho b_{fi}^\dagger - \frac{1}{2} \rho b_{fi}^\dagger b_{fi}^\dagger - \frac{1}{2} b_{fi}^\dagger b_{fi} b_{fi} \rho \right).
$$

Here $\Gamma_l$ is the transition rate at the contact $l$, and the operators $c_{lv}^\dagger$ and $c_{lv}$ create and annihilate an electron in dot $l$ and valley $v$, respectively. The sum of Eq. (B.7a) corresponds to the flow of electrons from lead $l$ into dot $l$, while Eq. (B.7b) describes the current in the opposite direction whereby electrons flow from the dot to the lead. The third term describes excitations within the DQD, i.e. incoherent interactions with a bosonic bath, such as phonons, that can induce transitions from one eigenstate of $\mathcal{H}$ to the other with the same total number of electrons in the DQD. The operator $b_{fi} = \sqrt{\tau_{fi}} \langle f | i \rangle | i \rangle$ takes the system from an initial state $| i \rangle$ to a final state $| f \rangle$ with a transition rate $\tau_{fi}$. A simple illustration of DQD energy levels and transport processes is provided by Fig. B.2.

We assume that the level broadenings, caused by the interaction with the leads and the bosonic bath, are smaller than the level splittings between the eigenstates of $\mathcal{H}$. This is the so-called secular approximation, which results in a steady-state density matrix $\rho$ diagonal in the eigenbasis of $\mathcal{H}$. This significantly simplifies the Lindblad equation (B.6), where the commutator vanishes and after taking the matrix representation of the remaining dissipative term in the eigenbasis of $\mathcal{H}$, we obtain
Figure B.2: Schematic energy ($E$) levels as functions of the detuning $\varepsilon$ and transport processes when a finite bias $V_{SD}$ is applied to the DQD. For simplicity, only the lower valley states are included in this illustration, and $t$ is the valley-preserving tunneling amplitude. Processes cycling through $|0\rangle$ belong to the lower FBT, while those cycling through $|LR\rangle$ belong to the upper FBT [Fig. 6.4(a)]. $\Gamma_l$ is the transition rate at the contact $l$, and $\gamma$ is the decay rate in the one-electron regime.

Redfield equations for the diagonal elements of the steady-state solution:

$$0 = \dot{\rho}_k = \sum_{j,l=\{L,R\}} \Gamma_l \left( \rho_{j}c_{jk}^{(l)} - \rho_{k}c_{kj}^{(l)} \right) + \rho_{j}c_{kj}^{(l)} - \rho_{k}c_{jk}^{(l)} + \sum_{i \neq k} \tau_{ki} \rho_i - \sum_{f \neq k} \tau_{fk} \rho_k .$$  

The expressions (B.8a)-(B.8c) are approximations of their respective counterparts (B.7a)-(B.7c). Here $\rho_k$ is shorthand for $\rho_{kk}$ which is the $k$th element of $\rho$ along the diagonal, and $c_{jk}^{(l)} = \sum_v |\langle j|c_{lv}|k\rangle|^2$, which can be finite only if there is one more electron in state $|k\rangle$ than in $|j\rangle$. Equation (B.8) applies to all quantum numbers $k$.  

133
We can extend this description toward finite temperatures in the leads with the following replacement rules in Eq. (B.8):

\[
\rho_m c_m^{(l)} \rightarrow \rho_m c_m^{(l)} n_{mn}
\]
\[
\rho_n c_n^{(l)} \rightarrow \rho_n c_n^{(l)} n_{mn},
\]

where \( n_{mn} = n_{FD}(E_m - E_n + (\nu_m - \nu_n)|e|V, T_e) \), \( E_m \) and \( E_n \) are the eigenenergies, \( \nu_m \) and \( \nu_n \) the number of electrons in the given eigenstates of \( \mathcal{H} \), and

\[
n_{FD}(\delta E, T_e) = \frac{1}{\exp(\delta E/(k_B T_e)) + 1}
\]

is the Fermi-Dirac distribution of the electrons in the lead, with \( k_B \) being the Boltzmann constant and \( T_e = 135 \text{ mK} \) being the electron temperature.

The \( \tau_{mn} \) transition rates in (B.8c) can be defined as \( \tau_{mn} = \gamma^{(l)} n_{BE}(E_m - E_n, T_e) \), where the prefactor \( \gamma \) in the one-electron subspace or \( \gamma' \) in the two-electron subspace is approximated to be independent of \( m \) and \( n \). The temperature dependent prefactor

\[
n_{BE}(\delta E, T_e) = \frac{1}{\exp(|\delta E|/(k_B T_e)) - 1} + \Theta(-\delta E)
\]

accounts for the Bose-Einstein statistics of the environmental thermal bath, which is assumed to be in thermal equilibrium with the electronic system and having an approximately constant density of states in the relevant energy window of the transitions. \( \Theta(\cdot) \) denotes the Heaviside function.
Equation (B.8) can be put in a more concise form, which also reflects the temperature dependence

$$0 = \dot{\rho}_k = \sum_{j \neq k} (\Gamma_{kj} \rho_j - \Gamma_{jk} \rho_k) + \sum_{j \neq k} (\tau_{kj} \rho_j - \tau_{jk} \rho_k), \quad (B.12)$$

where \( \Gamma_{jk} = \sum_{l = \{L,R\}} \Gamma_{ij} \left( c_{jk}^{(l)} + c_{kj}^{(l)} \right) n_{j}^{(l)} \) \( \quad (B.13) \)

denotes the total decay rate of the state \( k \) to state \( j \) with one electron hopping on or off the DQD. Note that depending on the direction of the hopping, either \( c_{jk}^{(l)} \) or \( c_{kj}^{(l)} \) will be zero. This set of classical rate equations can also be formulated as a matrix equation \( M \rho = 0 \), where \( \rho \) is a vector of the diagonal elements of the density matrix \( \rho \). The steady-state solution in the secular approximation is thus provided by the nullspace of the matrix \( M \), as a normalized vector of the probabilities \( \bar{\rho}_k \) for finding the system in its \( k \)th eigenstate.

Our experimental setup (Fig. 6.1) is designed in a way that the oscillating voltage generated inside the superconducting cavity is directly coupled to the DQD detuning parameter. The response of the system to a microwave probe field due to this electric dipole coupling can be determined using cavity input-output theory [168]. Assuming that the microwave field can only induce transitions between neighboring energy levels of the DQD, which proves to be justified in our case, the transmission coefficient \( A \) is given by Eq. (17) in Ref. [138]:

$$A = \frac{-i\sqrt{\kappa_1 \kappa_2}}{\omega_0 - \omega_R - i\gamma/2 + 2g_0 \sum_{n=1}^{3} d_{n,n+1} \chi_{n,n+1}}, \quad (B.14)$$

where

$$\chi_{n,n+1} = \frac{-2g_0 d_{n+1,n} (\bar{\rho}_n - \bar{\rho}_{n+1})}{E_{n+1} - E_n - \omega_R - i\gamma/2}. \quad (B.15)$$

Here \( \chi_{n,n+1} \) is the electric susceptibility of the DQD and \( d_{n,n+1} \) is the corresponding dipole matrix element pertaining to the \( n \to n+1 \) transition. The total cavity decay
rate is $\kappa = \kappa_1 + \kappa_2 + \kappa_i$, where $\kappa_1$ and $\kappa_2$ are the photon decay rates through the input and output ports and $\kappa_i$ is the internal photon decay rate. The probe frequency and the cavity resonance frequency are denoted by $\omega_0$ and $\omega_R$, and $g_0$ is the electric dipole coupling strength. Note that here we adopted the $\hbar = 1$ convention as in Ref. [138]. The summation in (B.14) goes over the possible next neighbor transitions in the one-electron regime with the eigenstates ordered with increasing eigenenergies $E_n$.

In Ref. [138], the assumption of thermal equilibrium was made and the $\rho_n$ probabilities were set by the Boltzmann distribution. This can be used only if no bias is applied to the leads. If a bias is applied, the steady-state solution can significantly differ from the Boltzmann distribution around or inside the FBTs, where the $|0\rangle$ and $|L^{(i)}R^{(i)}\rangle$ states may be appreciably occupied due to nonequilibrium transport. Note however that the $|0\rangle$ and $|L^{(i)}R^{(i)}\rangle$ states have no dipole moments and therefore their contributions to $A$ are only due to changes in the $\rho_n$ occupation probabilities. The contribution of the $|RR\rangle$ and $|LL\rangle$ states can be safely neglected everywhere in the range of parameters of this work.

The zero bias description deep in the one-electron regime in thermal equilibrium can still offer the simplest way to determine the valley splittings $E_L$ and $E_R$. Note that, in this case, we do not need the capacitor model since we can calculate $A$ as function of the detuning as in Ref. [138]. The excellent agreement between measurement and theory can be seen in Fig. 6.2 and in Fig. 6.3 for higher temperatures. The zero bias fitting reveals symmetrical valley splittings: $E_L = E_R = 50.5 \, \mu eV$, and it also provides us with the tunneling amplitudes: $t = 20.9 \, \mu eV$ and $t' = 15.0 \, \mu eV$, which completes the list of parameters in our Hamiltonian $\mathcal{H}$.

The unknown parameters left for the finite bias fitting are $\Gamma_L$, $\Gamma_R$ and $\gamma'$. The decay rate in the one-electron regime was measured independently to be $\gamma/2\pi = 30 \, \text{MHz}$. The theoretical predictions were obtained with $\Gamma_L = 25 \, \text{MHz}$, $\Gamma_R = 151$
MHz for $V_{SD} = -0.3$ mV (Fig. 6.5), and $\Gamma_L = 62$ MHz, $\Gamma_R = 132$ MHz for $V_{SD} = 0.3$ mV (Fig. 6.4), while $\gamma'/2\pi = 3$ MHz for both. Lastly, the calculated cavity responses have been convolved with a Gaussian of width $\sigma_{\epsilon} = 1.5 \mu eV$ to account for low frequency noise. This is the same broadening factor used for fitting zero-bias data in Fig. 6.2(c).
Appendix C

Theory for Landau-Zener-Stückelberg-Majorana Interferometry of Valley-Orbit States

C.1 Theoretical Model for Decoherence of the Valley-Orbit States

For all theory fits in Chapter 7, we have included two contributions to the total decoherence rate $\gamma = 1/T_2$ of the valley-orbit qubit (defined by the two lowest charge states having an energy difference $E_{\text{VO}} = E_1 - E_0$), such that $\gamma = \gamma_0 + \gamma_\phi$. Here $\gamma_0/2\pi = 4.1$ MHz is the decoherence rate extracted from the vacuum Rabi splitting at the charge sweet spot $\epsilon = 0$, where the first-order coupling between the noise on $\epsilon$ and the valley-orbit states is zero. Contributions to $\gamma_0$ may include both $T_1$ relaxation and second-order coupling to charge noise. $\gamma_\phi$ is the dephasing rate at $\epsilon \neq 0$, which can be estimated from the noise spectrum $S_{\epsilon}(f)$ by considering the decay law for free
Larmor precession over a time $t_p$,\footnote{103, 147},

$$F(t_p) = \exp \left[ -\left( \frac{\eta}{\hbar} \right)^2 \int_{\omega_0}^{\omega_1} S_\epsilon(\omega) \frac{\sin^2(\omega t_p/2)}{(\omega/2)^2} d\omega \right], \quad (\text{C.1})$$

where we have used the angular frequency $\omega = 2\pi f$ and $\eta = |dE_{\text{VO}}/d\epsilon|$ is the slope of the energy dispersion. Closer inspection of the integral reveals that the contribution with $\omega > \pi/t_p$ can be neglected safely. Then for DQD detuning noise with a PSD $S_\epsilon(f) = S_0/f^{1.4}$ and a sufficiently large upper cut-off $\omega_1 > \pi/t_p$, the above expression simplifies to \footnote{103}:

$$F(t_p) \approx \exp \left[ -\frac{1}{2} \left( \frac{\eta \sigma_\epsilon t_p}{\hbar} \right)^2 \right], \quad (\text{C.2})$$

where $\sigma_\epsilon = \left[ 2 \int_{\omega_0}^{\pi/t_p} S_\epsilon(\omega) d\omega \right]^{1/2} = \left[ 2 \int_{f_0}^{1/2t_p} S_\epsilon(f) df \right]^{1/2}$ is the rms amplitude of the noise. For $\beta > 1$, the dominating contribution stems from low frequencies such that $\sigma_\epsilon \approx \sqrt{5S_0f_0^{-0.2}}$. Choosing a lower cut-off frequency $f_0 = \omega_0/2\pi = 20$ Hz set by the measurement time of 50 ms, we obtain $\sigma_\epsilon \approx 0.41 \, \mu\text{eV}$ for the noise spectrum in Fig. 7.4(b). The dephasing rate is then estimated based on $F(t_p = 1/\gamma_\phi) = \exp(-1)$, which gives $\gamma_\phi \approx \eta \sigma_\epsilon / \sqrt{2\hbar}$. The maximum dephasing rate, which occurs at $|\epsilon| = 30 \, \mu\text{eV}$, is estimated to be $\gamma_\phi/2\pi \approx 6$ MHz. In contrast, for a charge qubit subject to the same noise spectrum, $\gamma_\phi/2\pi \approx 70$ MHz for $|\epsilon| \gg 2t_C$.

We note that choosing $f_0 = 1$ Hz, which corresponds to the time interval over which the histograms in Fig. 7.4(a) are taken, gives $\sigma_\epsilon \approx 0.74 \, \mu\text{eV}$. This value is close to the standard deviation of the noise $\delta_\epsilon = 0.87 \, \mu\text{eV}$ extracted from the histograms in Fig. 7.4(a), confirming that $S_\epsilon(f) = S_0/f^{1.4}$ closely models the noise spectrum in our device.

\section*{139}
C.2 Numerical Calculation of Landau-Zener-Stückelberg Interference

A detailed derivation for the dispersive response of a microwave cavity coupled to a two-level system under strong, periodic driving is given in Ref. [163]. Here we outline the essential ingredients for calculating the interference patterns in Fig. 7.3 and Fig. 7.5.

The central idea of dispersive readout is that a resonantly driven cavity acts on the DQD which in turn acts back to the cavity. The backaction leads to a frequency shift of the cavity which provides information on the DQD state. Within linear-response theory, the backaction is captured by the susceptibility (from here on, $t$ denotes time):

$$\chi(t, t') = -i\langle[Z(t), Z(t')]\rangle\theta(t - t'), \tag{C.3}$$

where $\theta$ is the Heaviside step function and $Z = \sigma_z$ the Pauli operator coupling the charge states to the cavity. If the DQD experiences an additional external driving, the susceptibility depends explicitly on both times. In the case of periodic driving, after a transient stage, $\chi(t, t') = \chi(t + T, t' + T)$, which implies that $\chi(t, t - \tau)$ with $\tau = t - t'$ is $T$-periodic in $t$.

The susceptibility allows one to calculate the cavity transmission $a_{out}/a_{in}$ via input-output theory [138, 163]:

$$\frac{a_{out}}{a_{in}} = \frac{-i\sqrt{\kappa_1 \kappa_2}}{\omega_c - \omega + g_c^2 \chi^{(0)}(\omega) - i(\kappa_1 + \kappa_2)/2}, \tag{C.4}$$

where $\omega_c = 2\pi f_c$, $\kappa_{1,2}$ are the photon loss rates at the input and output ports of the cavity, and $g_c$ is the charge-cavity coupling rate. We note that best fit to the data gives $g_c/2\pi = 19\,\text{MHz}$, which is larger than the coupling rate $g_{VO}/2\pi = 8\,\text{MHz}$ obtained from vacuum Rabi splitting. This difference is due to the valley-orbit nature
of the charge states. In contrast to the orbital d.o.f., the valley d.o.f. is assumed to have no direct coupling to the electric field of the cavity \cite{138}, thereby making the coupling rates of the valley-orbit states smaller compared to charge states based on pure DQD orbitals.

The impact of the DQD is contained in the $k = 0$ component of the Fourier transformed susceptibility

$$\chi^{(k)}(\omega) = \int_0^\infty \left[ \frac{1}{T} \int_0^T e^{i k \Omega t + i \omega t} \chi(t, t - \tau) dt \right] d\tau,$$

where for $k = 0$ the $t$-integration corresponds to the average over one driving period. In the present case of rather small driving frequencies $\Omega = 2\pi/T$, $\chi^{(0)}(\omega)$ can be evaluated within the adiabatic limit in which the stationary solutions of the Schrödinger equation read

$$|\psi_\alpha(t)\rangle = e^{i \varphi_\alpha(t)} |u_\alpha(t)\rangle,$$

where $|u_\alpha(t)\rangle$ are the adiabatic eigenstates of the DQD Hamiltonian. The phases obey the equation of motion

$$\partial_t \varphi_\alpha(t) = -\frac{E_\alpha(t)}{\hbar} + i \langle u_\alpha(t) | \partial_t | u_\alpha(t) \rangle.$$

Owing to time-reversal symmetry of our system, the adiabatic eigenstates are real. Then the second term in Eq. (C.7) vanishes, such that

$$\varphi_\alpha(t) = -\frac{1}{\hbar} \int_0^t E_\alpha(t') dt' = -\frac{\bar{E}_\alpha}{\hbar} t + \tilde{\varphi}_\alpha(t),$$

which we have separated into the $T$-periodic $\tilde{\varphi}_\alpha(t)$ and a contribution linear in time and proportional to the mean energy $\bar{E}_\alpha = \int_0^T E_\alpha(t) dt / T$.

Adiabaticity implies that for low temperatures and weak dissipation, the DQD will eventually reside in its adiabatic ground state. Then the commutator in Eq. (C.3)
provides a rotating and a counter-rotating term, where the latter may be neglected, such that
\[ \chi(t, t') = -i \sum_{\alpha > 0} \langle \psi_0(t) | Z | \psi_\alpha(t) \rangle \langle \psi_\alpha(t') | Z | \psi_0(t') \rangle \theta(t - t'). \] (C.9)

Inserting the adiabatic solutions of the Schrödinger equation into Eq. (C.5) and evaluating the Fourier integrals yields
\[ \chi^{(0)}(\omega) = \sum_{\alpha > 0, k} \frac{|Z_{\alpha 0, k}|^2}{\omega - (\bar{E}_\alpha - \bar{E}_0)/\hbar + k \Omega_s + i \bar{\gamma}/2}. \] (C.10)

The Fourier components
\[ Z_{\alpha \beta, k} = \frac{1}{T} \int_0^T \exp \left[ ik \Omega t - i \tilde{\varphi}_\alpha(t) + i \tilde{\varphi}_\beta(t) \right] Z_{\alpha \beta}(t) dt \] (C.11)
contain the transition matrix element between adiabatic states, \( Z_{\alpha \beta}(t) = \langle u_\alpha(t) | Z | u_\beta(t) \rangle \), and the periodic parts of the phases, \( \tilde{\varphi}_\alpha(t) \). The average decoherence rate \( \bar{\gamma} = \frac{1}{T} \int_0^T \gamma(\epsilon(t)) dt \) has been introduced phenomenologically.

For our experimental parameters, only the transition between the two lowest charge states contributes significantly to the DQD susceptibility (C.10), because all other transitions are either too high in energy or involve excited states that have negligible populations. Also, since the gate voltage \( V_{P1} \) is changed in the experiment to vary \( \epsilon \), we expect the valley splittings \( E_L \) and \( E_R \) to slightly vary as a function of \( \epsilon \) due to a changing electric field on each QD [18]. In the theory fits, we have assumed \( E_L = -0.004 \epsilon + 37.5 \mu eV \), \( E_R = -0.002 \epsilon + 38.3 \mu eV \). The calculated cavity transmission amplitude \( A = |a_{out}/a_{in}| \) is smoothed with a Gaussian having a standard deviation \( \sigma_\epsilon = 0.41 \mu eV \). The calculation shown in the right panel of Fig. 7.5(b) is done with a conventional two-level charge qubit having an energy dispersion \( E_{CQ} = \sqrt{\epsilon^2 + 4t_c^2} \), tunnel coupling \( t_C = 16.2 \mu eV \) and charge-cavity coupling rate \( g_c/2\pi = 8 \text{ MHz} \).
C.3 Analytical Approximations

C.3.1 Stationary Phase Approximation of DQD Susceptibility

For $\omega = \omega_c \approx \Delta / \hbar$ ($\Delta$ is the minimum value of $E_{V0}$ near $\epsilon = 0$), an analytical approximation for the DQD susceptibility $\chi^{(0)}(\omega)$ may be derived using stationary phase approximation (SPA). Considering only the two lowest adiabatic states, the relevant component of the DQD susceptibility is:

$$\chi^{(0)}(\omega) = -\frac{i}{T} \int_0^T \left[ \int_0^\infty \exp \left( i f(t, \tau) \right) \right.\left. \times Z_{01}(t)Z_{10}(t - \tau)d\tau \right] dt$$

(C.12)

with the phase

$$f(t, \tau) = \omega \tau - \frac{1}{\hbar} \int_{t-\tau}^t E(t')dt'$$

(C.13)

and $E(t) = E_1(t) - E_0(t)$. Owing to time-reversal symmetry, the matrix elements $Z_{\alpha\beta}(t) = \langle u_\alpha(t)|Z|u_\beta(t)\rangle$ can be chosen real and positive, such that the stationary phase condition is fully determined by $f(t, \tau)$ and reads:

$$\partial_\tau f = \omega - \frac{E(t - \tau)}{\hbar} = 0.$$  

(C.14)

$$\partial_t f = -E(t) + E(t - \tau) = 0.$$  

(C.15)

For $\omega = \omega_c = \Delta / \hbar$, solutions to these equations are listed in Table. The phases $\psi_1, \psi_2, \psi_+$ and $\psi_-$ are:

$$\psi_1 = \int_{t_1}^{t_0 + T} (\omega - E(t)/\hbar)dt,$$

(C.16)

$$\psi_2 = \int_{t_0}^{t_1} (\omega - E(t)/\hbar)dt,$$

(C.17)
Table C.1: Stationary points of the phase factor \( f(t, \tau) \) for \( \omega = \omega_c = \Delta / \hbar \), where \( t_{0,1} \) with \( 0 \leq t_0 \leq t_1 < T \) are the times at which \( \epsilon = 0 \) is passed. The different values of \( t - \tau \) are labeled such that the index \( n = 0, 1, 2, \ldots \) captures all stationary points with \( \tau \geq 0 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( t - \tau )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 )</td>
<td>( t_0 - nT )</td>
<td>( n\psi_+ )</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>( t_0 - (n + 1)T )</td>
<td>( n\psi_+ + \psi_1 )</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>( t_0 - nT )</td>
<td>( n\psi_+ + \psi_2 )</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>( t_1 - nT )</td>
<td>( n\psi_+ )</td>
</tr>
</tbody>
</table>

and \( \psi_\pm = \psi_1 \pm \psi_2 \).

In principle, the SPA here is quite complicated as the second derivatives of \( f(t, \tau) \) vanish as well. However, since the contributions are the same for all stationary points, this merely affects a pre-factor not relevant to the structure of the LZSM pattern. Therefore the qualitative behavior of the susceptibility follows already from summing the phase factor for all stationary points, where we have to consider that two points lie at the border \( \tau = 0 \) of the integration interval and, thus, contribute only half. Evaluating the resulting geometric series and introducing again a phenomenological decoherence \( \bar{\gamma} \), we obtain

\[
\chi^{(0)}(\omega) \propto \frac{\cos(\psi_1/2)\cos(\psi_2/2)}{\sin(\psi_+/2) + i\bar{\gamma}} \tag{C.18}
\]

with a mod-square:

\[
\left|\chi^{(0)}(\omega)\right|^2 \propto \frac{(1 + \cos\psi_1)(1 + \cos\psi_2)}{1 - \cos(\psi_1 + \psi_2) + 2\bar{\gamma}^2}. \tag{C.19}
\]

This form of the susceptibility predicts the resonance conditions (locations of each interference fringe and the individual nodes within each fringe) as follows:

- The denominator reaches a minimum (\( \sim 0 \) when \( \bar{\gamma} \) is small) when \( \psi_+ = 2\pi k \), where \( k \) is an integer. This is equivalent to the condition \( (\bar{E}_1 - \bar{E}_0)/\hbar - \omega = k\Omega \), which corresponds to the location of each interference fringe.
• The numerator vanishes if either $\psi_1$ or $\psi_2$ is an odd multiple of $\pi$, i.e. $\psi_{1,2} = (2l + 1)\pi$. On each interference fringe, both conditions are equivalent.

• $\psi_1$ varies from 0 to $2\pi k$ along each fringe, which means that $l$ can assume the values $0, 1, \ldots, k - 1$.

• Therefore, along the $k^{th}$ fringe, the numerator vanishes $k$ times, corresponding to the observed nodes within each fringe.

Within the triangle $\epsilon V_{ac} > |\epsilon_0|$, the SPA approximation is found to give an accurate prediction of the LZSM interference, as the mod-square susceptibility $|\chi^{(0)}(\omega)|^2$ gives resonance fringes and nodes identical to those found in numerically computed $A/A_0$. Outside the triangle, the DQD stays away from the avoided crossings and the LZSM dynamics disappears.

### C.3.2 Dressed State Interpretation

An alternative interpretation of the LZSM interference is based on including the cavity into the level dynamics. Then an effective two-level system is defined by the dressed states $|n, +\rangle$ and $|n + 1, -\rangle$, where $n$ is the number of photons in the cavity and $-(+)$ corresponds to the electron being in the ground ($1^{st}$ excited) charge state of the DQD. In the dispersive regime $E_{VO}/\hbar - f_c \geq 55 \text{ MHz} \gg g_{VO}/2\pi$, the energy difference between the two states is simply $E(t) - \hbar \omega$. The excited state population $\bar{P}$ for such a two-level system, as derived by Shevchenko et al. [153], to lowest order in $P_{LZSM}$ (the Landau-Zener transition probability as the DQD is swept through the avoided crossing at $\epsilon \approx 0$) reads:

$$\bar{P} = P_{LZSM} \frac{1 + \cos(\psi_1/2) \cos(\psi_2/2)}{\sin^2(\psi_1/2)} \sin^2(\psi_1/2)$$

$$= P_{LZSM} \frac{2 + \cos \psi_1 + \cos \psi_2}{1 - \cos(\psi_1 + \psi_2)}.$$

(C.20)
The qualitative difference to the SPA result is that the factors in the numerator of Eq. (C.19) now appear as a sum. As both factors are non-negative, one can draw from this expression also the conclusions listed above, and plots of both patterns can hardly be distinguished.

The dressed state interpretation is also appealing as it gives additional insight into the ability of the device in supporting LZSM interference at a low drive frequency $\Omega/2\pi = f_g = 50$ MHz. For a conventional charge qubit having an energy $E_{CQ} = \sqrt{\epsilon^2 + 4t_C^2}$, $E_{CQ}$ rapidly increases as $\epsilon$ is changed from zero. As a consequence, $\sin(\psi/2)$ rapidly oscillates as a function of $\epsilon_0$ at large drive amplitudes $eV_{ac}$. Quantum interference is therefore easily obscured by low frequency charge noise acting on the DQD level detuning. On the other hand, the valley-orbit states have an energy $E_{VO}$ that varies much less with $\epsilon$ and the visibility of their quantum interference is much less affected by low frequency noise on $\epsilon$. 
Bibliography


